Chapter 1 – Operations With Numbers

Part I – Negative Numbers

You may already know what negative numbers are, but even if you don't, then you have probably seen them several times over the past few days. If you look at a thermometer, for instance, there are numbers with a minus sign in front of them. Sometimes, football game statistics have numbers with a minus sign in front of them. These numbers are called <u>negative numbers</u>, and they are numbers that are less than zero.

Often, as in the case of football game statistics, negative numbers refer to a loss of something. For example, if you see a statistics that says "Passing yards: -21," then you would know that, when they tried to pass the ball, they were pushed back 21 yards instead of moving forward. As another example, electric bills sometimes have negative numbers on the line that tells you how much you owe. This means that, instead of you owing them money, they owe you that much money. Finally, the numbers that you are used to working with are called <u>positive numbers</u>. You can put a plus sign in front of these numbers if you want to, but you do not have to.

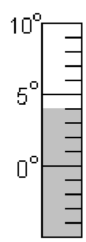
Problems:

1. Write the number represented: a loss of $35 ____________

2. Write the number represented: a gain of $21.34 ____________

3. What is one way of interpreting the number "+15"? ____________________________

4. What is one way of interpreting the number "-12"? ____________________________

5. Write the number represented: 

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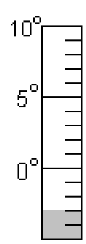
6. Write the number represented: 

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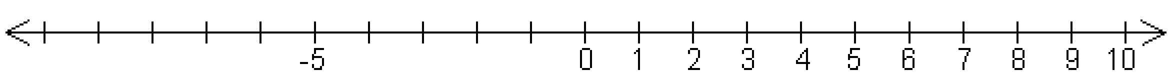
7. Fill in the missing numbers.



8. Suppose that a football player gains 20 yards, and then loses 3 yards. Write a number sentence and tell what the end result is. ______________________________

9. Suppose that a football player loses 6 yards on each of 3 plays. Write a number sentence and tell what the end result is. _____________________________________

10. Suppose that a person puts $200 in the bank and then writes a check for $50. Write a number sentence and tell what the end result is. _____________________________

11. Suppose that a person purchases a shirt for $25, then returns a shirt for $18, and then buys a sandwich for $2. Write a number sentence to describe his cash flow, and tell what the end result is. ____

Part II – Fraction Review

The following chart should help you in working with fractions.

Reduce	Change an improper fraction to a mixed number	Change a mixed number to an improper fraction	Add or subtract with like denominators (bottom numbers)	Add or subtract with unlike denominators	Multiply	Divide
Divide both the bottom and the top numbers by the biggest thing you can. Then, repeat if you can. Example: Reduce $\frac{16}{48}$. $\frac{16 \div 16}{48 \div 16} = \frac{1}{3}$ or, equivalently, $\frac{16 \div 2}{48 \div 2} = \frac{8 \div 8}{24 \div 8}$ Either way, $\frac{16}{48} = \frac{1}{3}$	Divide the top number by the bottom number. The answer is the whole number, and the remainder is the top number. The bottom number stays the same. Example: Change $\frac{65}{9}$ to a mixed number. 65 ÷ 9 = 7 remainder 2 $\frac{65}{9} = 7\frac{2}{9}$	Multiply the whole number by the bottom number, and add this answer to the top number. The bottom number stays the same. Example: Change $3\frac{5}{12}$ to an improper fraction. 3 x 12 = 36 36 + 5 = 41 $3\frac{5}{12} = \frac{41}{12}$	Change all mixed numbers to improper fractions. Then add or subtract the tops and keep the bottoms the same. Example: Simplify $\frac{3}{5} + \frac{1}{5}$. $\frac{3}{5} + \frac{1}{5} = \frac{4}{5}$	Change all mixed numbers to improper fractions. Then, you need to get a common denominator. Do this by finding the least common multiple of the bottom numbers. Next, change each of the fractions into equivalent fractions with this new common denominator by multiplying the top and bottom by the same number. Then, add the tops and keep the bottom the same. Example: Simplify $\frac{3}{5} + \frac{2}{7}$. LCM = 35 $\frac{3}{5} = \frac{?}{35}$ and $\frac{2}{7} = \frac{?}{35}$ $\frac{3 \times 7}{5 \times 7} = \frac{21}{35}$ $\frac{2 \times 5}{7 \times 5} = \frac{10}{35}$ $\frac{3}{5} + \frac{2}{7} = \frac{21}{35} + \frac{10}{35} = \frac{31}{35}$	Change all mixed numbers to improper fractions. Then multiply straight across. Example: Simplify $\frac{5}{8} \times \frac{12}{15}$. $\frac{5}{8} \times \frac{12}{15} = \frac{60}{120}$ $\frac{60 \div 60}{120 \div 60} = \frac{1}{2}$	Change all mixed numbers to improper fractions. Then copy down the first fraction, change the division sign to multiplication, and flip the second fraction. Example: Simplify $\frac{4}{9} \div \frac{5}{6}$. $\frac{4}{9} \div \frac{5}{6} = \frac{4}{9} \times \frac{6}{5}$ $\frac{4}{9} \times \frac{6}{5} = \frac{24}{45}$ $\frac{24 \div 3}{45 \div 3} = \frac{8}{15}$

