LIFEPAC®

MATH

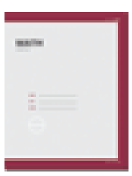
STUDENT BOOK

11th Grade | Unit 9

Alpha Omega
PUBLICATIONS

MATH 1109

COUNTING PRINCIPLES



LIFEPAC Test is located in the center of the booklet. Please remove before starting the unit.

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Counting Principles

Introduction

Counting is the oldest and most basic concept in math. Many important developments in counting principles have occurred in recent years. Computer technology has allowed solutions to previously impractical or unsolvable problems through these principles.

We shall begin this LIFEPAC® with a look at *progressions*, which represent real situations and are tools for more advanced math. *Permutations* and *combinations* are used in solving many problems that ask, "How many ways are possible?" These two concepts also provide the necessary background for the fascinating study of *probability*. Probability is one of the most important counting principles used in business and science.

Objectives

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIFEPAC. When you have finished this LIFEPAC, you should be able to:

1. Write the general term of a sequence.
2. Identify arithmetic and geometric series.
3. Use factorial notation.
4. Find the number of ways items can be arranged.
5. Find the number of ways a group can be subdivided.
6. Define probability.
7. Compare probabilities.

Survey the LIFEPAC. Ask yourself some questions about this study and write your questions here.

1. PROGRESSIONS

The **natural numbers** are those numbers you used when you first learned to count; that is, the positive integers. Notice that neither negative numbers nor zero nor fractions are included. The natural numbers begin with 1 but have no ending; hence, they are **infinite**.

The word *progression* can refer to either a **sequence** or a **series**. A sequence is an arrangement of quantities whose positions are based upon the natural numbers. A series is a summation of quantities based upon a sequence.

Section Objectives

Review these objectives. When you have completed this section, you should be able to:

1. Write the general term of a sequence.
2. Identify arithmetic and geometric sequences.

SEQUENCES

One type of progression is a sequence. After you have studied the definition of a sequence, the form for the general term of a sequence will be presented.

DEFINITIONS

Sequence: A group of numbers arranged in a definite order, with a specific first term.

Term: An individual quantity or number in a sequence.

Consider this sequence:

2, 4, 6, 8, 10

This sequence is different from

2, 6, 4, 8, 10

because the order is different even though the same numerals appear in both sequences.

Each individual quantity or number is called a **term**, and the terms are separated by commas. The ordering of the terms is a one-to-one **correspondence** between the terms and the natural numbers.

Although the position of a term in a sequence is given by a natural number, the term itself may be any number. The following sequences are all valid sequences.

Models: $1,\ 1\frac{1}{2},\ 2,\ 2\frac{1}{2},\ 3$

$-3,\ 0,\ 3,\ -2,\ 0,\ 2,\ -1$

$2a + 1,\ 2a + 2,\ 2a + 3,\ 2a + 4$

$\pi,\ \frac{1}{2}\pi,\ \frac{1}{3}\pi,\ \frac{1}{4}\pi,\ \frac{1}{5}\pi,\ \frac{1}{6}\pi$

$0.31,\ 0.61,\ 0.91,\ 1.21$

All the models given have had a last term; therefore, the sequences were **finite**. We can represent an infinite sequence by writing the first few terms followed by three periods. The sequence of positive odd integers could be written

$1,\ 3,\ 5,\ 7,\ \ldots$

We can also adapt this notation to finite sequences with many terms. For example, powers of 2 in order up to 256 are written

$2,\ 4,\ 8,\ 16,\ \ldots\ 256.$

Complete these activities.

1.1 The natural numbers are ______________________________ .

1.2 A sequence is ______________________________ .

1.3 Give an example of a finite sequence. ______________________________

1.4 Give an example of an infinite sequence. ______________________________

1.5 Why are the following numbers not a sequence? $\ldots -4,\ -2,\ 0,\ 2,\ 4,\ \ldots$

____

GENERAL TERM

Consider the following sequence:

$\frac{2}{3},\ \frac{3}{4},\ \frac{4}{5},\ \ldots$

We would probably expect that the fourth term of this sequence would be $\frac{5}{6}$. We say "probably" because to indicate a pattern simply by listing a few terms is not mathematically precise. If we only wrote three terms, we could not distinguish between the following two sequences:

$1,\ 2,\ 3,\ 4,\ 5,\ 6,\ \ldots$

and

$1,\ 2,\ 3,\ 1,\ 2,\ 3,\ \ldots$

Naturally, we want to be more precise. We can be more precise by writing the **general term** of the sequence.

The *general term* of a sequence is a formula that yields the value of a term when that term's position is substituted into it. In the sequence

$3,\ 5,\ 7,\ 9,\ \ldots\ 2n + 1,\ \ldots$

the first term is $(2 \cdot 1) + 1 = 3$, the second term is $(2 \cdot 2) + 1 = 5$, the 103rd term is 207, and so on. Computations of any term given the general term is almost trivial. Often a sequence is simply referred to by its general term since the sequence is completely described by the general term. Thus

$3,\ 6,\ 9,\ 12,\ \ldots\ 3n,\ \ldots$

is referred to as the sequence $3n$.

