Lesson 97: Area of a Circle

OBJECTIVES:

1. To understand the formula for the area of a circle

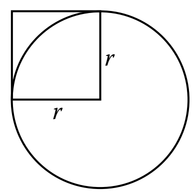
MATERIALS:

1. Worksheets 97-1 and 97-2, Area of a Circle
2. Drawing board, T-square, and triangles
3. 4-in-1 ruler
4. **Scissors**
5. *Math Card Games* book, F46

ACTIVITIES:

Worksheet 97-1. Complete the worksheet before reading further.

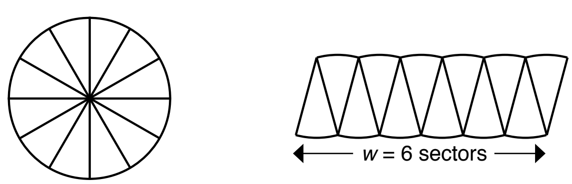
Thinking about the area of a circle. You found that the special square with side r fits in the circle a little more than three times. The actual number of times is π, which is a little more than 3. Since the area of the square is r^2, the area of a circle is $\pi \times r^2$, written πr^2 and read as "pi r squared." Are you surprised to find that pi is part of the area formula for a circle?



Worksheet 97-2. This worksheet is another way to think about finding the formula for the area of a circle. It is based on the circumference of a circle. Do the worksheet before reading further.

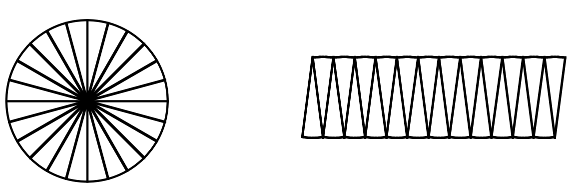
More thinking about the area of a circle. The second worksheet led you through a process to understand that the area of a circle is $\pi r \times r$, or πr^2.

The worksheet showed another way to think about the width of the parallelogram. See the figures below. The width is half the circumference of the circle because it is formed with half the sectors.



You know the circumference of a circle is $2\pi r$. So half the circumference is $\frac{2\pi r}{2}$, or πr. The height of each sector is r. Since the area of a parallelogram is wh, the area of a circle is $\pi r \times r$, or πr^2.

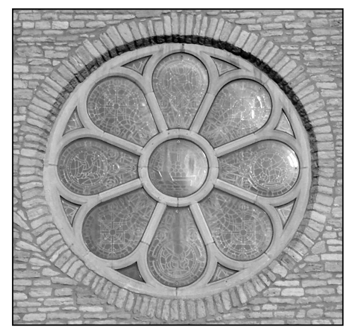
In the figures below, the same circle is divided into 24 sectors. Notice how much straighter the arcs appear.



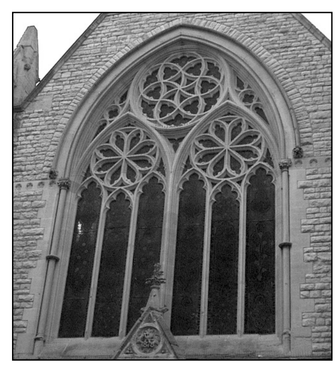
Today's game. Play the Percentages Memory game, found in the *Math Card Games* book, F46.

EXTRAS:

This square is special because $s = r$; the sides of the square equal the radius of the circle.



A circular window on a church.



This church has beautiful windows.

Jonathan's funny for the day: Why is pizza a circle, delivered in a square box, and cut into triangles?

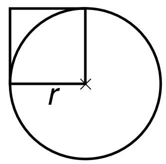
Imagine dividing the circle into a million parts, instead of 12 or 24, and arranging the sectors the same way. Then the sides of the parallelogram would look straight.

Name: ______________________________

Date: ______________________________

Worksheet 97-1,
Area of a Circle

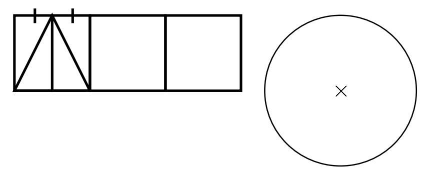
1. The radius of the circle is *r*. What is the area of the square? _______



How many total same size squares will cover the circle? Circle your best guess.

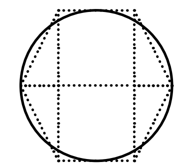
less than 3 more than 3

2. Using your drawing tools, construct three squares at the bottom of the page with sides equal to the radius of the large circle. Divide one of the squares into congruent triangles, as shown. Cut out the squares and triangles.



3. Now arrange the six pieces to cover the circle as shown. Does it look like the three squares would exactly cover the circle, or do you need a little more?

