

# Daniel and the Old Lion Hunter

No fewer than 120 of the descendants of the mathematical Bernoullis have been traced genealogically, and of this considerable posterity the majority achieved distinction—sometimes amounting to eminence....None were failures. The most significant thing about a majority of the mathematical members of this family...is that they did not deliberately choose mathematics as a profession but drifted into it in spite of themselves.

—E. T. Bell (1883–1960), British mathematician, *Men of Mathematics*

Of course these flying molecules must beat against whatever is placed among them, and the constant succession of these strokes is, according to our theory, the sole cause of what is called the pressure of air and other gases. This appears to have first been suspected by Daniel Bernoulli, but he had not the means which we now have of verifying the theory.

—James Clerk Maxwell (1831–1879), Scottish mathematician, “Molecules”

**D**aniel Bernoulli (ber-NOO-lee) wanted to be the Newton of the eighteenth century; he thought he could do it by studying fluids (by that he meant liquids and gases).

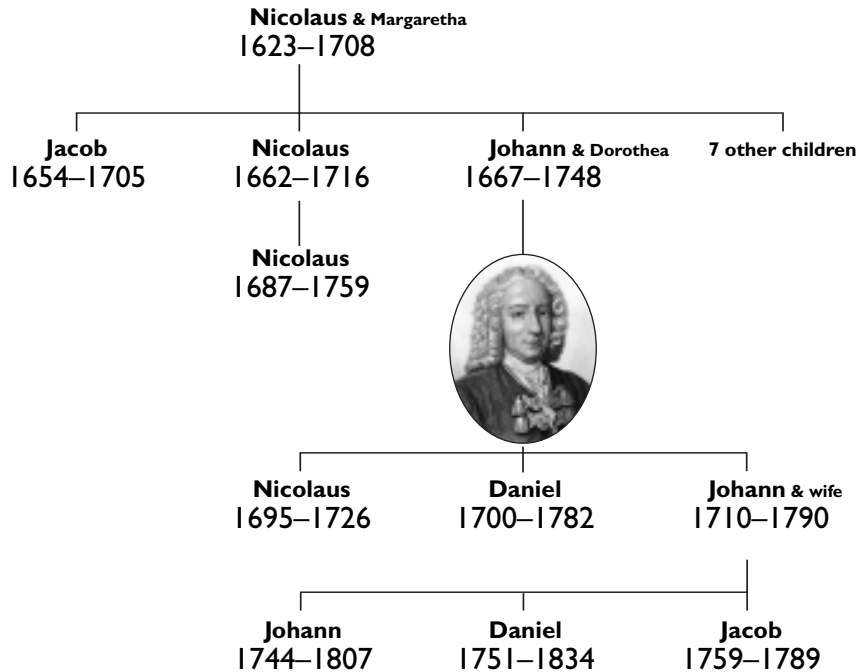
Daniel, who was born in 1700, had the genes to go for it. His father and an uncle were world-famous mathematicians. Another uncle, a cousin, an older brother, and some assorted relatives in this remarkable family were first-rate mathematicians. So it was no surprise that Daniel could deal with numbers at an amazingly early age. But his career wasn't as easy as you might think: his father, Johann, was not your normal, loving dad. He was jealous, nasty, and miserable—you'll see.

Who is the lion hunter in the chapter title? And who is the lion? There's a clue on page 179.

It is no accident that the words “ingenious” and “engineer” derive from the same root (*ingenium*: mental ability, cleverness, a naturally clever temperament).

—Lisa Jardine, English professor, *Ingenious Pursuits*

## THE BERNOULLI FAMILY



Almost all the Bernoullis were achievers; this family tree includes just the most prominent among them. Nicolaus (top), the patriarch, wasn't a mathematician or scientist. He was a spice merchant and druggist, like his father, Jacob the Elder. Nicolaus and his wife, Margaretha, had 10 children, including the feuding mathematician brothers, Jacob and Johann (the baby of that family). Johann and his wife, Dorothea, had three mathematician/scientist sons (including Daniel) and three grandsons who were mathematicians.