Whenever you give an answer in fractional form, **always** make sure that it is reduced.

Problems:

12. Reduce: $\frac{24}{36}$ = ________________

13. Change $2\frac{5}{7}$ to an improper fraction. ________________

14. Change to an improper fraction: $5\frac{3}{4}$ = ________________

15. Add: $\frac{7}{8}+\frac{3}{8}$ = ________________

16. Change $\frac{15}{7}$ to a mixed number. ________________

17. Multiply: $\frac{3}{4} \text{x} \frac{2}{5}$ = ________________

18. Add: $\frac{3}{10}+\frac{5}{10}$ = ________________

19. Change $\frac{34}{4}$ to a mixed number, and reduce the result. ________________

20. Subtract: $\frac{4}{5}-\frac{3}{10}$ = ________________

21. Reduce: $\frac{12}{15}$ = ________________

22. Multiply: $\frac{5}{6} \text{x} \frac{9}{10}$ = ________________

23. Divide: $21 \div \frac{5}{6}$ = ________________ Hint: You can always put a 1 on the bottom of a number to make it into a fraction.

24. Divide: $\frac{5}{9} \div \frac{4}{7}$ = ________________

25. Multiply: $4\frac{2}{5} \text{ x } 3\frac{1}{4}$ = ________________

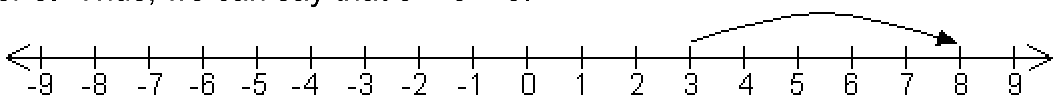
26. Multiply: $2\frac{3}{5} \text{ x } 6$ = ________________

27. Subtract: $3\frac{2}{5}-1\frac{5}{8}$ = ________________

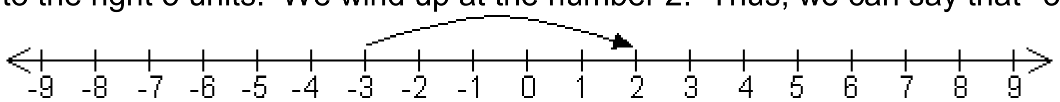
28. Divide: $\frac{2}{15} \div 1\frac{5}{6}$ = ________________

Part III – Operations With Positive and Negative Numbers

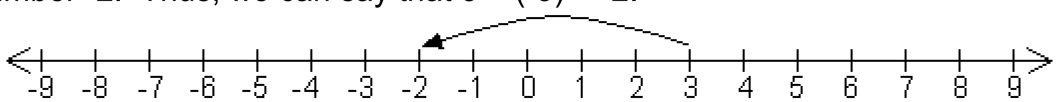
We will now discuss adding positive and negative numbers. Let us first consider the number sentence 3 + 5. To find the answer to this problem, we will look at the number line. We start at the number 3 and then move to the right 5 units. We wind up at the number 8. Thus, we can say that 3 + 5 = 8.



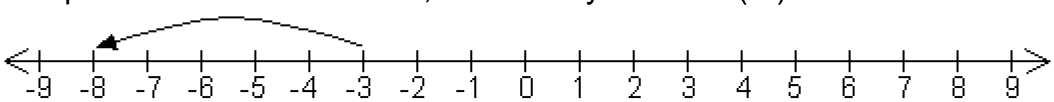
Now, let's consider the number sentence -3 + 5. We start at the number -3 and then move to the right 5 units. We wind up at the number 2. Thus, we can say that -3 + 5 = 2.



