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LESSON 5: Solving Equations

GOAL: To learn how to simplify and solve equations and equations with squared variables

WORDS TO KNOW

- constant
- cubed
- denominator
- distance formula
- exponent
- formula
- numerator
- square root
- squared
- squared variable
- reciprocal

Solving Equations by Subtracting and Adding

When you solve an equation, you want to get the variable by itself. This way, you can find the value of the variable. Look at the equation below.

Example 1

\[ x + 6 = 13 \]

In this equation, you want to get the \( x \) by itself. To do this, you must get rid of the 6. Notice that 6 is being added to \( x \). To get rid of the 6, do the opposite operation: Subtract 6. This is called doing the inverse operation. The word \textit{inverse} means “opposite.” If you subtract 6 from the left side, you must subtract 6 from the right side, too. Then both sides of the equation remain the same.

\[ x + 6 - 6 = 13 - 6 \]

\[ x = 7 \]

By subtracting the same number from both sides of the equation, you get \( x \) by itself: \( x = 7 \). The solution is 7.
Example 2

Try another equation: \( m - 30 = 67 \). In this equation, you want to get rid of the 30 so the \( m \) will be by itself. Notice that 30 is being subtracted from \( m \). Do the inverse operation: Add 30. Do this to both sides of the equation.

\[
m - 30 + 30 = 67 + 30
\]
\[
m = 97
\]

When you solve for a variable, you can check to see if your answer is correct. Just put your answer in place of the variable in the original equation. Then see if the equation still works. Look at the example below.

\[
x + 2 = 7
\]
\[
x + 2 - 2 = 7 - 2
\]
\[
x = 5
\]

Now check your answer.

\[
x + 2 = 7
\]
\[
5 + 2 = 7
\]
\[
7 = 7
\]

Yes, your answer is correct!
PRACTICE 17: Solving Equations by Subtracting and Adding

Solve each equation below. Write your answer on the line after each equation.

1. \( x + 3 = 24 \)  \( x = \) ______
2. \( z + 450 = 2467 \)  \( z = \) ______
3. \( h - .5 = 4 \)  \( h = \) ______
4. \( z - 36 = 683 \)  \( z = \) ______
5. \( q - 70 = 120 \)  \( q = \) ______
6. \( m - 12 = 45 \)  \( m = \) ______

Solving Equations by Dividing and Multiplying

Equations can also be solved by using division and multiplication.

Example 1

Look at the following equation.

\[ 9x = 27 \]

In the equation above, you must get rid of the 9 to get the \( x \) by itself.

Notice that 9 is being multiplied by \( x \). To get rid of 9, do the inverse operation: Divide by 9. Do this to both sides of the equation.

\[
\frac{9x}{9} = \frac{27}{9}
\]

\( x = 3 \)
You now know that you must get a variable by itself to find its value and solve an equation. You have learned that when a number is added to a variable in an equation, you must subtract that number from both sides of the equation. When a number is subtracted from a variable, you must add that number to both sides of the equation. When a variable is multiplied by a number in an equation, you must divide both sides by that number. Lastly, when a variable is divided by a number in an equation, you must multiply both sides by that number.

Look at the example below.

**Example 2**

\[
\frac{x}{3} = 9
\]

To get the \(x\) by itself, you must multiply both sides of the equation by 3.

\[
\frac{3}{1} \left( \frac{x}{3} \right) = 3(9)
\]

\[
\frac{3x}{3} = 27
\]

\[x = 27\]

**PRACTICE 18: Solving Equations by Dividing and Multiplying**

Solve each equation on the next page. Write your answer on the line after each equation.

**Example:** \[6z = 24\]

\[
\frac{6z}{6} = \frac{24}{6}
\]

\[z = 4\]
<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>$10x = 150$</td>
<td>$x = \phantom{000}$</td>
</tr>
<tr>
<td>2.</td>
<td>$84 = 3m$</td>
<td>$m = \phantom{000}$</td>
</tr>
<tr>
<td>3.</td>
<td>$\frac{n}{4} = 12$</td>
<td>$n = \phantom{000}$</td>
</tr>
<tr>
<td>4.</td>
<td>$\frac{f}{7} = 3$</td>
<td>$f = \phantom{000}$</td>
</tr>
<tr>
<td>5.</td>
<td>$\frac{s}{18} = 5$</td>
<td>$s = \phantom{000}$</td>
</tr>
<tr>
<td>6.</td>
<td>$\frac{y}{9} = 3$</td>
<td>$y = \phantom{000}$</td>
</tr>
</tbody>
</table>

**IN REAL LIFE**

Suppose it is your friend’s birthday. You are in charge of collecting money for the birthday cake. You know that the cake costs $12.00. You also know that there are four people (including yourself) who want to help pay. How much should each person pay? If you think of this problem as an algebraic equation, it might look like this: $4x = 12$. $x$ is the amount of money each person must pay. It is the unknown number. Now solve the equation. Divide both sides by 4 to get the $x$ by itself. $\frac{4x}{4} = \frac{12}{4}$. $x = 3$. Now you know that each person should pay $3.00!

**Solving Two-Step Equations**

In some equations, the variable is part of two or more different operations.

Look at the example below.

**Example 1**

$$4x - 8 = 12$$
In this equation, 4 is multiplied by \( x \). Then 8 is subtracted from the product of 4 and \( x \). You still need to get the variable by itself. What do you do first?

**Rule for Solving Two-Step Equations**

If there are parentheses—( )—in an equation, always do the operation in parentheses first. When there is more than one operation and there are no parentheses, do the steps below:

1. Take care of any addition or subtraction operations.
2. Take care of any multiplication or division operations.

**Example 2**

Let’s work through the following equation.

\[
4x - 8 = 12
\]

**Step 1.** First, take care of any addition or subtraction operations. Add 8 to both sides.

\[
4x - 8 + 8 = 12 + 8
\]

\[
4x = 20
\]

**Step 2.** Take care of any multiplication or division operations.

\[
\frac{4x}{4} = \frac{20}{4}
\]

\[
x = 5
\]

Now let’s look at another example.

**Example 3**

\[
\frac{x}{3} + 65 = 120
\]
Step 1. Take care of any addition or subtraction operations. Subtract 65 from both sides of the equation.

\[
\frac{x}{3} + 65 - 65 = 120 - 65
\]

\[
\frac{x}{3} = 55
\]

Step 2. Take care of any multiplication or division operations. Multiply both sides by 3 to solve for \(x\).

\[
\frac{x}{3} \times 3 = 55 \times 3
\]

\[
x = 165
\]

**PRACTICE 19: Solving Two-Step Equations**

Solve each equation below. Write your answer on the line.

Example:

\[
2z - 5 = 11
\]

\[
2z - 5 + 5 = 11 + 5
\]

\[
2z = 16
\]

\[
\frac{2z}{2} = \frac{16}{2}
\]

1. \[
\frac{w}{7} + 12 = 33
\]

\[
w = \underline{22}
\]

2. \[
\frac{m}{6} - 8 = 14
\]

\[
m = \underline{108}
\]

3. \[
2p + 11 = 47
\]

\[
p = \underline{18}
\]

4. \[
7 + \frac{t}{6} = 15
\]

\[
t = \underline{72}
\]