The Bernoulli story begins in Belgium with Jacob the Elder (Daniel's great-grandfather), who was a Huguenot, which means he was a Protestant follower of John Calvin—not a safe thing to be in seventeenth-century France or Belgium. Huguenots were branded, maimed, and sometimes killed by the majority Catholics. Many left for Switzerland or the Americas, which were havens for religious outcasts. Jacob fled to Basel, Switzerland in 1622, where he established a spice and drug business and prospered.

His son Nicolaus, also a prosperous druggist, has high hopes for his children. (He has 10 kids.) Jacob and Johann are expected to carry on the family traditions: Jacob is to study religion; Johann is to be a druggist. But both



"Which Bernoulli do you wish to see—  
'Hydrodynamics' Bernoulli, 'Calculus' Bernoulli,  
'Geodesic' Bernoulli, 'Large Numbers' Bernoulli, or  
'Probability' Bernoulli?"



are fascinated with numbers and natural philosophy. Jacob, almost 13 years older than his brother, writes in his diary, “Against my father’s will I study the stars.”

Johann goes into business, but he doesn’t have a talent for it, and soon even his father can see that. So Nicolaus agrees to let Johann study medicine; it has a connection to the pharmacy business. But like his brother Jacob, Johann is secretly studying numbers. “I’ve now turned to mathematics,” he writes in his diary, “for which I feel a special joy.”

When Daddy Nicolaus finds out what his sons are doing, he is furious. He says they will have to find jobs; he is not going to support their studies any longer. In 1676 Jacob leaves for Geneva, and 7 years later he becomes a professor of mathematics at the University of Basel. Brother Johann becomes chair of the mathematics department at Groningen University in the Netherlands in 1695.

Meanwhile, the German mathematician Gottfried Wilhelm Leibniz (LYB-nits) publishes a paper on calculus (in 1684). As you know (I hope), Isaac Newton already worked out calculus in 1666—but he still hasn’t published his results. Two great minds have independently found the same thing, which is not so unusual. But Newton feels his ideas have

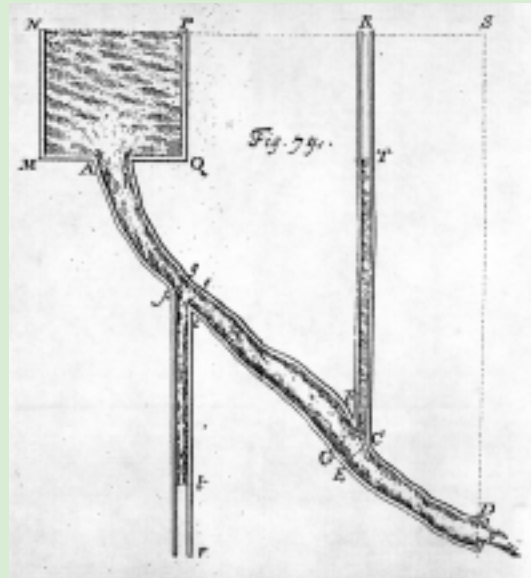


Think of Gottfried Leibniz as a universal genius, and you won’t be wrong. He wrote about math, philosophy, science, history, law, and politics, and has been called the Aristotle of the Seventeenth Century.



### EULER: A LEAGUE OF HIS OWN

Outstanding as the Bernoullis were in mathematics, they aren't even in the same league as Leonhard Euler (say "oiler"), who was Johann's student and Daniel's friend. If this were a book on mathematics, there would be several chapters on Euler. But it isn't, so you'll have to find out about him on your own. As it happens, he was a kind, generous man (in contrast to grumpy Johann Bernoulli and miserable, secretive Isaac Newton). Euler loved to tell stories to his 13 children. William Dunham, in a book called *Journey Through Genius*, says, "The legacy of Leonhard Euler... is unsurpassed in the long history of mathematics.... Euler's collected works fill over 70 large volumes, a testament to the genius of this unassuming Swiss citizen who changed the face of mathematics so profoundly."



Daniel Bernoulli and Leonhard Euler found a way to measure the pressure of a fluid flowing in a pipe (A to D, above). The greater the pressure, the higher the liquid rose (T) in the glass tube (C). Doctors began measuring blood pressure by sticking a tube into a patient's artery.

been stolen. Some years later, Newton accuses Leibniz, saying that Leibniz had seen some of his (Newton's) work; he releases their correspondence. Leibniz retaliates by suggesting that it was Newton who actually used his (Leibniz's) methods.