Now, let's consider the number sentence 3 + (-5). We start at the number 3 and then move to the *left* 5 units (we move to the left because of the negative sign). We wind up at the number -2. Thus, we can say that 3 + (-5) = -2.



Finally, let's consider the number sentence -3 + (-5). We start at the number -3 and then move to the left 5 units (again, we move to the left because of the negative sign). We wind up at the number -8. Thus, we can say that -3 + (-5) = -8.

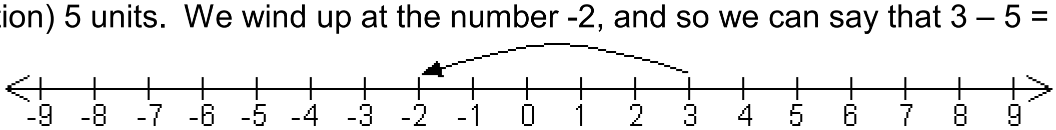


If you think about these examples and the mathematics behind them, you can come up with the following rules for adding positive and negative numbers: (1) If the signs are both positive or both negative, add the two numbers and keep the same sign, and (2) If you want to add one positive and one negative, subtract the numbers and keep the sign of the larger number. Study the following examples.

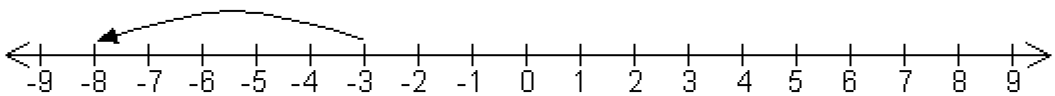
-5 + (-1) = -6	58 + 84 = 142
5 + 1 = 6	-58 + (-84) = -142
-5 + 1 = -4	58 + (-84) = -26
5 + (-1) = 4	-58 + 84 = 26
5 + (-5) = 0	-58 + 58 = 0

Now, let's talk about how to subtract positive and negative numbers. In general, minus and negative signs mean "reverse your direction."

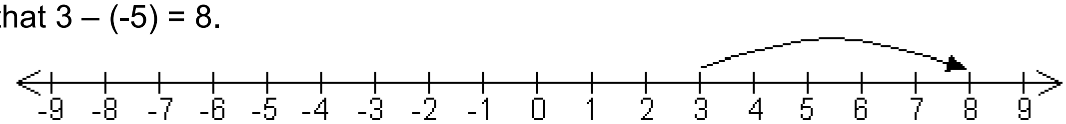
Let's consider the number sentence 3 – 5. We start at the number 3, and then we find out we must move to the left (because the minus sign tells us to reverse the direction) 5 units. We wind up at the number -2, and so we can say that 3 – 5 = -2.



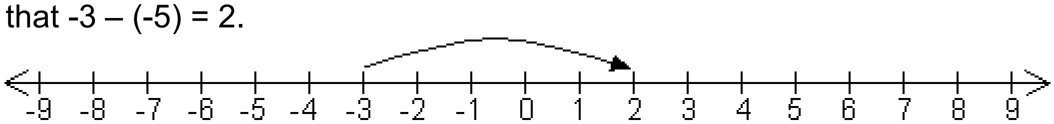
Now, let’s consider the number sentence -3 – 5. This time, we start at the number -3, and then we find out we must move to the left (because the minus sign tells us to reverse the direction) 5 units. We end up at the number -8, and so we know that -3 – 5 is equal to -8.



Next, we will look at a trickier example: 3 – (-5). We start out at the number 3 again. The subtraction sign tells us to move to the left, but then, before we find out how many units to move, we see that we must reverse our direction again and move to the right. We see that we must move to the right 5 units. Since we wind up at the number 8, we say that 3 – (-5) = 8.



Finally, we will consider the number sentence -3 – (-5). This time, we start out at -3. The subtraction sign tells us to move to the left, but then, before we find out how many units to move, we see that we must reverse our direction again and move to the right. We see that we must move to the right 5 units. Since we wind up at the number 2, we know that -3 – (-5) = 2.



These examples and the mathematics behind them give rise to the following rule when subtracting positive and negative numbers: Change the subtraction sign to addition and change the sign of the next number. You then have an addition problem, and so you then need to follow the rules for addition that were discussed earlier. Study the following examples.

