

MODULE #1: Measurement and Units

Introduction

What is chemistry? That's a very good question. Chemistry is, quite simply, the study of **matter**. Of course, this definition doesn't do us much good unless we know what matter is. So, in order to understand what chemistry is, we first need to define matter. A good working definition for matter is:

Matter - Anything that has mass and takes up space

If you have a problem with the word "mass," don't worry about it. We will discuss this concept in a little while. For right now, you can replace the word "mass" with the word "weight." As we will see later, this isn't quite right, but it will be okay for now.

If matter is defined in this way, almost *everything* around us is matter. Your family car has a lot of mass. That's why it's so heavy. It also takes up a lot of space sitting in the driveway or in the garage. Thus, your car must be made of matter. The food you eat isn't as heavy as a car, but it still has some mass. It also takes up space. So food must be made up of matter as well. Indeed, almost everything you see around you is made up of matter because nearly everything has mass and takes up space. There is one thing, however, that has no mass and takes up no space. It's all around you right now. Can you think of what it might be? What very common thing that is surrounding you right now has no mass and takes up no space?

You might think that the answer is "air." Unfortunately, that's not the right answer. Perform the following experiments to see what I mean.

EXPERIMENT 1.1

Air Has Mass

Supplies:

- A meterstick (A yardstick will work as well; a 12-inch ruler is not long enough.)
 - Two 8-inch or larger balloons
 - Two pieces of string long enough to tie the balloons to the meterstick
 - Tape
 - Safety goggles
1. Without blowing them up, tie the balloons to the strings. Be sure to make the knots loose so that you can untie one of the balloons later in the experiment.
 2. Tie the other end of each string to each end of the meterstick. Try to attach the strings as close to the ends of the meterstick as possible.
 3. Once the strings have been tied to the meterstick, tape them down so that they cannot move.
 4. Go into your bathroom and pull back the shower curtain so that a large portion of the curtain rod is bare. Balance the meterstick (with the balloons attached) on the bare part of the shower curtain rod. You should be able to balance it very well. If you don't have a shower curtain rod or you are having trouble using yours, you can use any surface that is adequate for delicate balancing.
 5. Once you have the meterstick balanced, stand back and look at it. The meterstick balances right now because the total mass on one side of the meterstick equals the total mass on the other side of

the meterstick. In order to knock it off balance, you would need to move the meterstick or add more mass to one side. You will do the latter.

6. Have someone else hold the meterstick so that it does not move. In order for this experiment to work properly, the meterstick must stay stationary.
7. While the meterstick is held stationary, remove one of the balloons from its string (do not untie the string from the meterstick), and blow up the balloon.
8. Tie the balloon closed so that the air does not escape, then reattach it to its string.
9. Have the person holding the meterstick let go. If the meterstick was not moved while you were blowing up the balloon, it will tilt toward the side with the inflated balloon as soon as the person lets it go. This is because you added air to the balloon. Since air has mass, it knocks the meterstick off balance. Thus, air does have mass!
10. Clean up your mess.

EXPERIMENT 1.2

Air Takes Up Space

Supplies:

- A tall glass
- A paper towel
- A sink full of water
- Safety goggles

1. Fill your sink with water until the water level is high enough to submerge the entire glass.
2. Make sure the inside of the glass is dry.
3. Wad up the paper towel and shove it down into the bottom of the glass.
4. Turn the glass upside down and be sure that the paper towel does not fall out of the glass.
5. Submerge the glass upside down into the water, being careful not to tip the glass at any time.
6. Wait a few seconds and remove the glass, still being careful not to tilt it.
7. Pull the paper towel out of the glass. You will find that the paper towel is completely dry. Even though the glass was submerged in water, the paper towel never got wet. Why? When you tipped the glass upside down, there was air inside the glass. When you submerged it in the water, the air could not escape the glass. Thus, the glass was still full of air. Since air takes up space, there was no room for water to enter the glass, so the paper towel stayed dry.
8. Repeat the experiment, but this time be sure to tip the glass while it is underwater. You will see large bubbles rise to the surface of the water, and when you pull the glass out, you will find that it has water in it and that the paper towel is wet. This is because you allowed the air trapped inside the glass to escape when you tilted the glass. Once the air escaped, there was room for the water to come into the glass.
9. Clean up your mess.

Now that you see that air does have mass and does take up space, have you figured out the correct answer to my original question? What very common thing that is surrounding you right now has no mass and takes up no space? The answer is *light*. As far as scientists can tell, light does not have any mass and takes up no space. Thus, light is not considered matter. Instead, it is pure energy.

Everything else that you see around you, however, is considered matter. Chemistry, then, is the study of nearly everything! As you can imagine, studying nearly everything can be a very daunting task. However, chemists have found that even though there are many forms of matter, they all behave according to a few fundamental laws. If we can clearly understand these laws, then we can clearly understand the nature of the matter that exists in God's creation.

Before we start trying to understand these laws, however, we must first step back and ask a more fundamental question. *How* do we study matter? Well, the first thing we have to be able to do in order to study matter is to measure it. If I want to study an object, I first must learn things like how big it is, how heavy it is, and how old it is. In order to learn these things, I have to make some measurements. The rest of this module explains how scientists measure things and what those measurements mean.

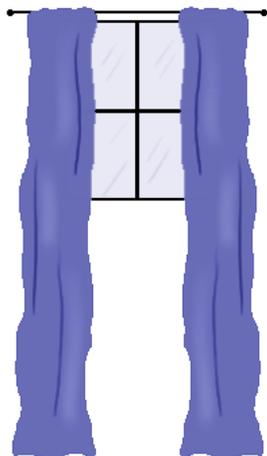
Units of Measurement

Let's suppose I'm making curtains for a friend's windows. I ask him to measure the window and give me the dimensions so that I can make the curtains the right size. My friend tells me that his windows are 50 by 60, so that's how big I make the curtains. When I go over to his house, it turns out that my curtains are more than twice as big as his windows! My friend tells me that he's certain he measured the windows correctly, and I tell my friend that I'm certain I measured the curtains correctly. How can this be? The answer is quite simple. My friend measured the windows in *centimeters*. I, on the other hand, measured the curtains in *inches*. Our problem was not caused by one of us measuring incorrectly. Instead, our problem was the result of measuring with different **units**.

When we are making measurements, the units we use are just as important as the numbers that we get. If my friend had told me that his windows were 50 centimeters by 60 centimeters, then there would have been no problem. I would have known exactly how big to make the curtains. Since he failed to do this, the numbers that he gave me (50 by 60) were essentially useless. Please note that a failure to indicate the units involved in measurements can lead to serious problems. For example, the Mars Climate Orbiter, a NASA spacecraft built for the exploration of Mars, vanished into during an attempt to put the craft into orbit around the planet. In an investigation that followed, NASA determined that a units mix-up had caused the disaster. One team of engineers had used metric units in its designs, while another team had used English units. The teams did not indicate the units they were using, and as a result, the designs were incompatible.

In the end, then, scientists should never simply report numbers. They must always include units with those numbers so that everyone knows exactly what those numbers mean. That will be the rule in this chemistry course. If you answer a question or a problem and do not list units with the numbers, your answer will be considered incorrect. In science, numbers mean nothing unless there are units attached to them.

Window illustration by
Megan Whitaker



These curtains are too long for this window because the window was measured in centimeters, but the curtains were made assuming the measurements were in inches.

FIGURE 1.1

Two Consequences of Not Using Units Properly

Mars Climate Orbiter image
Courtesy of NASA/JPL/Caltech



The Mars Climate Orbiter did not successfully make it into orbit because two of the engineering teams involved used different units in their designs.

Since scientists use units in all of their measurements, it is convenient to define a standard set of units that will be used by everyone. This system of standard units is called the **metric system**. If you do not fully understand the metric system, don't worry. By the end of this module, you will be an expert at using it. If you do fully understand the metric system, you can probably skip ahead to the section labeled "Converting Between Units."

The Metric System

There are many different things that we need to measure when studying nature. First, we must determine how much matter exists in the object that we want to study. We know that there is a lot more matter in a car than there is in a feather, since a car is heavier. In order to study an object precisely, however, we need to know *exactly* how much matter is in the object. To accomplish this, we measure the object's **mass**. In the metric system, the unit for mass is the **gram**. If an object has a mass of 10 grams, we know that it has 10 times the matter that is in an object with a mass of 1 gram. To give you an idea of the size of a gram, the average mass of a housefly is just about 1 gram. Based on this fact, we can say that a gram is a rather small unit. Most of the things that we will measure will have masses of 10 to 10,000 grams. For example, this book has a mass of about 2,300 grams.

Now that we know what the metric unit for mass is, we need to know a little bit more about the concept itself. I said in the beginning that we could think of mass as weight. That's not exactly true. Mass and weight are two different things. Mass measures how much matter exists in an object. Weight, on the other hand, measures how hard gravity pulls on that object.

For example, if I were to get on my bathroom scale and weigh myself, I would find that I weigh 170 pounds. However, if I were to take that scale to the moon and weigh myself, I would find that I only weighed only 28 pounds there. Does that mean I'm thinner on the moon than I am at home? Of course not. It means that on the moon, gravity is not as strong as it is in my house.

On the other hand, if I were to measure my mass at home, I would find it to be 77,000 grams. If I were to measure my mass on the moon, it would still be 77,000 grams. That's the difference between mass and weight. Since weight is a measure of how hard gravity pulls, an object weighs different amounts depending on where that object is. Mass, on the other hand, is a measure of how much matter is in an object and does not depend on where that object is.

Unfortunately, there are many other unit systems in use today besides the metric system. In fact, the metric system is probably not the system with which you are most familiar. You are probably most familiar with the English system. The unit of pounds comes from the English system. However, pounds are not a measure of mass; they are a measure of weight. The metric unit for weight is called the **Newton**. The English unit for mass is (believe it or not) called the **slug**. Although we will not use the slug often, it is important to understand what it means, especially when you study physics.