____

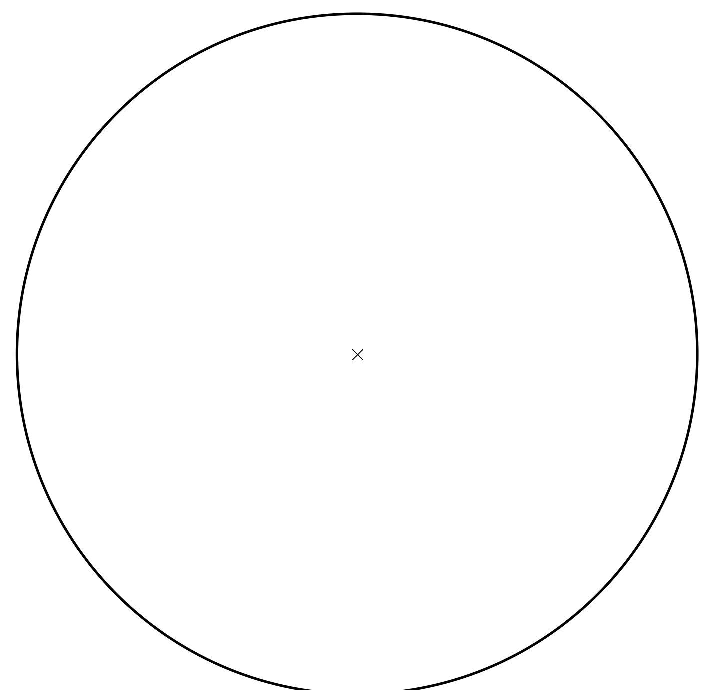


4. What symbol associated with circles is a little more than 3? _______

5. Using a symbol, how many squares do you need to cover the circle? _________

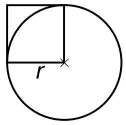
6. Using your answers to Questions 1 and 5, what is the area of a circle?

____



CONTINUE READING THE LESSON.

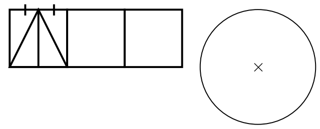
1. The radius of the circle is r. What is the area of the square? r^2



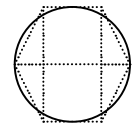
How many total same size squares will cover the circle? Circle your best guess.

less than 3   more than 3

2. Using your drawing tools, construct three squares at the bottom of the page with sides equal to the radius of the large circle. Divide one of the squares into congruent triangles, as shown. Cut out the squares and triangles.



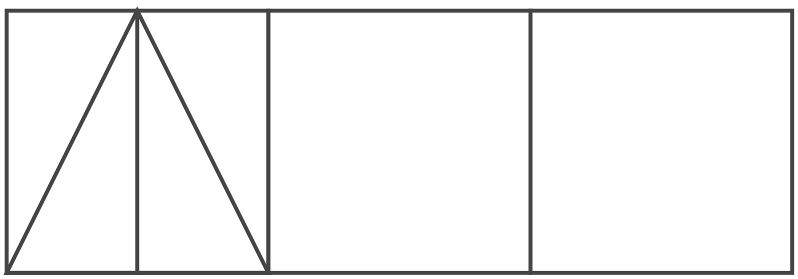
3. Now arrange the six pieces to cover the circle as shown. Does it look like the three squares would exactly cover the circle, or do you need a little more? a little more

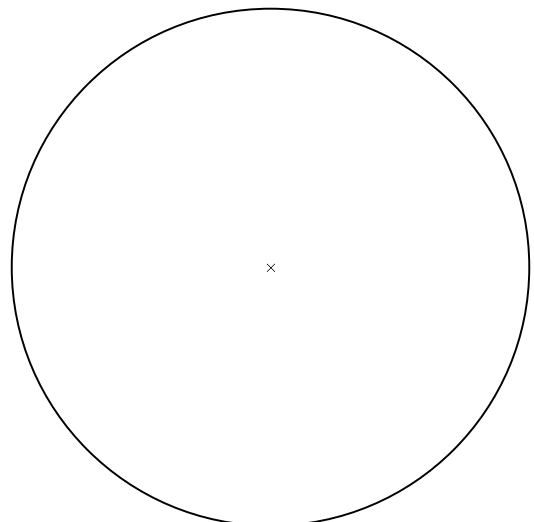


4. What symbol associated with circles is a little more than 3? π

5. Using a symbol, how many squares do you need to cover the circle? π

6. Using your answers to Questions 1 and 5, what is the area of a circle? πr^2





CONTINUE READING THE LESSON.

NOTES: For Problem 1, if the student answered as $r \times r$, which is correct, guide them to restating $r \times r$ as r^2.

Discovery is important for understanding. Yes, the student could simply memorize the formula for the area of a circle, however, going through this process helps the student see WHY the area of a circle is πr^2. This understanding will help the information be remembered for future use.