7 – 13 = 7 + (-13) = -6	32 – 14 = 32 + (-14) = 18
-7 – 13 = -7 + (-13) = -20	-32 – 14 = -32 + (-14) = -46
7 – (-13) = 7 + (+13) = 20	32 – (-14) = 32 + (+14) = 46
-7 – (-13) = -7 + (+13) = 6	-32 – (-14) = -32 + (+14) = -18
7 – 7 = 7 + (-7) = 0	-32 – (-32) = -32 + (+32) = 0

Now, let’s look at how to multiply positive and negative numbers. Let us first look at how to multiply 4 x 5. You can think of this as gaining $4 five times (or, equivalently, gaining $5 four times). This means you gained $20, and so we say 4 x 5 = 20.

Next, we will look at -4 x 5. You can think of this as losing $4 five times, which means you lost a total of $20. Thus, we say that -4 x 5 = -20.

Similarly, you can think of 4 x (-5) as meaning that you lost $5 four times. This would mean that you lost $20, and so we say 4 x (-5) = -20.

Finally, we will look at how to multiply -4 x (-5). There is not really a good way to understand this problem, and so we will just give you a rule: When you multiply 2 negatives, the answer is always positive. Thus, -4 x (-5) = 20.

These examples give rise to the following two rules: (1) When you multiply 2 numbers with the same sign, the answer will be positive, and (2) When you multiply 2 numbers with different signs, the answer will be negative. Study the following examples.

8 x 3 = 24

-8 x 3 = -24

5 x (-2) = -10

-5 x (-2) = 10

The rules for division are exactly the same as the rules for multiplication. Study the examples below.

18 ÷ 3 = 6

18 ÷ (-3) = -6

-24 ÷ 4 = -6

-24 ÷ (-4) = 6

The following chart summarizes the rules for adding and subtracting positive and negative numbers. The words across the top indicate the sign in the middle.

<table>
<tr><th></th><th>Addition</th><th>Subtraction</th><th>Multiplication or Division</th></tr>
<tr><td>2 positives or 2 negatives (like signs)</td><td>Add the numbers and keep the same sign.</td><td rowspan="2">Change the subtraction sign to addition and change the sign of the next number.</td><td>Multiply or divide the numbers. The sign is positive.</td></tr>
<tr><td>1 positive and 1 negative (unlike signs)</td><td>Subtract the numbers and keep the sign of the larger number.</td><td>Multiply or divide the numbers. The sign is negative.</td></tr>
</table>

Problems – Simplify each of the following.