There is more to measurement than just grams, however. We might also want to measure how big an object is. For this, we must use the metric system's unit for distance, which is the **meter**. You are probably familiar with a yardstick. Well, a meter is just slightly longer than a yardstick. The English unit for distance is the **foot**. What about inches, yards, and miles? We'll talk about those a little later.

We also need to be able to measure how much space an object occupies. This measurement is commonly called "volume" and is measured in the metric system with the unit called the **liter**. The main unit for measuring volume in the English system is the **gallon**. To give you an idea of the size of a liter, it takes just under four liters to make a gallon.

Finally, we have to be able to measure the passage of time. When studying matter, we will see that it has the ability to change. The shape, size, and chemical properties of certain substances change over time, so it is important to be able to measure time so that we can determine how quickly the changes take place. In both the English and metric systems, time is measured in **seconds**.

Since it is very important for you to be able to recognize which units correspond to which measurements, Table 1.1 summarizes what you have just read. The letters in parentheses are the commonly used abbreviations for the units listed.

TABLE 1.1
Physical Quantities and Their Base Units

Physical Quantity	Base Metric Unit	Base English Unit
Mass	gram (g)	slug (sl)
Distance	meter (m)	foot (ft)
Volume	liter (L)	gallon (gal)
Time	second (s)	second (s)

Manipulating Units

Now, let's suppose I asked you to measure the width of your home's kitchen using the English system. What unit would you use? Most likely, you would express your measurement in feet. However, suppose instead I asked you to measure the length of a sewing needle. Would you still use the foot as your measurement unit? Probably not. Since you know the English system already, you would probably recognize that inches are also a unit for distance and, since a sewing needle is relatively small, you would use inches instead of feet. In the same way, if you were asked to measure the distance between two cities, you would probably express your measurement in terms of miles, not feet. This is why I used the term "Base English Unit" in Table 1.1. Even though the English system's normal unit for distance is the foot, there are alternative units for length if you are trying to measure very short or very long distances. The same holds true for all English units. Volume, for example, can be measured in cups, pints, and ounces.

This concept exists in the metric system as well. There are alternative units for measuring small things as well as alternative units for measuring big things. These alternative units are called "prefix units" and, as you will soon see, prefix units are much easier to use and understand than the alternative English units! The reason that prefix units are easy to use and understand is that they always have the same relationship to the base unit, regardless of what physical quantity you are interested in measuring. You will see how this works in a minute.

In order to use a prefix unit in the metric system, you simply add a prefix to the base unit. For example, in the metric system, the prefix "centi" means one hundredth, or 0.01. So, if I wanted to measure the length of a sewing needle in the metric system, I would probably express my measurement with the centimeter unit. Since a centimeter is one hundredth of a meter, it can be used to measure relatively small things. On the other hand, the prefix "kilo" means 1,000. So, if I want to measure the distance between two states, I would probably use the kilometer. Since each kilometer is 1,000 times longer than the meter, it can be used to measure long things.

Now, the beauty of the metric system is that these prefixes mean the same thing *regardless of the physical quantity that you want to measure!* So, if I were measuring something with a very large mass (such as a car), I would probably use the kilogram unit. One kilogram is the same as 1,000 grams. In the same way, if I were measuring something that had a large volume, I might use the kiloliter, which would be 1,000 liters.

Compare this incredibly logical system of units to the chaotic English system. If you want to measure something short, you use the inch unit, which is equal to one twelfth of a foot. On the other hand, if you want to measure something with small volume, you might use the quart unit, which is equal to one fourth of a gallon. In the English system, every alternative unit has a different relationship to the base unit, and you must remember all of those crazy numbers. You have to remember that there are 12 inches in a foot, 3 feet in a yard, and 5,280 feet in a mile, while at the same time remembering that for volume there are 8 ounces in a cup, 2 cups in a pint, 2 pints in a quart, and 4 quarts in a gallon.

In the metric system, all you have to remember is what the prefix means. Since the "centi" prefix means one hundredth, then you know that 1 centimeter is one hundredth of a meter, 1 centiliter is one hundredth of a liter, and 1 centigram is one hundredth a gram. Since the "kilo" prefix means

1,000, you know that there are 1,000 meters in a kilometer, 1,000 grams in a kilogram, and 1,000 liters in a kiloliter. Doesn't that make a lot more sense?

Another advantage to the metric system is that there are many, many more prefix units than there are alternative units in the English system. Table 1.2 summarizes the most commonly used prefixes and their numerical meanings. The prefixes in boldface type are the ones that we will use over and over again. You will be expected to have those three prefixes and their meanings memorized before you take the test for this module. Once again, the commonly used abbreviations for these prefixes are listed in parentheses.

TABLE 1.2
Common Prefixes Used in the Metric System

PREFIX	NUMERICAL MEANING
micro (μ)	0.000001
milli (m)	0.001
centi (c)	0.01
deci (d)	0.1
deca (D)	10
hecta (H)	100
kilo (k)	1,000
Mega (M)	1,000,000

Remember that each of these prefixes, when added to a base unit, makes an alternative unit for measurement. So, if you wanted to measure the length of something small, the only unit you could use in the English system would be the inch. However, if you used the metric system, you would have all sorts of options for which unit to use. If you wanted to measure the length of someone's foot, you could use the decimeter. Since the decimeter is one tenth of a meter, it measures things that are only slightly smaller than a meter. On the other hand, if you wanted to measure the length of a sewing needle, you could use the centimeter, because a sewing needle is significantly smaller than a meter. Or, if you want to measure the thickness of a piece of paper, you might use the millimeter, since it is one thousandth of a meter, which is a *really* small unit.

So you see that the metric system is more logical and versatile than the English system. That is, in part, why scientists use it as their main system of units. The other reason that scientists use the metric system is that most countries in the world use it. With the exception of the United States, almost every other country in the world uses the metric system as its standard system of units. Since scientists in the United States frequently work with scientists from other countries around the world, it is necessary that American scientists use and understand the metric system. Throughout all of the modules of this chemistry course, the English system of measurement will only be presented for illustration purposes. Since scientists must thoroughly understand the metric system, it will be the main system of units that we will use.

Converting Between Units

Now that we understand what prefix units are and how they are used in the metric system, we must become familiar with converting between units within the metric system. In other words, if you measure the length of an object in centimeters, you should also be able to convert your answer to any

other distance unit. For example, if I measure the length of a sewing needle in centimeters, I should be able to convert that length to millimeters, decimeters, meters, etc. Accomplishing this task is relatively simple as long as we remember a trick we can use when multiplying fractions. Suppose I asked you to complete the following problem:

$$\frac{7}{64} \times \frac{64}{13} =$$

There are two ways to figure out the answer. The first way would be to multiply the numerators and the denominators together and, once you had accomplished that, simplify the fraction. If you did it that way, it would look something like this:

$$\frac{7}{64} \times \frac{64}{13} = \frac{448}{832} = \frac{7}{13}$$

You could get the answer much more quickly, however, if you remember that when multiplying fractions, common factors in the numerator and the denominator cancel each other out. Thus, the 64 in the numerator cancels with the 64 in the denominator, and the only factors left are the 7 in the numerator and the 13 in the denominator. In this way, you reach the final answer in one less step:

$$\frac{7}{\cancel{64}} \times \frac{\cancel{64}}{13} = \frac{7}{13}$$

We will use the same idea in converting between units. Suppose I measure the length of a pencil to be 15.1 centimeters, but suppose the person who wants to know the length of the pencil would like me to tell him the answer in meters. How would I convert between centimeters and meters? First, I would need to know the relationship between centimeters and meters. According to Table 1.2, “centi” means 0.01. So, 1 centimeter is the same thing as 0.01 meters. In mathematical form, we would say:

$$1 \text{ centimeter} = 0.01 \text{ meter}$$

Now that we know how centimeters and meters relate to one another, we can convert from one to another. First, we write down the measurement that we know:

15.1 centimeters

We then realize that any number can be expressed as a fraction by putting it over the number one. So we can rewrite our measurement as:

$$\frac{15.1 \text{ centimeters}}{1}$$

Now we can take that measurement and convert it into meters by multiplying it with the relationship we determined above. We have to do it the right way, however, so that the units work out properly. Here’s how we do it:

$$\frac{15.1 \cancel{\text{centimeters}}}{1} \times \frac{0.01 \text{ meters}}{1 \cancel{\text{centimeters}}} = 0.151 \text{ meters}$$

So, 15.1 centimeters is the same as 0.151 meters. There are two reasons this conversion method, called the **factor-label method**, works. First, since 0.01 meters is the same as 1 centimeter, multiplying our measurement by 0.01 meters over 1 centimeter is the same as multiplying by one. Since nothing changes when we multiply by one, we haven't altered the value of our measurement at all. Second, by putting the 1 centimeters in the denominator of the second fraction, we allow the centimeters unit to cancel (just like the 64 canceled in the previous discussion). Once the centimeters unit has canceled, the only thing left is meters, so we know that our measurement is now in meters.

This is how we will do all of our unit conversions. We will first write the measurement we know in fraction form by putting it over one. We will then find the relationship between the unit we have and the unit to which we want to convert. We will then use that relationship to make a fraction that, when multiplied by our first fraction, cancels out the unit we have and replaces it with the unit we want to have. We will see many examples of this method, so don't worry if you are a little confused right now.

It may seem odd to you that words can be treated exactly the same as numbers. Measuring units, however, have just that property. Whenever a measurement is used in any mathematical equation, the units for that measurement must be included in the equation. Those units are then treated the same way numbers are treated. I will come back to this point in an upcoming section of this module.

We will be using the factor-label method for many other types of problems throughout this course, so it is very, very important for you to learn it. Also, since we will be using it so often, we should start abbreviating things so that they will be easier to write down. We will use the abbreviations for the base units that have been listed in Table 1.1 along with the prefix abbreviations listed in Table 1.2. Thus, kilograms will be abbreviated as "kg," while milliliters will be abbreviated as "mL."