DEFINITION

General term of a sequence: A formula that yields the value of a term when the term's position in the sequence is substituted into the formula.

To write the general term from a partial listing of terms of a sequence or from a pattern described in words or phrases, however, is *not* trivial. A unique general term may not exist for a partial listing of terms, since any formula that fits this partial list is a correct one. Usually, however, the pattern is clear from the context.

The natural numbers are

$1,\ 2,\ 3,\ 4,\ \ldots\ n,\ \ldots$

and the odd positive integers are

$1,\ 3,\ 5,\ 7,\ \ldots\ 2n - 1,\ \ldots$

Notice the general term was obtained by subtracting 1 from the general term for the even positive integers. The sequence

$-1,\ 1,\ -1,\ 1,\ -1,\ 1,\ \ldots$

has $(-1)^n$ as its general term. Thus, we can use either $(-1)^n$ or $(-1)^{n+1}$ as a factor in the general term of any sequence that alternates in sign. Which factor we use depends on whether the first term is negative or positive.

If you determine that subtracting any two successive terms of a particular sequence always gives a constant, then the general term of that sequence will have that constant as a factor of n.

$7,\ 10,\ 13,\ 16,\ \ldots\ 3n + 4,\ \ldots$

If the result of dividing successive terms of a particular sequence is a constant, then the general term of that sequence will have that constant raised to a power of n.

$3,\ 9,\ 27,\ 81,\ \ldots\ 3^n$

HINTS FOR FINDING THE GENERAL TERM

A.	Alternating sign:	$(-1)^n$ or $(-1)^{n+1}$ as a factor
B.	Constant difference:	constant as a factor of n
C.	Constant ratio:	constant raised to a power of n
D.	Common factor:	factor in general term

Complete these activities.

1.6 Write an example of two sequences that have the same first four terms, but different terms thereafter.

____

____

1.7 What are the sixth and seventh terms of the following sequences?

a. $5,\ 8,\ 11,\ 14,\ \ldots\ 3n + 2,\ \ldots$ ____________ ____________

b. $a,\ -a,\ a,\ -a,\ a,\ \ldots\ (a)(-1)^{n+1},\ \ldots$ ____________ ____________

c. $1,\ \frac{1}{8},\ \frac{1}{27},\ \frac{1}{64},\ \ldots\ \frac{1}{n^3},\ \ldots$ ____________ ____________

d. $4,\ 16,\ 64,\ 256,\ \ldots\ 4^n,\ \ldots$ ____________ ____________

1.8 Write an example of an alternating sequence.

____

1.9 Write a general term for these sequences.

a. $2,\ 4,\ 8,\ 16,\ \ldots$ ______________________

b. $50,\ 100,\ 150,\ 200,\ \ldots$ ______________________

c. $6a,\ 3a,\ 0,\ -3a,\ -6a,\ \ldots$ ______________________

d. $-0.1,\ +0.2,\ -0.3,\ +0.4,\ \ldots$ ______________________

e. $\frac{4}{3},\ \frac{5}{4},\ \frac{6}{5},\ \frac{7}{6},\ \ldots$ ______________________

SERIES

A **series** is a summation of terms of a sequence. For example,

$$1,\ 3,\ 5,\ 7,\ \ldots\ 2n - 1,\ \ldots\ 13$$

is a sequence; and

$$1 + 3 + 5 + 7 + 9 + 11 + 13$$

is the associated series.

DEFINITION

Series (plural, *series*): The summation of the terms of a sequence.

NOTATION

A term of a series or the general term of a series is the same as the corresponding term in its sequence. A series may be finite or infinite just as a sequence. We indicate infinite series in a similar manner to sequences:

$$3 + 6 + 9 + \ldots 3n + \ldots$$

A convenient notation for summation is the Greek letter sigma

$$\sum$$

followed by the general term. The notation shown

$$\sum_{n=1}^{5} (4n - 3) = 1 + 5 + 9 + 13 + 17$$

is read, "The sum of $(4n - 3)$ as n varies from 1 to 5." An infinite series in summation notation, as shown,

$$\sum_{n=1}^{\infty} (4n - 3) = 1 + 5 + 9 + \ldots (4n - 3) + \ldots$$

uses the symbol ∞ for infinity and is read, "The sum of $(4n - 3)$ as n increases without end."

Complete these activities.

1.10 Define *series*.

____

____

1.11 Write an example of

a. an infinite series ______________________________

b. a finite series ______________________________

1.12 Write the two examples in Problem 1.11 using the summation notation.

a. ____

b. ____

1.13 Write out the series given by

$$\sum_{n=1}^{6} \frac{n}{2n + 1}$$ ______________________________

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