29. -16 + 4 = ________________

30. 4 – (-9) = ________________

31. 3 x 4 = ________________

32. 5 + (-21) = ________________

33. 64 – 59 = ________________

34. 14 ÷ (-2) = ________________

35. -31 – 6 = ________________

36. -28 ÷ 7 = ________________

37. -8 – (-17) = ________________

38. 8 x (-4) = ________________

39. 6 + 19 = ________________

40. -10 + 6 = ________________

41. -9 x (-4) = ________________

42. 48 – 9 = ________________

43. 14 ÷ 7 = ________________

44. -5 x 4 = ________________

45. 7 – 29 = ________________

46. -18 + (-6) = ________________

47. -27 ÷ (-9) = ________________

48. -25 ÷ 5 = ________________

49. 19 + (-2) = ________________

50. 4 x 6 = ________________

51. -5 x 7 = ________________

52. 81 – (-5) = ________________

53. 12 ÷ (-3) = ________________

54. -7 x (-7) = ________________

55. -17 + 8 = ________________

56. 15 + (-6) = ________________

57. -32 ÷ 8 = ________________

58. -28 – (-15) = ________________

59. $\frac{4}{5} + \left(-\frac{3}{5}\right) =$ ________________

60. $\frac{8}{9} \text{ x } \left(-\frac{3}{4}\right)$ = ________________

61. $\frac{7}{8} \div \left(-3\frac{2}{5}\right)$ = ________________

Hint: $-3\frac{2}{5} = -\left(3\frac{2}{5}\right) = -\frac{17}{5}$

62. $1\frac{2}{5} - 4\frac{1}{3}$ = ________________

63. $1\frac{7}{10} - \left(-\frac{4}{5}\right)$ = ________________

64. $-2\frac{1}{2} \div \left(-\frac{5}{6}\right)$ = ________________

65. $-\frac{6}{7} - \frac{2}{5}$ = ________________

66. $\frac{5}{9} - \left(-2\frac{2}{3}\right)$ = ________________

67. $-\frac{5}{6} + 1\frac{1}{4}$ = ________________

68. $-5\frac{2}{5} \text{ x } \frac{3}{8}$ = ________________

69. $-3\frac{8}{9} - \left(-\frac{7}{9}\right)$ = ________________

70. $-\frac{4}{9} + \frac{3}{5}$ = ________________

71. $-\frac{3}{5} \text{ x } \left(-\frac{7}{9}\right)$ = ________________

72. $-\frac{5}{8} + \left(-\frac{4}{15}\right)$ = ________________

73. $-\frac{4}{5} \text{ x } \left(-2\frac{7}{8}\right)$ = ________________

74. $-1\frac{6}{7} \div \frac{3}{5}$ = ________________

75. $-3\frac{1}{8} \div 4\frac{2}{3}$ = ________________

Part IV – Review of Decimals

Before we talk about different operations with decimals, we will review two concepts.

- If a decimal is not stated in a number, you should always assume that it goes at the end. Also, you can always put zeros after the decimal as long as you put them after the last number. For example, all of the following numbers are equal to each other.

  14   14.   14.0   14.000000

- In long division, the number inside the division symbol is called the <u>dividend</u>, and the number outside the division symbol is called the <u>divisor</u>.

We can now discuss the primary operations with decimals.

Add or subtract with decimals	Multiply with decimals	Divide with decimals	Convert a fraction to a decimal	Convert a decimal to a fraction
Line up the decimal points, and add zeros on the ends if you need to. Example 1: Subtract: 126 – 4.23 $\begin{array}{r} 126.00 \\ -\quad 4.23 \\ \hline 121.77 \end{array}$	Ignore the decimal points and multiply the numbers like you normally would. Then count up the total number of digits behind the decimals in the problem. This is how many digits should be behind the decimal point in your answer. Example 2: Multiply: 6.24 x 0.3 $\begin{array}{r} 6.24 \\ \times\ \ 0.3 \\ \hline 1.872 \end{array}$ (Note that there are a total of 3 digits behind the decimal in the problem and in the answer.)	Bring the decimal in the dividend straight up. However, if the divisor has a decimal in it, you will first need to move it over to the end. When you do this, move the decimal in the dividend over the same number of spaces. Example 3: Divide: 28 ÷ 3.5 We must begin by moving the decimals as shown below. $3.5\overline{)28.0}$ Now, we can divide as we normally would. $\begin{array}{r} 8. \\ 35\overline{)280.} \\ -280 \\ \hline 0 \end{array}$	Divide the top number by the bottom number. Add a decimal and zeros onto the end of the dividend if necessary. Example 4: Convert $\frac{12}{5}$ to a decimal. $\begin{array}{r} 2.4 \\ 5\overline{)12.0} \\ -10 \\ \hline 20 \\ -20 \\ \hline 0 \end{array}$ Therefore, $\frac{12}{5} = 2.4$.	Read the number and write what you say as a fraction. Example 5: Convert 1.63 to a fraction. To read this number correctly, you say, “one and sixty-three hundredths.” Therefore, $1.63 = 1\frac{63}{100}$.

Example 6: Subtract: $9 - 12.4$.

We must begin by noting that we cannot subtract these two numbers without talking about negative numbers because the 12.4 is bigger than the 9. Therefore, we must recall our rule for subtraction. We must change the subtraction sign to addition and change the sign of the next number. Therefore, our problem becomes $9 + (-12.4)$. Now, since the rule for adding two numbers with different signs says we must subtract the two numbers and keep the sign of the larger, our final answer is -3.4. (Note that, when you use your pencil and paper to subtract these two numbers, you must do $12.4 - 9$ because you cannot do $9 - 12.4$.)

Example 7: Divide: $-0.08 \div 2$.

First of all, note that the sign in our final answer will be negative because we are dividing with two numbers that have different signs. Now, we can divide the two numbers like we normally would.

$$\begin{array}{r} 0.04 \\ 2\overline{)0.08} \\ \underline{-8} \\ 0 \end{array}$$

Thus, our final answer is -0.04.

Example 8: Subtract: $-3\frac{4}{5} - 7.12$.