Since the factor-label method is so important in our studies of chemistry, let's see how it works in another example:



EXAMPLE 1.1

A student measures the mass of a rock to be 14,351 grams. What is the rock's mass in kilograms?

First, we use the definition of "kilo" to determine the relationship between grams and kilograms:

$$1 \text{ kg} = 1,000 \text{ g}$$

Then we put our measurement in fraction form:

$$\frac{14,351 \text{ g}}{1}$$

Then we multiply our measurement by a fraction that contains the relationship noted above, making sure to put the 1,000 g in the denominator so that the unit of grams will cancel out:

$$\frac{14,351 \text{ g}}{1} \times \frac{1 \text{ kg}}{1,000 \text{ g}} = 14.351 \text{ kg}$$

So 14,351 grams is the same as 14.351 kilograms.

Because we will use it over and over again, you must master this powerful technique. Also, you will see towards the end of this module that the factor-label method can become extremely complex; therefore, it is very important that you take the time now to perform the following “on your own” problems. Once you have solved the problems on your own, check your answers using the solutions provided at the end of the module.

ON YOUR OWN

- 1.1 A student measures the mass of a book as 12,321 g. What is the book’s mass in kg?
- 1.2 If a glass contains 0.121 L of milk, what is the volume of milk in mL?
- 1.3 On a professional basketball court, the distance from the three-point line to the basket is 640.08 cm. What is this distance in meters?

Converting Between Unit Systems

As you may have guessed, the factor-label method can also be used to convert *between systems* of units as well as within systems of units. Thus, if a measurement is done in the English system, the factor-label method can be used to convert that measurement to the metric system, or vice-versa. In order to be able to do this, however, we must learn the relationships between metric and English units. Although these relationships are important, we will not use them very often, so you need not memorize them.

TABLE 1.3
Relationships Between English and Metric Units.

Measurement	English/Metric Relationship
Distance	1 inch = 2.54 cm
Mass	1 slug = 14.59 kg
Volume	1 gallon = 3.78 L

We can use this information in the factor-label method the same way we used the information in Table 1.2. Study the following example to see what I mean.



EXAMPLE 1.2

The length of a tabletop is measured to be 37.8 inches. How many cm is that?

To solve this problem, we first put the measurement in its fraction form:

$$\frac{37.8 \text{ in}}{1}$$

We then multiply this fraction by the conversion relationship so that the inches unit cancels:

$$\frac{37.8 \cancel{\text{in}}}{1} \times \frac{2.54 \text{ cm}}{1 \cancel{\text{in}}} = 96.012 \text{ cm}$$

So a measurement of 37.8 inches is equivalent to 96.012 cm.

Give yourself a little more practice with the factor-label method by answering the following “on your own” problems:

ON YOUR OWN

1.4 How many slugs are there in 123.5 kg?

1.5 If an object occupies 3.2 gallons of space, how many liters of space does it occupy?

More Complex Unit Conversions

Now that we have seen some simple applications of the factor-label method, let’s look at more complex problems. For example, suppose I measure the volume of a liquid to be 4,523 centiliters but would like to convert this measurement into kiloliters. This is a more complicated problem because we do not have a direct relationship between cL and kL. In all of the previous examples, we knew a relationship between the unit we had and the unit to which we wanted to convert. In this problem, however, no such relationship exists.

We do, however, have an *indirect* relationship between the two units. Although we don’t know how many cL are in a kL, we do know how many cL are in a L and how many L are in a kL. We can use these two relationships in a two-step conversion. First, we can convert centiliters into liters:

$$\frac{4,523 \cancel{\text{cL}}}{1} \times \frac{0.01 \text{ L}}{1 \cancel{\text{cL}}} = 45.23 \text{ L}$$

Then we can convert liters into kiloliters:

$$\frac{45.23 \cancel{\text{L}}}{1} \times \frac{1 \text{ kL}}{1,000 \cancel{\text{L}}} = 0.04523 \text{ kL}$$

We are forced to do this two-step process because we do not have a direct relationship between two prefix units in the metric system. However, we can always convert between two prefix units if we first convert to the base unit. In order to speed up this kind of conversion, we can take these two steps and combine them into one line:

$$\frac{4,523 \cancel{\text{cL}}}{1} \times \frac{0.01 \cancel{\text{L}}}{1 \cancel{\text{cL}}} \times \frac{1 \text{ kL}}{1,000 \cancel{\text{L}}} = 0.04523 \text{ kL}$$

You will be seeing mathematical equations like this one as we move through the subject of chemistry, so it is important for you to be able to understand what's going on in it. The first fraction in the equation above represents the measurement that we were given. Since we have no relationship between the unit we were given and the unit to which we will convert, we first convert the given unit to the base unit. This is accomplished with the second fraction in the equation. When the first fraction is multiplied by the second fraction, the "cL" unit cancels and is replaced by the "L" unit. The third fraction then cancels the "L" unit and replaces it with the "kL" unit, which is the unit we wanted. This, then, gives us our final answer. Try to follow this reasoning in Example 1.3.



EXAMPLE 1.3

The mass of an object is measured to be 0.030 kg. What is the object's mass in mg?

We have no direct relationship between milligrams and kilograms, but we do have an indirect relationship between the two. First, we know that

$$1 \text{ kg} = 1,000 \text{ g}$$

We also know that

$$1 \text{ mg} = 0.001 \text{ g}$$

So, we need to take our original measurement in fraction form and multiply it with both of these relationships. We must do it in such a way as to cancel out the kg and replace it with g, and then cancel out the g and replace it with mg:

$$\frac{0.030 \cancel{\text{kg}}}{1} \times \frac{1,000 \cancel{\text{g}}}{1 \cancel{\text{kg}}} \times \frac{1 \text{ mg}}{0.001 \cancel{\text{g}}} = 30,000 \text{ mg}$$

The object's mass is 0.030 kg, which is the same as 30,000 mg.

Once again, the factor-label method is one of the most important tools you can learn for the study of chemistry (and physics, for that matter). Thus, you must become a veritable expert at it. Try your hand at a few more "on your own" problems so that you can get some more practice.

ON YOUR OWN

- 1.6 A balloon is blown up so that its volume is 1,500 mL. What is its volume in kL?
- 1.7 If the length of a race car track is 2.0 km, how many cm is that?
- 1.8 How many Mg are there in 10,000,000 mg?

Derived Units

I mentioned previously that units can be used in mathematical expressions in the same way that numbers can be used. Just as there are rules for adding, subtracting, multiplying, and dividing numbers, there are also rules governing those operations when using units. You will have to become very adept at using units in mathematical expressions, so I want to discuss those rules now.

Adding and Subtracting Units: When adding and subtracting units, the most important thing to remember is that the units you are adding or subtracting must be identical: You cannot add grams and liters. The result would not make sense physically. Since gram is a mass unit and liter is a volume unit, there is no way you can add or subtract the two. You also cannot add or subtract kilograms and grams. Even though both units measure mass, you are not allowed to add or subtract them unless they are *identical*. Thus, if I did want to add or subtract them, I would have to convert the kilograms into grams. Alternatively, I could convert the grams into kilograms. It doesn't matter which way I go, as long as the units I add or subtract are identical.

Once I have identical units, I can add and subtract them using the rules of algebra. Since $2x + 3x = 5x$, we know that $2 \text{ cm} + 3 \text{ cm} = 5 \text{ cm}$. In the same way, $3.1 \text{ g} - 2.7 \text{ g} = 0.4 \text{ g}$. So, when adding or subtracting units, you add or subtract the numbers they are associated with and then simply carry the unit along in the answer.

Multiplying and Dividing Units: When multiplying and dividing units, it doesn't matter whether or not the units are identical. Unlike addition and subtraction, you are allowed to multiply or divide any unit by any other unit. In algebra,

$$3x \cdot 4y = 12xy$$

When multiplying units,

$$3 \text{ kg} \cdot 4 \text{ mL} = 12 \text{ kg} \cdot \text{mL}$$

Similarly, in algebra,

$$6x \div 2y = 3 \frac{x}{y}$$

When dividing units,

$$6 \text{ g} \div 2 \text{ mL} = 3 \frac{\text{g}}{\text{mL}}$$

So, when multiplying or dividing units, you multiply or divide the numbers and then do exactly the same thing to the units.

Let's use the rules we've just learned to explore a few other things about units. First, let's see what happens when we multiply measurements that have the same units. Suppose I wanted to measure the surface area of a rectangular table. From geometry, we know that the area of a rectangle is the length times the width. So, let's suppose we measure the length of a table to be 1.1 meters and the width to be 2.0 meters. Its area would be

$$1.1 \cdot 2.0 = 2.2$$

What would the units be? In algebra, we would say that

$$1.1x \cdot 2.0x = 2.2x^2$$

Thus,

$$1.1 \text{ m} \cdot 2.0 \text{ m} = 2.2 \text{ m}^2$$

This tells us that m^2 is a unit for area.

Let's take this one step further and suppose we measured the length, width, and height of a small box to be 1.2 cm, 3.1 cm, and 1.4 cm, respectively. What would the volume of the box be? From geometry, we know that volume is length times width times height, so the volume would be:

$$1.2 \text{ cm} \cdot 3.1 \text{ cm} \cdot 1.4 \text{ cm} = 5.208 \text{ cm}^3$$

Thus, cm^3 (usually called "cubic centimeters," or "cc's") is a unit for volume. If you've ever listened to doctors or nurses talking about how much liquid to put in a hypodermic needle when administering a shot, they usually use "cc's" as the unit. When a doctor tells a nurse, "Give the patient 4 cc's of penicillin," he or she is telling the nurse to inject a 4 cm^3 volume of penicillin into the patient.

Wait a minute. Wasn't the metric unit for volume the liter? Well, yes, but another metric unit for volume is the cm^3 . Additionally, m^3 (cubic meters) and km^3 (cubic kilometers) are also possible units for volume. This is a very important point. Often, several different units exist for the same measurement. The units you use will depend, to a large extent, on what information you are given in the first place. We'll see more about this fact later.