Name: ______________________________________

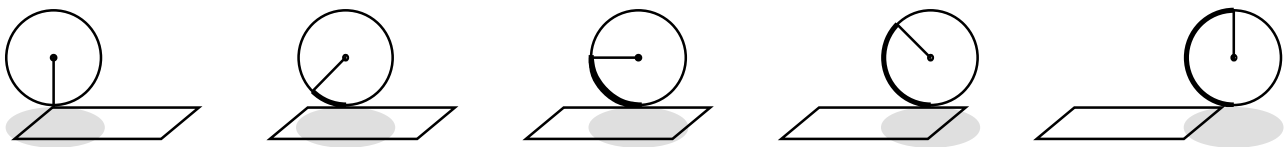
Date: ____________________________

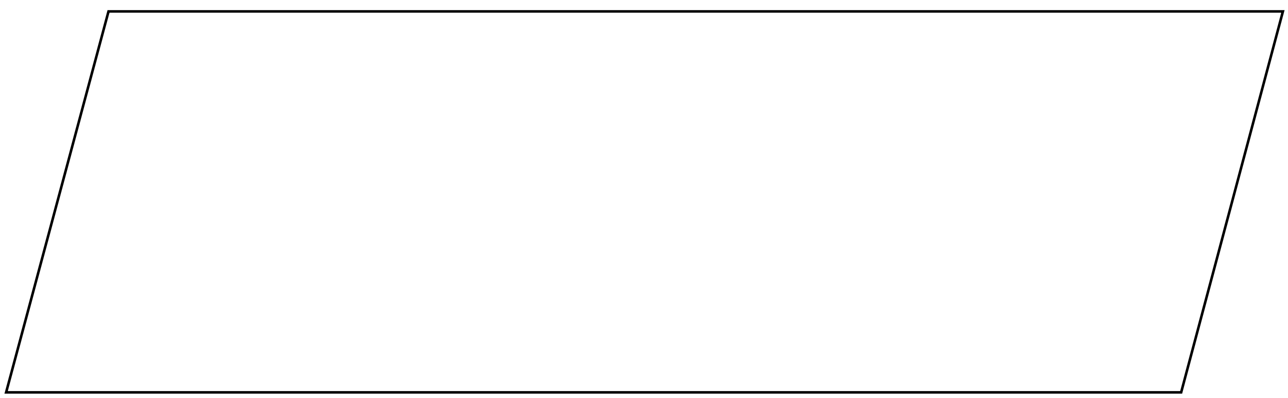
Worksheet 97-2,
Area of a Circle

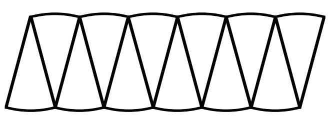
7. Use your drawing tools to divide the large circle below into 12 equal sectors. Then cut out the circle.

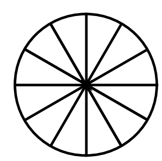
8. On the back side of the circle, draw a radius. (It needn't be exact.) Roll the circle along the top edge of the parallelogram below as shown. The radius of the circle is *r*. What is the width of the parallelogram?