To work this problem, we will start by changing $-3\frac{4}{5}$ to a decimal. You can do this one of two ways: (1) by noting that $\frac{4}{5} = 0.8$ (as discussed in the chart on the previous page), and thus $-3\frac{4}{5} = -3.8$, or (2) by noting that $-3\frac{4}{5} = -\frac{19}{5}$, which becomes -3.8 when you change it to a decimal. Either way, our problem becomes $-3.8 - 7.12$. Now, we must remember our rule for subtraction again. We must change the subtraction sign to addition and change the sign of the next number. Therefore, our problem now becomes $-3.8 + (-7.12)$. Next, since the sign in the middle is now addition and the two numbers have the same sign, we must add the two numbers together and keep the negative sign. Hence, we can now say that $-3\frac{4}{5} - 7.12 = -10.92$.

Example 9: Add: $-9.2 + \frac{4}{7}$.

To work this problem, we could change the $\frac{4}{7}$ to a decimal like we did in the last problem. However, when we do this, we find that, as a decimal, $\frac{4}{7}$ is a nasty, ugly number. Therefore, we will change the -9.2 to a fraction instead. To read this number, we say, “negative nine and two-tenths.” Thus, -9.2 = $-9\frac{2}{10}$ (or, equivalently, $-9\frac{1}{5}$). So, our problem now becomes $-9\frac{1}{5}+\frac{4}{7}$. When you use your rules for adding and subtracting fractions that we discussed earlier in this chapter, you should find that the final answer is $-8\frac{22}{35}$.

Problems – Simplify each of the following.

76. 4.5 + 12 = _______________

77. 620 – 5.71 = _______________

78. 8 – 1.759 = _______________

79. 3.14 x 7 = _______________

80. 5.41 x 1.2 = _______________

81. 2.7 x 5.121 = _______________

82. 5.4 ÷ 3 = _______________

83. 3.4 ÷ 0.4 = _______________

84. 6.8 + (-1.43) = _______________

85. -3.2 + (-54) = _______________

86. -2 – (-3.51) = _______________

87. 5.6 – 82 = _______________

88. -32 – 5.3 = _______________

89. -8.12 – 63.4 = _______________

90. $-3\frac{1}{2} - 4.45 =$ _______________

91. $-6.2 \times (-0.5) =$ _______________

92. $\frac{1}{5} \times (-1.4) =$ _______________

93. $-0.4 \times 2\frac{1}{3} =$ _______________

94. $8 \div (-0.5) =$ _______________

95. $-2\frac{1}{6} \div (-0.7) =$ _______________

96. $3.2 - 14 =$ _______________

97. $5.6 \div 1\frac{2}{9} =$ _______________

98. $-3.49 - (-7.5) =$ _______________

99. $\frac{3}{8} - (-0.2) =$ _______________

100. $0.2 \div \left(-2\frac{1}{3}\right) =$ _______________

101. $4 - 7.26 =$ _______________

102. $-2.5 - 4\frac{1}{6} =$ _______________

103. $15.2 - 42 =$ _______________

Part V – Exponents

Sometimes, when you are working math problems, you will see a little raised number above the rest of the line. These numbers are called exponents or powers. They tell you to multiply the number by itself that many times. For example, let's look at the problem below.

Example 1: Simplify 5^3.

In this case, the 5 is called the base, and the 3 is called the exponent. This problem tells you to multiply 5 by itself 3 times. In other words, it wants you to figure out what $5 \times 5 \times 5$ equals. Therefore, we say $5^3 = 125$.

Now, let's look at how to read these numbers. We read them by first saying the base, then "to the," and then the power. For example, we read "4^{15}" by saying "four to the fifteenth power." Sometimes, however, we replace the words "to the second power" with "squared" and "to the third power" with "cubed." For example, we could read "5^2" as "five to the second power" or "five squared." We could read "5^3" as "five to the third power" or "five cubed."

Before you get started with the problems, there is only one more piece of information you need to know: Anything (except zero) to the zero power will always equal 1. For example, $4^0 = 1$ and $105^0 = 1$.