Units like cm^3 are called **derived units** because they are derived from the basic units that make up the metric system. It turns out that many of the units you will use in chemistry are derived units. I'll talk about one very important physical quantity with derived units in an upcoming section of this module, but first I want to make sure that you understand exactly how to use derived units in mathematical equations.

Let's suppose I would like to take the volume that we previously determined for the box and convert it from cubic centimeters to cubic meters. You might think the conversion would look something like this:

$$\frac{5.208 \text{ cm}^3}{1} \times \frac{0.01 \text{ m}}{1 \text{ cm}}$$

Unfortunately, even though this conversion might look correct, there is a major problem with it. Remember what the factor-label method is designed to accomplish. In the end, the old units are supposed to cancel out, leaving the new units in their place. The way this conversion is set up, however, the old units *do not cancel!* When I take these two fractions and multiply them together, the cm in the denominator does not cancel out the cm³ in the numerator. When multiplying fractions, the numerator and denominator must be *identical* in order for them both to cancel. Thus, the cm in the denominator above must be replaced with a cm³.

How is this done? Actually it's quite simple. Just take the second fraction and raise it to the third power:

$$\frac{5.208 \text{ cm}^3}{1} \times \left(\frac{0.01 \text{ m}}{1 \text{ cm}} \right)^3$$

That way, the m becomes m³, the cm becomes cm³, the 0.01 becomes 0.000001, and the 1 stays as 1:

$$\frac{5.208 \text{ cm}^3}{1} \times \frac{0.000001 \text{ m}^3}{1 \text{ cm}^3} = 0.000005208 \text{ m}^3$$

Now, since both the numerator and denominator have a unit of cm³, that unit cancels and is replaced with the m³. So, a volume of 5.208 cm³ is equivalent to a volume of 0.000005208 m³.

Since cubic meters, cubic centimeters and the like are measurements of volume, you might have already guessed that there must be a relationship between these units and the other volume units we discussed earlier. In fact, 1 cm³ is the same thing as 1 mL. This is a very important relationship, and it is something you will have to know before you can finish this module. So, commit it to memory now:

1 cubic centimeter is the same as 1 milliliter.

Let's combine this fact with the mathematics we just learned and perform a very complicated unit conversion. If you can understand this next example and successfully complete the "on your own" problems that follow it, then you have mastered the art of unit conversion. If things are still a bit shaky for you, don't worry. There are plenty of practice problems at the end of this module to help you shore up your unit conversion skills!



EXAMPLE 1.4

The length, width, and height of a small box are measured to be 1.1 in, 3.2 in, and 4.6 in, respectively. What is the box's volume in liters?

In order to solve this problem, we first use the geometric equation for the volume of a box:

$$V = l \cdot w \cdot h$$

Inserting our numbers:

$$V = 1.1 \text{ in} \cdot 3.2 \text{ in} \cdot 4.6 \text{ in} = 16.192 \text{ in}^3$$

Now that we know the volume, we just have to convert from in^3 to L. This is a little more difficult than it sounds, however. Since we have no direct relationship between in^3 and L, we must go through a series of conversions to get to the desired unit. First, we can convert our unit from the English system to the metric system by using the relationship

$$1 \text{ in} = 2.54 \text{ cm}$$

To do this, however, we will have to cube the fraction we multiply by so that we have cm^3 and in^3 !

$$\frac{16.192 \text{ in}^3}{1} \times \left(\frac{2.54 \text{ cm}}{1 \text{ in}} \right)^3 = \frac{16.192 \cancel{\text{in}^3}}{1} \times \frac{16.387 \text{ cm}^3}{1 \cancel{\text{in}^3}} = 265.338 \text{ cm}^3$$

Now that we have the metric volume unit, we can use the fact that a cm^3 is the same as a mL:

$$265.338 \text{ cm}^3 = 265.338 \text{ mL}$$

Now we can convert from mL to L:

$$\frac{265.338 \cancel{\text{mL}}}{1} \times \frac{0.001 \text{ L}}{1 \cancel{\text{mL}}} = 0.265338 \text{ L}$$

The volume, then, is 0.265338 L.

ON YOUR OWN

1.9 A braggart tells you that he walks 100,000 cm each day. He expects you to be impressed with such a big number. Should you be impressed? Convert the distance measurement to miles in order to determine whether or not to be impressed. (HINT: Earlier in this module, you were told how many inches are in a foot and how many feet are in a mile. You must use those numbers to solve this problem.)

1.10 How many cm^3 are in 0.0045 kL?

1.11 The area of a room is 16 m^2 . What is the area of the room in mm^2 ?

Making Measurements

Now that we've learned so much about measurement units, we need to spend a little time learning how to make measurements. After all, being able to manipulate units in mathematical equations isn't going to help us unless we can make measurements with those units to begin with. In order to learn how to make measurements properly, let's start with something simple: a ruler.

Suppose I wanted to measure the length of a small ribbon with an English ruler. I would make my measurement something like this:

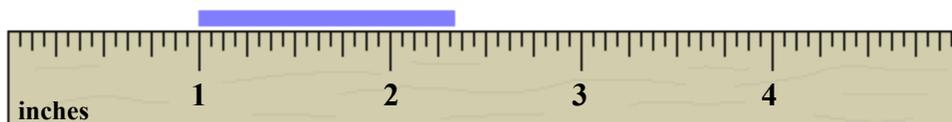


Illustration by Megan Whitaker

First, notice that I did not start my measurement at the beginning of the ruler. Instead, I lined up the ribbon with the first inch mark. The reason I did this is that it is slightly more accurate. It is very difficult to line the edge of a ruler up with the edge of the object you are measuring. This is especially true when the ruler is old and the edges are worn. So, the first rule for measuring with a ruler is to start at "1," not "0."

Now, how would you read this measurement? Well, first you need to see what the scale on the ruler is. If you count the number of dashes between 1 inch and 2 inches, you will find that there are 15 of them. That means that every dash is worth one sixteenth of an inch. This is because 15 dashes break up the area between 1 inch and 2 inches into 16 equal regions.

Now that we know the scale is marked off in sixteenths of an inch, we can see that the ribbon is a little bigger than 1 and $\frac{5}{16}$ of an inch. Is that the best we can do? Of course not! Because the edge of the ribbon falls between $\frac{5}{16}$ ($\frac{10}{32}$) and $\frac{6}{16}$ ($\frac{12}{32}$) of an inch, we can estimate that it is approximately $\frac{11}{32}$. Thus, the proper length of the ribbon is 1 and $\frac{11}{32}$ inches. Generally, chemists do not like fractions in their final measurements, so we will convert $\frac{11}{32}$ into its decimal form to get a measurement of 1.34375 inches. Later on we will see that this measurement has far too many digits in it, but for right now we will assume that it is okay.

Let's take that same ribbon and now measure it with a metric ruler:



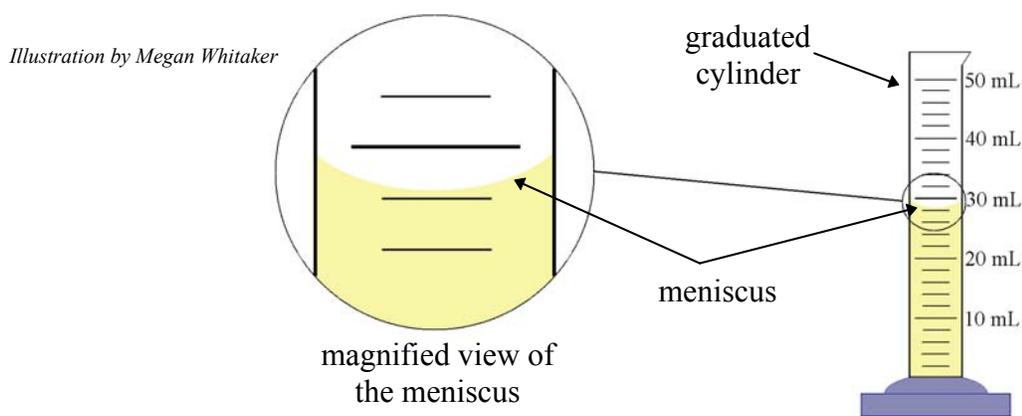
Illustration by Megan Whitaker

Now what measurement do we get? Well, there are 9 small dashes between each cm mark; therefore, the scale of this ruler is one tenth of a cm, or 0.1 cm. This is typical of metric rulers. They are almost always marked off in tenths since the prefixes in the metric system are all multiples of 10. If you think about it, 1 mm = 0.1 cm, so you could also say that each small dash is 1 mm. Clearly, the ribbon is between 3.4 cm and 3.5 cm long. Using our method of approximating between the dashes, we would say that the ribbon is 3.41 cm long.

Whenever you are using a measuring device that has a scale on it, be sure to use it the way we have used the rulers here. First, determine what the dashes on the scale mean. Then, try to estimate in between the dashes if the object you are measuring does not exactly line up with a dash. That gives you as accurate a measurement as possible. You should always strive to read the scale to *the next decimal place* if possible. In the metric ruler, for example, the scale is marked off in 0.1 cm, so you should read the ruler to 0.01 cm, as I discussed above.

One physical quantity that chemists measure quite a lot is volume. Since chemists spend a great deal of time mixing liquids, volume is often a very important factor and must be measured. When chemists measure volume, one of the most useful tools is the **graduated cylinder**. This device looks a lot like a glass rain gauge. It is a hollow glass cylinder with markings on it. These markings, called graduations, measure the volume of liquid that is poured into the cylinder.