____

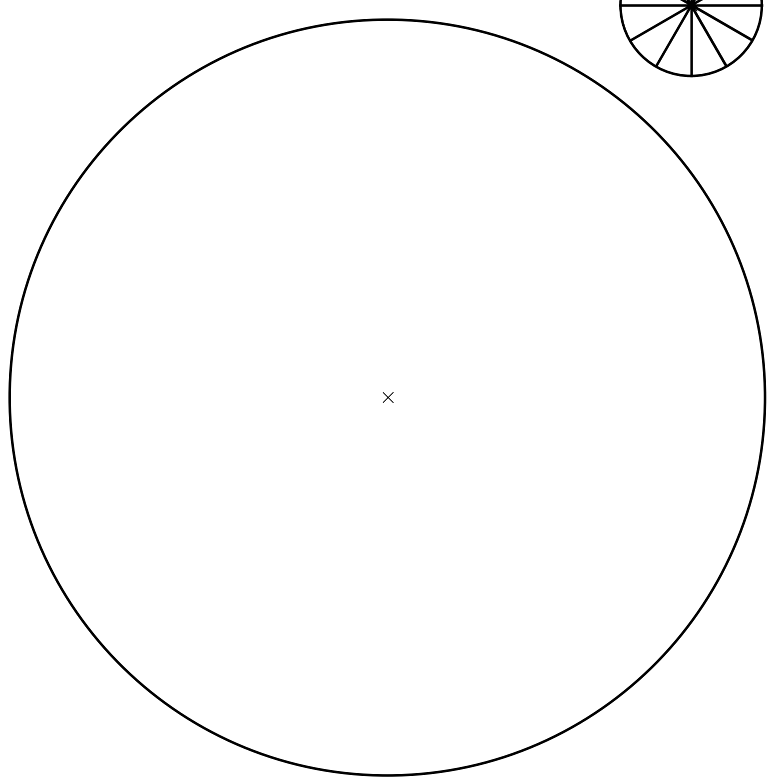








9. Cut apart the 12 sectors of your circle. Arrange them on the parallelogram as shown above. Tape if desired.



10. The height of the parallelogram is *r*.

Based on Problem 8, what is the area of the parallelogram? _________

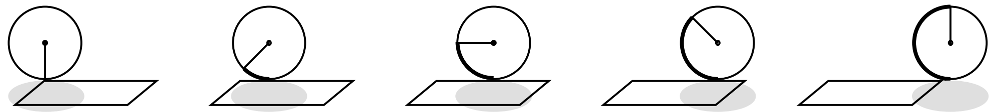
What is the area of a circle? ________

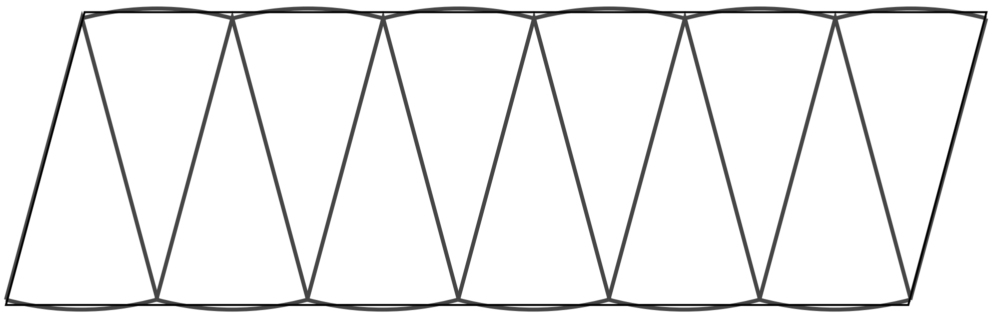
CONTINUE READING THE LESSON.

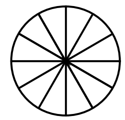
7. Use your drawing tools to divide the large circle below into 12 equal sectors. Then cut out the circle.

8. On the back side of the circle, draw a radius. (It needn't be exact.) Roll the circle along the top edge of the parallelogram below as shown. The radius of the circle is *r*. What is the width of the parallelogram?

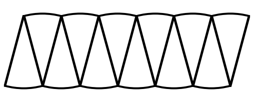
πr

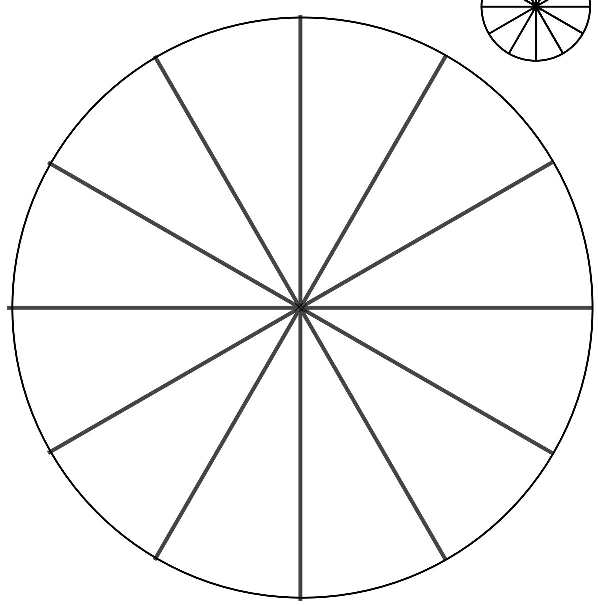






9. Cut apart the 12 sectors of your circle. Arrange them on the parallelogram as shown below. Tape if desired.





10. The height of the parallelogram is *r*.

Based on Problem 8, what is the area of the parallelogram? πr^2

What is the area of a circle? πr^2

CONTINUE READING THE LESSON.

NOTES: When dividing the circle into 12 sectors, remind the student to use their T-square and 30-60 triangle to draw the lines. If they are uncertain, use the small figure as a guide, playing around with the triangle until they find the correct angle to use.

Problem 8 has the student roll the circle. It may also be laid flat on the table and rotated along the edge.

For the width of the parallelogram in relation to the circle, the circumference of the circle is $2\pi r$. Since half the circle rolled across the width, the width is $\frac{1}{2} \times 2\pi r$, or πr.

If the student says the width is half of the circumference of the circle, ask them what is the formula for the circumference, then calculate half of $2\pi r$, which would be πr.

Problem 10 might be answered as $\pi r \times r$, which is correct. Guide the student to restating $r \times r$ as r^2, which then gives the area of a circle as πr^2.

DICTIONARY TERMS: none