Problems:

104. $4^2 =$ ________________

105. $3^4 =$ ________________

106. $9^2 =$ ________________

107. $5^1 =$ ________________

108. $64^0 =$ ________________

109. $72^0 =$ ________________

110. $38^1 =$ ________________

111. $1^{20} =$ ________________

112. $\left(\frac{4}{5}\right)^2 =$ ________________

113. $\left(\frac{1}{2}\right)^3 =$ ________________

114. Fill in the blank with <, >, or =: 1^8 ___ 1^{15}

115. Fill in the blank with <, >, or =: 5^3 ___ 3^5

Part VI – The Order of Operations

What would you get if you were asked to find out what 3 + 1 x 4 equals? If you are like most people, you probably got 16. You probably added 3 + 1, and then multiplied by 4. Now, try this problem on a calculator. If you have a scientific calculator, then the calculator told you 7. It multiplied 1 x 4 and then added 3 to that. How do we resolve these conflicting answers?

The answer is that we have a set of rules to follow called the order of operations, which is summarized in the following list. (The phrase "Please excuse my dear Aunt Sally" is only a tool to help you remember the order of operations; it has no real relevance to math or to this lesson.)

P	Please	Parentheses
E	Excuse	Exponents
M/D	My Dear	Multiplication and Division in order from left to right
A/S	Aunt Sally	Addition and Subtraction in order from left to right

Scientific calculators follow the order of operations, and that is why they often give an answer that is different from the one you would expect to get when working this type of problem. The order of operations says that we must get rid of all parentheses by simplifying whatever is in parentheses before we try to do anything else. Then, we must get rid of all the exponents. Then, we can move on to multiplication and division. We do these operations in order from left to right. Finally, we can finish the problem by adding and subtracting in order from left to right.

Example 1: Simplify $1 – (4 + 1) \times 6 \div 2 + 3$.

In order to work this problem, you would need to follow the steps below.

Steps	Explanations of Steps
$1 – (4 + 1) \times 6 \div 2 + 3 = 1 – (5) \times 6 \div 2 + 3$	Always simplify everything in parentheses first.
$= 1 – 30 \div 2 + 3$	We have no exponents, but we do have multiplication and division. The multiplication comes first, so we do it next.
$= 1 – 15 + 3$	We must get rid of all the multiplication and division before we can move on to addition and subtraction.
$= -14 + 3$	The subtraction comes before the addition in the problem, and so we must do the subtraction next.
$= -11$	The only thing left to do is add.

Whenever you have more than one operation to do inside a set of parentheses, you must follow this same order of operations to simplify whatever is inside the parentheses.

Example 4: Simplify $1 – (3 – 5 \times 2)$.

The steps you use to work this problem are described below.

Steps	Explanations of Steps
$1 – (3 – 5 \times 2) = 1 – (3 – 10)$	We must simplify whatever is inside the parentheses first, and, inside these parentheses, we must do the multiplication before the subtraction.
$= 1 – (-7)$	Again, we must simplify whatever is in parentheses before moving on to anything else.
$= 8$	The only thing left to do is subtract.

Sometimes, you will see problems with parentheses inside of parentheses. In these problems, simplify the innermost parentheses first, and then move on to the next innermost set of parentheses. Also, to make the problems easier to understand, we will change some of the sets of parentheses to symbols that look like "{" and "}" or "[" and "]".

Example 3: Simplify $-5 + \{-7 – 3 \times [1 – 45 \div (2 – 5)^2]\}^3$.

The steps you use to work this problem are described below.

Steps	Explanations of Steps
$-5 + \{-7 – 3 \times [1 – 45 \div (2 – 5)^2]\}^3$ $= -5 + \{-7 – 3 \times [1 – 45 \div (-3)^2]\}^3$	We must work from the innermost set of parentheses outward: $2 – 5 = -3$.
$= -5 + \{-7 – 3 \times [1 – 45 \div 9]\}^3$	We must continue working from the innermost set of parentheses outward. Within the next innermost set of parentheses, the order of operations tells us to square the (-3).
$= -5 + \{-7 – 3 \times [1 – 5]\}^3$	Within the next innermost set of parentheses, we must perform the multiplication and division before we can do addition or subtraction.
$= -5 + \{-7 – 3 \times [-4]\}^3$	Since the innermost set of parentheses now only contains one operation, we know we must now subtract: $1 – 5 = -4$.
$= -5 + \{-7 – (-12)\}^3$	Now, within the next set of innermost parentheses, we must multiply before adding or subtracting.
$= -5 + \{5\}^3$	We can now subtract: $-7 – (-12) = 5$.
$= -5 + 125$	We must take care of the exponents before adding or subtracting.
$= 120$	The only thing left to do is add.