In the last experiment you will perform in this module, you will use a graduated cylinder (or a suitable substitute) to measure volume; thus, you need to be aware of how to do this. When liquid is poured into a cylinder, the liquid tends to creep up the edges of the cylinder. This is because there are attractive forces between the liquid and the cylinder. Thus, liquid poured into a graduated cylinder does not have a flat surface. Instead, it looks something like this:



The curved surface of the yellow liquid is called the **meniscus** (muh nis' kus). In order to determine the volume of the liquid that is in any graduated cylinder, you must read the level of the liquid from the *bottom of the meniscus*. Looking at the graduated cylinder pictured above, there are 4 dashes in between each marking of 10 mL. Thus, the dashes split the distance between the 10 mL marks into 5 divisions. This means that each dash must be worth 2 mL. If you look at the bottom of the meniscus in the drawing above, you will see that it is between 28 and 30 mL. Is the volume 29 mL then? No, not quite.

In the two examples you saw before, it was hard to guess how far between the dashes the object's edge was because the dashes were very close together. In this example, the dashes are farther apart, so we can be a bit more precise in our final answer. In order for the volume to be 29 mL, the bottom of the meniscus would have to be exactly halfway between 28 and 30. Clearly, the meniscus is much closer to 28 than to 30. So the volume is really between 28 and 29, and probably a little closer to 28. I would estimate that the proper reading is 28.3 mL. It could be as low as 28.1 or as high as 28.5, so 28.3 is a good compromise. Remember, you need to try to read the scale to the next decimal place. Since the scale is marked off in increments of 2 mL, you must read try to read the answer to 0.1 mL. Experiments 1.3 and 1.4 will give you some practice at this kind of estimation.

Accuracy, Precision, and Significant Figures

Now that we've learned a little bit about making measurements, we need to talk about when measurements are good and when they are not. In chemistry, we can describe a measurement in two ways. We can talk about its **accuracy** or we can talk about its **precision**. Believe it or not, these two words mean two entirely different things to a chemist:

Accuracy - An indication of how close a measurement is to the true value

Precision - An indication of the scale on the measuring device that was used

In other words, the more correct a measurement is, the more accurate it is. On the other hand, the smaller the scale on the measuring instrument, the more precise the measurement.

For example, let's go back to the ribbon we measured earlier. Using a ruler with a scale marked off in 0.1 cm, we got a length of 3.45 cm. What does that number mean? It means that the length of the ribbon, as far as we could tell, was somewhere between 3.445 and 3.454 cm long. Since both of those numbers round to 3.45, any length within that range is consistent with our measurement. We could not determine the length of the ribbon any better than that because our ruler was not precise enough to do any better.

On the other hand, suppose we found a ruler whose scale was marked off in 0.01 cm. With that ruler, we could get a measurement of 3.448 cm, because estimating between the dashes gives us one more decimal place. Thus, since the ruler is marked off in hundredths, we can get a measurement out to the thousandths place. Now this measurement is still consistent with our previous one, because it rounds up to 3.45 cm. That extra digit in the thousandths place, however, "nails down" the length better. It would be impossible to obtain so precise a measurement from the ruler we used in the example above, so the new ruler provided us with a way of being more precise. Thus, the precision of your measurement depends completely on the measuring device you use. The smaller the difference in the dashes on the scale, the more precise your answer will be.

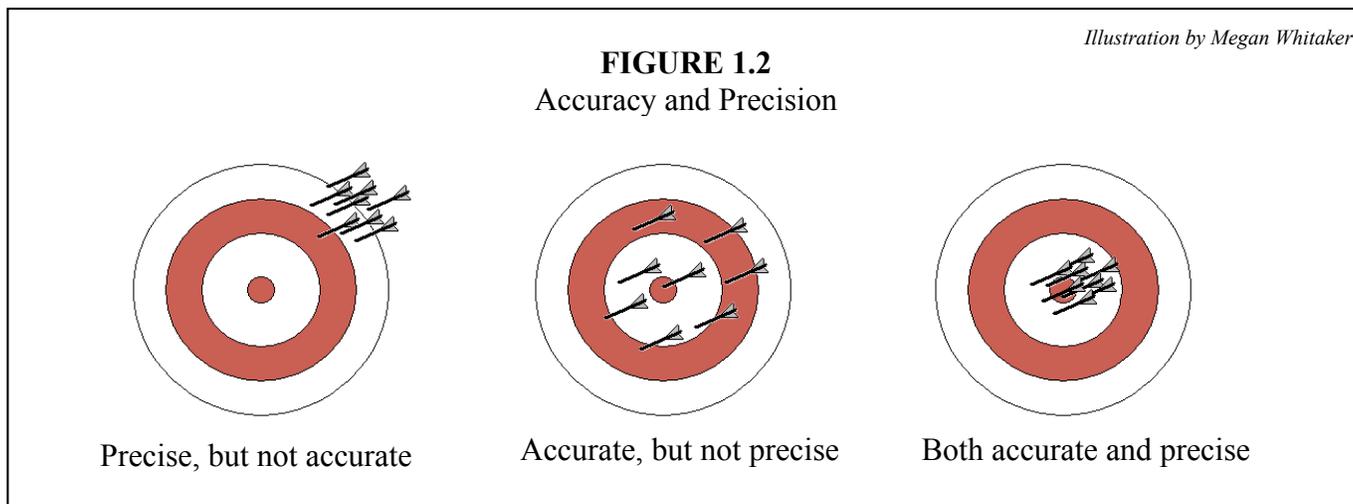
However, suppose you used the second ruler improperly. Maybe you read the scale incorrectly or didn't line the ribbon up to the ruler very well and ended up getting a measurement of 3.118 cm. Even though this measurement is more *precise* than the one we made with the first ruler, it is significantly less accurate because it is way off of the correct value of 3.448 cm. Thus, the accuracy of your measurement depends on how carefully and correctly you used the measuring device. In other words, a measurement's precision depends upon the instrument, whereas a measurement's accuracy depends upon the person doing the measurement.

Since a measurement's precision depends on the instrument used, the only way to improve your precision is to get a better instrument. However, there are other ways to improve your accuracy. First, you can make sure you understand the proper methods of using each instrument at your disposal. Second, you can practice making measurements, which will help you in being careful.

The most practical way to help your accuracy in measurement, however, is to make your measurement several times and average the results. This tends to average out all of the little differences between measurements. An even better way of assuring accuracy is to have several

different people make the measurements and average all of their answers together. The more individual measurements you can make, the more accurate the average of them will be.

Let me illustrate this whole idea of accuracy and precision in another way. Suppose you were throwing darts on a dart board. Here are three possible outcomes and the way that I would characterize them:



The first target on the left has all of the darts clumped together but way off the bull's eye of the target. If I made several measurements with a precise device, but I used the device wrongly every time, or if the device had a flaw, I would get a bunch of measurements very close to one another, but they would be far from the true value (the bull's eye). Thus, like the result on the dart board, I would have a lot of precision, but not much accuracy.

In the middle target, the darts are surrounding the bull's eye, but they are far from one another. This is what would happen if I used a measuring device that was not very precise, but I used it correctly. Because the estimation between the marks on the scale would be harder to do if the marks were far from each other, I would get a lot of different measurements from estimating between the dashes on the scale. However, if I used the device properly, and if the device was good, the average measurement would be accurate. Thus, like the result on the dart board, I would have accuracy, but not much precision.

In the target on the right side of the figure, I have the darts in a tight clump right around the bull's eye. This, then, is an illustration of measurements that are both accurate and precise. They are accurate because they average out to the correct value (the bull's eye). They are precise because they are very close to one another, indicating a precise measuring device was used.

Since accuracy and precision are very important, we need to know how to evaluate the accuracy and precision of our measurements. The way to determine the accuracy of a measurement is to compare it to the correct value. If you have no idea what the correct value is, determining your measurement's accuracy is a little more difficult. It is not impossible, but it is so complex that we will not discuss it in this course.

Determining the precision of a measurement, however, is quite easy. In order to determine the precision of a measurement, we merely need to look at its **significant figures**. If two instruments measure the same thing, the one which gives you a significant figure in the smallest decimal place is the more precise instrument. Of course, this fact does us no good if we do not know what a significant figure is:

A digit within a number is considered to be a significant figure if:

- i. It is non-zero OR**
- ii. It is a zero that is between two significant figures OR**
- iii. It is a zero at the end of the number and to the right of the decimal point**

Counting significant figures will be very important in our ability to understand measurements, so you must have a firm grasp of this concept. Read through example 1.5 and follow the logic. After that, try the “on your own” problems that follow.

EXAMPLE 1.5

Count the significant figures in each of the following numbers:

a. 3.234

b. 6.016

c. 105.340

d. 0.00450010

(a) Since every digit is non-zero in this number, every digit is a significant figure. Thus, there are four significant figures in this number.

(b) The three non-zero digits are all significant figures and the zero is also significant because it is between two significant figures. Thus, there are four significant figures in this number.

(c) The four non-zero digits are all significant figures. The zero between the 1 and the 5 is significant because it is between two significant figures. The last zero is also significant because it is at the end of the number and to the right of the decimal point. Thus, there are six significant figures in the number.

(d) The first three zeros are not significant because they are not between two significant figures and they are not at the end of the number. The three non-zero digits are all significant figures as are the zeros between the 5 and the 1. The zero at the end is also a significant figure. Thus, there are six significant figures in the number.

ON YOUR OWN

1.12 How many significant figures are in the following measurements?

a. 3.0220 cm

b. 0.0060 m

c. 1.00450 L

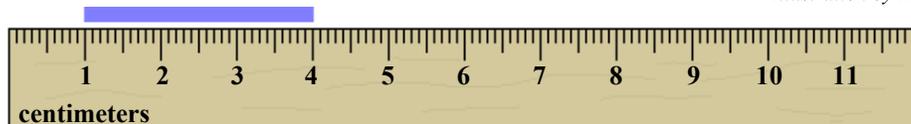
d. 61.054 kg

In the end, then, the precision of our instrument determines the number of significant figures we can report in a measurement. In our graduated cylinder example, we decided that we could reasonably approximate our measurement to somewhere between 28 and 29 mL. If we can do that, then we can report our answer to the nearest tenth of a mL, giving us a three-significant-figure answer of 28.3 mL. If, instead, our graduated cylinder had been marked off in tenths of a mL, we could probably approximate the measurement to somewhere between 28.2 and 28.3 mL, allowing us to have a four-significant-figure answer like 28.26 mL.