Problems – Simplify each of the following.

116. $6 + 8 – 3 =$ ____________________ 117. $2 – 4 \times 2 \div 2 =$ _________________

118. $24 \div 3 \times 2 + 2 =$ ______________

119. $20 - (3 + 2 \times 2) =$ ______________

120. $8 - (6 + 3) =$ ______________

121. $8 - 6 + 3 =$ ______________

122. $7 \times 3 - 15 \div 3 =$ ______________

123. $5 \times 2 \div (4 - 2) =$ ______________

124. $16 \div 8 \div 2 =$ ______________

125. $1 - 24 \times (6 \div 2) =$ ______________

126. $5 + 1^2 =$ ______________

127. $(3 + 1)^2 =$ ______________

128. $5 + (-2)^4 =$ ______________

129. $5 - 2^4 =$ ______________

130. $(-2)^4 =$ ______________

131. $-2^4 =$ ______________

Hint: Note that only the 2, and not the negative sign, is being raised to the fourth power.

132. $1 - 3 \times (1 + 1)^2 =$ ______________

133. $7 + 18 \div (4 - 1)^2 =$ ______________

134. $-5 \times 2 + 4 \times (-3)^3 =$ ______________

135. $(-3)^2 - 4^3 + 5 =$ ______________

136. $1 - 28 \div (-2)^2 + 3 =$ ______________

137. $-6 + 3 \times (2 - 5) =$ ______________

138. $-5^2 =$ ______________

139. $-2^2 + 32 \div 2^3 =$ ______________

140. $3^{(2 + 1)} - 4 \times 2 =$ ______________

141. $18 \div \{6 - [2 \times (-3) + 3]\} =$ ________

142. $-3 - [9 + 18 \div 3 - (1 + 2)^2] =$ ______

143. $-54 + 12 \times \{8 - [3 + 2 \times (-1)]\} =$ ____

144. $17 + 4 \times \{3 + [1 - (2 + 1)]^2 - 5\} =$ ____

145. $16 - [15 \div (2 + 1)] \times 4^2 =$ ________

146. $4 - \{3 + 5 \times [4 - 1 \times (2 + 3) + 5^2]\}$
$=$ ______________

147. $1 + \{-64 + [4 \times (0 - 1)^2]^3\} - [2 \times (3 + 4)]^0$
$=$ ______________

Part VII – More on Operations With Numbers

Sometimes, we want to talk about values that change, or vary. When we do this, we often use variables, or letters or symbols that represent a quantity that can change. For example, we can say x + 3 represents some number plus 3. As another example, suppose a long-distance telephone company charges 25¢ to connect you and 5¢ for every minute you spend talking. Then, to talk about how much a long-distance company charges for a phone call, we can say $c = 0.25 + 0.10 \times n$, where c represents the cost of the phone call and n represents the number of minutes the person spent talking. We will talk more about this subject in the next chapter.

We will no longer use the symbol "x" to mean multiply because x is the most commonly used variable, and things often get confusing when we use the symbol "x" to refer to both a variable and the operation of multiplication. Instead, we will now use a dot, parentheses, or, if a variable is multiplied by a number or another variable, nothing at all. Study the following examples.

3 • 4 = 3 times 4
3(4) = 3 times 4
3 • 4 • 2 = 3 times 4 times 2
3(4)(2) = 3 times 4 times 2
3y = 3 times y
3xy = 3 times x times y
34 = thirty-four

Notice that, in each of the above examples, if a number is multiplied by one or more variables, then the variable(s) are listed *after* the numbers. Also, the variables are listed in alphabetical order. These are conventions that will be used, for the most part, throughout the rest of the book.

Finally, in working the problems below, remember your order of operations (parentheses, then exponents, then multiplication/division, and then addition/subtraction).

Problems – Simplify each of the following.

148. 5(2) • 4 = ________________

149. 5(2 + 4) = ________________

150. (2 + 4)5 = ________________

151. $(2 + 4)5^2$ = ________________

152. 3 • 4(5 – 7) = ______________

153. $4 + 18 \div 1 \cdot (-3)^2$ = ______________

154. 3 – 6(2 + 1) = ________________

155. $6 - 5 \cdot (3 + 3)^2 - 6$ = ____________

156. 8 + 14 ÷ 2 • 2 – 1 = _____________

157. 30 ÷ 5 • (4 + 2) = _______________

Chapter Notes