In the example which used an English ruler, I said that our final answer, 1.34375 inches, had too many digits in it. Now hopefully you can see why. According to this number, our ruler was precise enough to measure distance of one hundred thousandth of an inch! That's far too much precision. Most English rulers are precise to, at best, 0.01 inches. Thus, the proper length of the ribbon is 1.34 in.

Now, suppose we had another ribbon to measure :

Illustration by Megan Whitaker



How would we report its measurement? Would we say that this ribbon is 3 cm long? Actually, that measurement is not quite right. The ribbon does seem to end right on the 4 cm line, so there is no need to do any approximations here. Why, then, is 3 cm a wrong answer for the length of the ribbon?

The problem with reporting the length of the ribbon as 3 cm is that you are not being as precise as you can be. Since the ruler's scale is marked off in 0.1 cm, you can safely report your answers to the hundredths of a cm, because if the ribbon's edge had fallen between two dashes, you could have approximated as we did above. Thus, the precision of the ruler is to the hundredths place. Therefore, if the object's edge falls right on one of the dashes, do not throw away your precision. Report this length as 3.00 cm. This tells someone who reads the measurement that the ribbon's length was measured to a precision in the hundredths of a cm.

If you report the length as 3 cm, that means the ribbon could be as short as 2.5 cm or as long as 3.4 cm. Both of those measurements round to 3. But the ruler you used was much more precise. It determined the length of the ribbon to be 3.00 cm. Thus, the ribbon is somewhere between 2.995 and 3.004 cm long. Keeping all of the significant figures that you can is a very important part of doing chemistry experiments. You will get some practice at this in experiments 1.3 and 1.4.

Reporting the precision of a measurement is just as important as reporting the number itself. Why? Well, let's suppose that the NO-WEIGHT COOKIE CO. just produced a diet cookie that they claim has only 5 Calories per cookie. In order to confirm this claim, researchers did several careful experiments and found that, in fact, there were 5.4 Calories per cookie. Does this result mean that the NO-WEIGHT COOKIE CO. lied about the number of Calories in its cookies? No. When the company reported that there are 5 Calories per cookie, the precision of their claim indicated that there could be anywhere from 4.5 to 5.4 Calories per cookie. The researchers' finding was more precise than the company's claim, but, nevertheless, the company's claim was accurate.

Scientific Notation

Since reporting the precision of a measurement is so important, we need to be able to develop a notation system that allows us to do this no matter what number is involved. As numbers get very large, it becomes more difficult to report their precision properly. For example, suppose you measured the distance between two cities as 100.0 km. According to our rules of precision, reporting 100.0 km as the distance means that your measuring device was marked off in units of 1 km, and you estimated between the marks to come up with 100.0 km. However, suppose your measurement wasn't that precise. Suppose the instrument you used could only determine the distance to within 10 km? How could you write down a distance of 100 km and indicate that the precision was only to within 10 km?

The answer to this question lies in the technique of **scientific notation**. In scientific notation, we write numbers so that no matter how large or how small they are, they always have a decimal point in them. The way we do this is to remember that a number can be represented in many, many different ways. The number 4, for example, could be written as "2 x 2" or "4 x 1" or simply "4." Each one of these are appropriate representations of the number "4." In scientific notation, we always represent a number as a something times a power of 10. For example, "50" could be written in scientific notation as "5 x 10." The number "150" could be written as "1.5 x 100."

Do you see why this helps us in writing down the precision of our original measurement? Instead of writing the distance as 100 km, I could write it as 1.0 x 100. How does this help? Well, according to our rules of significant figures, the zero in 1.0 is significant, because it is at the end of the number and to the right of the decimal. Thus, by writing down our measurement this way, we indicate that the zero was actually measured and that the measurement is precise to within 10 km. There is no way to do that with normal, decimal notation, because neither of the zeros in 100 are significant. Scientific notation, then, gives us a way to *make* zeros significant if they need to be. If our measurement of 100 km was precise to within 1 km, we could indicate that by reporting the measurement as 1.00 x 100 km. Since both zeros in 1.00 are significant, this tells us that both zeros were measured, so our precision is within 1 km.

Now, since numbers that we deal with in chemistry can be very big or very small, we use one piece of mathematical shorthand in scientific notation. Recall from algebra that "100" is the same as "10²." We will use this shorthand to make the numbers easier to write down. In the end, then, scientific notation always has a number with a decimal point right after the first digit times a 10 raised to some power.

One other advantage of using scientific notation is that you can use it to simplify the job of recording very large or very small numbers. For example, there are roughly 20,000,000,000,000,000,000 particles in each breath of air that you take. Numbers like that are very common in chemistry. In scientific notation, the number would look like 2 x 10²². That's much easier to write down!

How did I know that I needed to raise the 10 to the 22nd power? I saw that in order to get the decimal point right after the two, I would have to move it to the left 22 digits. Moving the decimal 22 digits is equivalent to multiplying by 10²². So, when putting a large number into scientific notation, all you need to do is count the number of spaces the decimal point needs to move, and raise the 10 to that power.

(c) We must move the decimal point 9 places and make the number smaller, so the answer is 0.00000000189.

(d) Since the exponent is not listed, we assume it's "1." That means that we move the decimal point 1 place so that the number gets bigger, so the answer is 30. Once again, there is no way to indicate that the zero is significant, as it is in the original number.

ON YOUR OWN

1.13 Convert the following numbers into scientific notation.

- (a) 26,789,000 (b) 123 (c) 0.00009870 (d) 0.980

1.14 Convert the following numbers from scientific notation to decimal form.

- (a) 3.456×10^{14} (b) 1.2341×10^3 (c) 3.45×10^{-5} (d) 3.1×10^{-1}

Using Significant Figures in Mathematical Problems

Now that we have the ability to write down any measurement with its proper precision, there is only one more topic on significant figures that we need to discuss. We need to know how to use our concepts of significant figures when we work mathematical problems. Suppose I had two measurements and I wanted to add them together. Since each measurement has its own precision, the final answer would also have a certain precision. How do I know what the precision of my answer is?

For example, suppose I measured the total length of a knife to be 25.46 cm. Later on, someone else measured the length of the knife handle with a less precise ruler and got 7.8 cm. If I wanted to determine the length of the knife's blade, I could either go and measure it, or I could say that the blade's length was the total length of the knife minus the length of the handle, or $25.46 \text{ cm} - 7.8 \text{ cm}$. If I do the subtraction, I get 17.66 cm. It turns out, however, that this answer is too precise. Since the knife handle was measured with a less precise ruler, when I use its measurement in a calculation, the answer is also less precise. In the end, the proper answer would be 17.7 cm.

In order to add, subtract, multiply, or divide measurements, we have to learn two rules about using significant figures in mathematical equations. You will be using these rules over and over again throughout this course, so you will be expected to know them:

***Adding and Subtracting with Significant Figures:* When adding and subtracting measurements, round your answer so that it has the same precision as the *least precise* measurement in the equation.**

Multiplying and Dividing with Significant Figures: When multiplying and dividing measurements, round the answer so that it has the *same number of significant figures as the measurement with the fewest significant figures.*

Here's how these rules work:



EXAMPLE 1.7

A student measures the mass of a jar that is filled with sand and finds it to be 546.2075 kg. The jar has a note on it which says “when empty, this jar has a mass of 87.61 kg.” What is the mass of the sand that is in the jar?

Since 546.2075 kg is the mass of both the jar and the sand, and since 87.61 kg is the mass of the jar alone, the mass of the sand must be the difference between the two:

$$546.2075 \text{ kg} - 87.61 \text{ kg} = 458.5975 \text{ kg}$$

However, since the precision of the jar's mass only goes out to the hundredths place, that's the best we can do in our final answer. Thus, the mass of the sand is 458.60 kg. Note that this number has more significant figures than 87.61. That doesn't matter, however, because in addition and subtraction, you *do not* count significant figures; you only look at precision.

A person runs 3.012 miles in 0.430 hours. What is the person's average speed?

We can get the person's average speed by taking the distance traveled divided by the time:

$$\text{Speed} = 3.012 \text{ miles} \div 0.430 \text{ hours} = 7.004651163 \frac{\text{miles}}{\text{hours}}$$

The 3.012 miles has 4 significant figures while 0.430 hours has 3. Thus, our final answer must have 3 significant figures, making it 7.00 $\frac{\text{miles}}{\text{hour}}$.

Now that we have learned these rules, you will be expected to use them *in all further mathematical operations!* Whether you are working an “on your own” problem, a practice problem, a test problem, or an experiment, you will be expected to use these rules. In the examples and answers for all previous problems, these rules have not been followed, but they will from now on. By the time you finish the next couple of modules, keeping track of significant figures and precision should be second nature to you.

There is one point that I must make about significant figures before you get some practice using the rules. When making unit conversions, you might be tempted to round everything to one significant figure, because of the conversion relationships. For example, when you convert 121 g into kg, you use the following equation:

$$\frac{121 \text{ g}}{1} \times \frac{1 \text{ kg}}{1,000 \text{ g}}$$

Note that the “1 kg,” the “1” in the denominator of the first fraction, and “1,000 g” all look like they have only one significant figure. Thus, you might be tempted to round your answer to one significant figure. However, that would not be right. The reason is simple--these three numbers all come from definitions. They are actually infinitely precise. The “1 kg” is really “1.000... kg,” and the “1,000 g” is really “1,000.000... g.” This is because *exactly* 1 kilogram is defined to be *exactly* 1,000 g. In the same way, the “1” on the bottom of the first fraction is really “1.000...,” because it is an integer. In the end, then, the *only* number that has a limited number of significant figures is the 121 g (it is a measurement), so the answer is 0.121 kg.

In general, then, the prefixes used in the metric system as well as the integers that we use in fractions are infinitely precise and thus have an infinite number of significant figures. As a result, we ignore them when determining the significant figures in a problem. This is a very important rule:

The definitions of the prefixes in the metric system and the integers we use in fractions are not considered when determining the significant figures in the answer.

Get some practice making measurements, using them in mathematical equations, and keeping track of significant figures by performing the following experiment:

EXPERIMENT 1.3

Comparing Conversions to Measurements

Supplies:

- Book (not oversized)
- Metric/English ruler or rulers
- Safety goggles

Note: A sample set of calculations is available in the solutions and tests guide. It is with the solutions to the practice problems.

1. Lay the book on a table and measure its length in inches. Read the ruler as I showed you in the measurement section above, estimating any answer that falls in between the markings on the scale. Once you do that, convert the fraction to a decimal (as we did in the measurement section above) and round it to the hundredths place, because that’s the precision of an English ruler.
2. Measure the width of the book in the same way.
3. Now that you have the length and width measured, multiply them together to get the surface area of the book. Since you are multiplying inches by inches, your area unit should be in². Remember to count the significant figures in each of the measurements and round your final answer so that it has the same number of significant figures as the measurement with the least number of significant figures.
4. Now take the length measurement and use the relationship given in Table 1.3 to convert it into cm. Do the same thing to the width measurement, making sure to keep the proper number of significant figures. Note that the relationship between inches and centimeters is *exact*, so the “2.54 cm” should not be taken into account when considering the significant figures, because 1 inch is exactly 2.54 cm.
5. Now use your metric ruler to measure the length and width of the book in centimeters. Once again, do it just like I showed you in the measurements section above. If the scale of the ruler is marked

off in 0.1 cm, then your length and width measurements should be written to the hundredths of a centimeter. Compare these answers to the length and width you calculated by converting from inches. They should be the nearly the same. If they are different by only a few percent, there is no problem. However, if they differ by more than a few percent, recheck your measurements and your conversions.

6. Finally, multiply the length and width measurements you took with the metric ruler to calculate the surface area of the book in cm^2 . Use the relationship between inches and centimeters to convert your answer into in^2 . Remember, since you are using a derived unit, the conversion is more complicated. You might want to review Example 1.4.
7. Now compare the converted value for the surface area to the one you calculated in step (3) using your English measurements. Once again, they should be equal or close to equal. If not, you have either measured wrongly or made a mistake in your conversion.
8. Clean up your mess.

Density

Before we leave this module, I would like to introduce one very important quantity that is measured quite frequently in chemistry: **density**. The definition of density is as follows:

Density - An object's mass divided by the volume that the object occupies

This definition for density can be mathematically represented as:

$$\rho = \frac{m}{V} \quad (1.1)$$

In the equation above, the “ ρ ” is the Greek letter “rho” and is typically the symbol used to represent density. The letter “ m ” stands for the mass of the object, and the letter “ V ” stands for the object's volume. From this equation, we can determine the units that describe density. Since mass is measured in grams and the most common volume unit in chemistry is mL, the equation says that the units for density will be the unit one gets when grams are divided by mL, or $\frac{\text{grams}}{\text{mL}}$. In words, we would call

this unit “grams per mL.” Since mL and cm^3 are equivalent, we often see density expressed in grams per cm^3 . These, of course, are not the only units for density. Any mass unit divided by any volume unit is a possible unit for density, but these two are the most common in chemistry.

Density is a very important quantity in chemistry. It gives you an idea of how tightly packed the matter is in an object. For example, suppose you had a ball made out of plastic and another ball of precisely the same size made out of lead instead. Which ball would be heavier? The lead ball would be much heavier than the plastic ball, because there would be a lot more matter in the lead ball than in the plastic ball. This is because matter is packed very tightly together in lead, but matter is packed pretty loosely in plastic. Thus, for the same size ball, the lead ball is much heavier.

Density gives us a way to determine exactly how heavy the lead ball is compared to the plastic ball. Lead has a density of 11.4 grams per mL, whereas most plastic has a density of less than 1 gram per mL. Let's suppose we had a sample of plastic with a density of 0.57 grams per mL. Since the

density of lead is 20 times ($11.4 \div 0.57$) bigger than the density of the sample of plastic, we know that the matter inside of lead is packed 20 times tighter than the matter inside of the sample of plastic.

It also turns out that every substance on earth has its own unique density. This is nice because it provides a way for us to identify a substance if we don't know exactly what it is. For example, years ago people used density to determine whether or not something that looked golden was really made out of pure gold. They would measure the substance's mass, measure its volume and then divide the two. If the result turned out to be 19.3 grams per mL (the density of pure gold), then they knew it was pure gold because no other substance on earth has that density. If the density was not 19.3 grams per mL, then they knew it was not pure gold.

Before showing you some examples of how to use density in problems, I want you to perform another experiment. This experiment will teach you something very important about the physical meaning of density.

EXPERIMENT 1.4 The Density of Liquids

Note: A sample set of calculations is available in the solutions and tests guide. It is with the solutions to the practice problems.

Supplies:

- Water
 - Vegetable oil
 - Something that measures the volume of a liquid, preferably in mL or cm^3 . A graduated cylinder would be ideal, but measuring cups will work as well.
 - Maple syrup (Natural syrup does not work as well as something like Mrs. Butterworth's[®].)
 - A large glass
 - A mass scale, preferably one that reads in grams. (The scale should not go much over 500 grams, or it will be very difficult for you to read the mass of the objects in this experiment.)
 - Safety goggles
1. First, measure the mass of the graduated cylinder, or whatever you have that measures the volume of a liquid. Be sure to write it down with the correct precision. If you are using a standard mass scale from a grocery store, its scale is probably marked off in units of 10 grams. Thus, you should be able to report the mass to a precision of 1 g.
 2. Next, measure out 50.0 mL (1/4 cup if you are using measuring cups) of syrup. Now put the graduated cylinder (with the syrup in it) back on the scale and measure the total mass. Subtract the mass of the graduated cylinder from this number (using our rules for significant figures) to get the mass of the table syrup by itself. This method of measuring mass is called the **difference method**. Chemists often call it "measuring the mass by difference."
 3. Now that you have the mass of the table syrup, and you know that its volume was 50.0 mL (because that's what you measured out), divide the mass by the volume to get the density. Be sure to follow our significant figure rules when you do this! Finally, pour the syrup into the tall glass. Repeat this procedure for both the water and the vegetable oil.
 4. Once you have measured the density of all three substances, look at the tall glass. You should see that the table syrup is all at the bottom of the glass, the water forms a layer above that, and the vegetable oil is all in one layer on top!
 5. Clean up your mess.

What explains the layering you saw in the glass? Look at the densities you calculated from your measurements. Which substance has the largest density? The syrup. Where was the syrup? It was on the bottom of the glass. In the same way, the vegetable oil has the smallest density and was at the top of the glass, while the water's density is in between the two, so it stays in between the syrup and the vegetable oil. If we think about what density means, this should make sense. Density tells us how tightly packed the matter is inside a substance. Syrup's matter is very tightly packed, so it falls through the more loosely packed water and vegetable oil until it reaches the bottom of the glass. Water falls through the more loosely packed matter in vegetable oil but then is stopped by the more tightly packed matter in syrup. The vegetable oil could not fall through either water or syrup, so it stayed at the top of the glass.



(The multimedia CD has a video demonstration of the fact that even gases have density.)

Now that you have a better understanding of what density means, follow the example below and then do the “on your own” problems after it.



EXAMPLE 1.8

A gold miner has just found a nugget of pure gold. He measures its dimensions and then calculates its volume to be 0.125 L. Knowing that the density of gold is 19.3 grams per mL, calculate the mass of the miner's nugget.

Our equation for density is:

$$\rho = \frac{m}{V}$$

The problem is, the equation is not much good to us the way it is currently written. Since we would like to calculate the mass of the nugget, we need an equation that starts “m=.” So we use algebra to rearrange the equation so that it reads:

$$m = \rho \cdot V$$

Now we can simply stick in our numbers for volume and density, and we will get the mass, right? There's only one small problem. If we use the numbers as they are given in the problem, we would not get the correct answer. Why? If we did that, it would look like:

$$m = 19.3 \frac{\text{g}}{\text{mL}} \times 0.125 \text{ L}$$

According to our rules for multiplying numbers with units, we multiply the numbers, and then we do the same with the units. In the end, since we are calculating mass, we know that our answer should end up with the units of grams. If we multiply these two units together, however, we don't get grams. In order to get grams, the mL in the denominator would have to cancel with the L in the volume

measurement. That doesn't work. Instead, we need to change one of the units. It doesn't matter which, but I will choose to change the volume measurement into mL.

$$\frac{0.125 \cancel{\text{L}}}{1} \times \frac{1 \text{ mL}}{0.001 \cancel{\text{L}}} = 125 \text{ mL}$$

Now that I have the proper units, I can put everything into the equation:

$$m = 19.3 \frac{\text{g}}{\cancel{\text{mL}}} \times 125 \cancel{\text{mL}} = 2412.5 \text{ g}$$

We're still not quite done, however. According to our rules of significant figures, we can only have 3 significant figures in our final answer, because there are 3 significant figures in each of our numbers. Thus, the answer is 2,410 g. In scientific notation, the answer is $2.41 \times 10^3 \text{ g}$. Either expression is correct, as they both have 3 significant figures.

This problem illustrates two important things about studying chemistry. First, many chemistry problems require a great deal of algebra in order to solve them. Thus, you must be pretty familiar with algebra to be successful at chemistry. Second, you must always watch your units. Be sure that units cancel when they should and do not cancel when they shouldn't. The best way to do this is to write down all of your units in every equation. That way, you can see if they work out once you do the math. Try your hand at the following "on your own" problems to see how well you grasp this. You will have to use algebra, watch your units, and keep track of significant figures.

ON YOUR OWN

1.15 The density of silver is 10.5 grams per cm^3 . If a jeweler makes a silver bracelet out of 0.081 kg of silver, what is the bracelet's volume in mL?

1.16 A gold miner tries to sell some gold that he found in a nearby river. The person who is thinking about purchasing the gold measures the mass and volume of one nugget. The mass is 0.319 kg and the volume is 0.065 liters. Is this nugget really made out of gold? (Remember that the density of gold is 19.3 grams per mL)

Now that you have completed reading this module and have done the "on your own" problems, it is time for you to shore up your newfound skills and knowledge with the practice problems and review questions at the end of this module. As you go through them, check your answers with the solutions provided, and be sure you understand any mistakes you made. If you need more practice problems, you can find them in Appendix B of this book. Once you are confident in your abilities, take the test. You should try to score at least 70% on the test. If you do not, you should probably spend time reviewing this module before you proceed to the next one.

ANSWERS TO THE “ON YOUR OWN” PROBLEMS

$$1.1 \frac{12,321 \cancel{\text{g}}}{1} \times \frac{1 \text{ kg}}{1,000 \cancel{\text{g}}} = \underline{12.321 \text{ kg}}$$

$$1.2 \frac{0.121 \cancel{\text{L}}}{1} \times \frac{1 \text{ mL}}{0.001 \cancel{\text{L}}} = \underline{121 \text{ mL}}$$

$$1.3 \frac{640.08 \cancel{\text{cm}}}{1} \times \frac{0.01 \text{ m}}{1 \cancel{\text{cm}}} = \underline{6.4008 \text{ m}}$$

$$1.4 \frac{123.5 \cancel{\text{kg}}}{1} \times \frac{1 \text{ sl}}{14.59 \cancel{\text{kg}}} = \underline{8.46 \text{ sl}}$$

$$1.5 \frac{3.2 \cancel{\text{gal}}}{1} \times \frac{3.78 \text{ L}}{1 \cancel{\text{gal}}} = \underline{12.096 \text{ L}}$$

$$1.6 \frac{1,500 \cancel{\text{mL}}}{1} \times \frac{0.001 \cancel{\text{L}}}{1 \cancel{\text{mL}}} \times \frac{1 \text{ kL}}{1,000 \cancel{\text{L}}} = \underline{0.0015 \text{ kL}}$$

$$1.7 \frac{2.0 \cancel{\text{km}}}{1} \times \frac{1,000 \cancel{\text{m}}}{1 \cancel{\text{km}}} \times \frac{1 \text{ cm}}{0.01 \cancel{\text{m}}} = \underline{200,000 \text{ cm}}$$

$$1.8 \frac{10,000,000 \cancel{\text{mg}}}{1} \times \frac{0.001 \cancel{\text{g}}}{1 \cancel{\text{mg}}} \times \frac{1 \text{ Mg}}{1,000,000 \cancel{\text{g}}} = \underline{0.01 \text{ Mg}}$$

$$1.9 \frac{100,000 \cancel{\text{cm}}}{1} \times \frac{1 \cancel{\text{in}}}{2.54 \cancel{\text{cm}}} \times \frac{1 \cancel{\text{ft}}}{12 \cancel{\text{in}}} \times \frac{1 \text{ mile}}{5280 \cancel{\text{ft}}} = \underline{0.621371 \text{ miles}}$$

Since that's not even 1 mile, you should not be impressed by the braggart.

$$1.10 \frac{0.0045 \cancel{\text{kL}}}{1} \times \frac{1,000 \cancel{\text{L}}}{1 \cancel{\text{kL}}} \times \frac{1 \text{ mL}}{0.001 \cancel{\text{L}}} = 4,500 \text{ mL} = \underline{4,500 \text{ cm}^3} \text{ (mL and cm}^3 \text{ equivalent)}$$

1.11 The relationship between m and mm is easy:

$$1 \text{ mm} = 0.001 \text{ m}$$

Thus, to set up the conversion, we start with:

$$\frac{16 \text{ m}^2}{1} \times \frac{1 \text{ mm}}{0.001 \text{ m}}$$

This expression, however *does not cancel* m^2 . There is a m^2 on the top of the first fraction and only a m on the bottom of the second fraction. In order to cancel m^2 (which we must do to get the answer), we have to square the conversion fraction:

$$\frac{16 \text{ m}^2}{1} \times \left(\frac{1 \text{ mm}}{0.001 \text{ m}} \right)^2$$

Then we get:

$$\frac{16 \cancel{\text{m}^2}}{1} \times \frac{1 \text{ mm}^2}{0.000001 \cancel{\text{m}^2}} = \underline{16,000,000 \text{ mm}^2}$$

1.12 (a) All three non-zero digits are significant figures as are both zeros. One zero is between two significant figures while the other is at the end of the number to the right of the decimal. Thus, there are five significant figures.

(b) The first three zeros are not significant because they are not between two significant figures. The six is a significant figure, as is the last zero because it is at the end of the number to the right of the decimal. Thus, there are two significant figures.

(c) All digits are significant figures here. The first two zeros are between significant figures and the last one is at the end of the number to the right of the decimal. Thus, there are six significant figures.

(d) All digits are significant figures. The zero is between two significant figures. Thus, there are five significant figures.

1.13 (a) 2.6789×10^7 (b) 1.23×10^2 (c) 9.870×10^{-5} (d) 9.80×10^{-1}

1.14 (a) 345,600,000,000,000 (b) 1234.1 (c) 0.0000345 (d) 0.31

1.15 Since we are trying to calculate volume, we must first use algebra to rearrange the equation so that it begins “V=.” If you do not understand how this is done, go back to your algebra book and review these skills. You cannot be successful in this chemistry course without a command of algebra! The rearranged equation looks like:

$$V = \frac{m}{\rho}$$

We can't put our numbers into the equation yet, however, because the units would not work out if we did. In the end, we need an answer with a volume unit, so the mass units must cancel. Right now, however, density is in grams per cm³ while mass is in kg. So we first have to change kg into grams:

$$\frac{0.081 \text{ kg}}{1} \times \frac{1,000 \text{ g}}{1 \text{ kg}} = 81 \text{ g}$$

Now we can take 81 grams and divide by 10.5 grams per cm³. Remember from math that dividing is the same as inverting and multiplying, so:

$$V = m \times \frac{1}{\rho} = 81 \text{ g} \times \frac{1 \text{ cm}^3}{10.5 \text{ g}} = 7.7 \text{ cm}^3 = \underline{7.7 \text{ mL}}$$

The answer was rounded to 7.7 because there are only 2 significant figures in the mass measurement, and the volume unit was changed to mL since cm³ and mL are the same thing.

1.16 Since we only know gold's density in the units of grams per mL, our mass has to be in grams and our volume has to be in mL:

$$\frac{0.319 \text{ kg}}{1} \times \frac{1,000 \text{ g}}{1 \text{ kg}} = 319 \text{ g}$$

$$\frac{0.065 \text{ L}}{1} \times \frac{1 \text{ mL}}{0.001 \text{ L}} = 65 \text{ mL}$$

Now we can take these numbers and divide them to get the density:

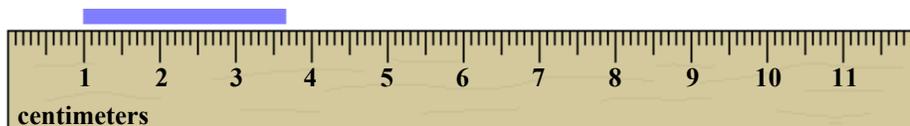
$$\rho = \frac{319 \text{ g}}{65 \text{ mL}} = 4.9 \frac{\text{g}}{\text{mL}}$$

The answer can have only 2 significant figures, since 65 has only 2 significant figures. Since the density did not work out to be 19.3 grams per mL, then we know that this is not gold. In fact, this density is equal to the density of iron pyrite, also known as “fool's gold” because it looks a lot like gold but has little value.

REVIEW QUESTIONS FOR MODULE #1

- Which of the following contains no matter?
 - A rock
 - A balloon full of air
 - A balloon full of helium
 - A lightning bolt
- List the base metric units used to measure length, mass, time, and volume.
- In the metric system, what does the prefix “centi” mean?
- Which has more liquid: a glass holding 0.5 kL or a glass holding 120 mL?
- How long is the bar in the picture below?

Illustration by Megan Whitaker



- Two students measure the mass of a 502.1 gram object. The first student measures the mass to be 496.8123 grams. The second measures the mass to be 501 grams. Which student was more precise? Which student was more accurate?
- How many significant figures are in the following numbers?
 - 0.0120350
 - 10.020
 - 12
 - 3.40×10^3
- A student measures the mass of an object as 2.32 grams and its volume as 34.56 mL. The student then calculates the density to be 0.067129629. There are two things wrong with the student's value for density. What are they?
- Why does ice float on top of water?
- Lead has a density of 11.4 grams per mL, whereas gold has a density of 19.3 grams per cc. If I were to make two identical statues, one out of gold and the other out of lead, which would be heavier?

PRACTICE PROBLEMS FOR MODULE #1

Be sure to use the proper number of significant figures in ALL of your answers!

1. Convert 1.2 mL into L.
2. Convert 34.50 km into m.
3. Convert 0.045 km into cm.
4. If an object has a volume of 34.6 mL, how many kL of space does it occupy?
5. A box is measured to be 2.3 m by 4.2 m by 3.5 m. What is its volume in cubic centimeters?
6. A nurse injects 34.5 cc of medicine into a patient. How many liters is that?
7. Convert the following decimal numbers into scientific notation:
 - a. 123.45
 - b. 0.0003040
 - c. 6,100,000
 - d. 0.1234
8. Convert the following numbers back into decimal:
 - a. 6.54×10^3
 - b. 3.450×10^{-3}
 - c. 3.56×10^7
 - d. 4.050×10^{-7}
9. Lead has a density of 11.4 grams per mL. If I make a statue out of 3.45 L of lead, what is the statue's mass?
10. Gold has a density of 19.3 grams per cc. If a gold nugget has a mass of 45.6 kg, what is its volume?