Along come the two Bernoulli brothers. They read Leibniz's paper on calculus and at first have a hard time understanding it. (For more on calculus, see pages 160–163.)

The Bernoullis study Leibniz again and again until they figure out calculus for themselves; then they get involved in the Newton-versus-Leibniz argument—on Leibniz's side. To them (and some others), it seems to be British science against the science of the European continent. Johann takes a leading role in the controversy, writing with no modesty, "When in England war was declared against Monsieur Leibniz for the honor of the first invention of the new

## TAKING A GAMBLE WITH MATH

Probability, a branch of mathematics, began with gambling. Pierre de Fermat (of the famous Last Theorem), Blaise Pascal, and the Bernoullis wanted to know the mathematical odds of winning at the card table. Probability didn't tell them for certain that they would or wouldn't draw an ace; it just told them how likely it was. A deck of 52 cards has 4 aces, so the odds of the first drawn card being an ace are 4 in 52 (or 1 in 13). If 20 cards have been played and not an ace among them, those odds improve to 4 in 32 (1 in 8).

Always keep in mind that probability is about the *likelihood* of outcomes, not the *certainty*. If there are only 4 cards left in the deck, and no aces have been played, you can predict with certainty that the next card will be an ace—but you're not using probability; you're using fact.

Probability is central to the physics that deals with the complex world inside atoms. We can't determine the action of an individual particle, but with a large number of atoms, predictions based on probability become very accurate.



Card games were widely popular in seventeenth-century France. Here, workers are making playing cards by hand in about 1680.

calculus of the infinitely small, I alone . . . kept at bay. . . the entire English army.”

Fighting Isaac Newton doesn't seem to be enough for them: the brothers take to belittling each other's work. One prestigious mathematics journal prints their vitriol until finally the editor has had enough.

But the brothers also do serious mathematics. People of their time are fascinated with games of chance. When you roll of a pair of dice, can mathematics predict what numbers you'll roll? No. But looking for patterns in cards and in dice makes the Bernoullis realize that overall patterns can be described statistically. And that leads them to help develop a language of mathematics that deals with probabilities. (Probabilities will become essential in understanding the



action of atoms.) The Bernoulli work in this field is 100 years ahead of the times.

Now, let's skip to Daniel's birth. He is the son of Johann, the younger brother. When he is five, his uncle Jacob dies. "This unexpected news bowled me over," says his daddy, Johann, who adds, "and then it entered my thoughts immediately. . . that I could succeed to my brother's position." So the family moves from Holland to Switzerland. Daniel grows up in Basel, where Johann is a university professor.

Johann begins acting just as his father, Nicolaus, had. Johann decides his son Daniel should become a merchant and enter the family pharmacy business. But Daniel wants to study mathematics. He fails as a pharmacist. (Does this sound familiar?)

Johann then insists that his son Daniel go to medical school, but he does allow him to study mathematics on the side, and he does answer his questions. And since Johann is one of the best mathematicians in the world, Daniel gets very good training.

One of the things that preoccupies the great professor Johann Bernoulli is a little-studied phenomenon called *vis viva* in Latin. It means "living force" and is what we call the energy of motion. No one understands it, but Daniel is fascinated. *Vis viva* is invisible but clearly powerful.

What do editors do? Here's an example: On the opposite page (line 5), I used the word *vitriol*. If you read the last chapter, you know what it means. But it wasn't the first word I chose to describe the Bernoullis' angry words. I called them "diatribes." This is what my editor wrote: "Joy, I find that, back then, a diatribe was just a discourse or study; it came to mean a bitter criticism in English in 1804. Rant would be an excellent choice because it's from Shakespeare, with a Dutch root." Good comment, but somehow *rant* didn't work for me. I tried *screed*, *harangue*, *tirade*, *censure*, *denunciation*, *attack*, and *invective*—but decided a word with a double meaning—like *vitriol*—was just what I wanted. That's what good editors do (like good teachers): they make you think.



That's the city of Basel, Switzerland (left), flanked by mountains and halved by the mighty river Rhine. Note how many (30!) churches appear in this 1740 engraving.

In a paint sprayer, high-speed air from a horizontal tube draws paint up a vertical tube. How? Try this: Snip a narrow straw *almost* all the way through and bend it at a 90° angle. Put the bottom end in water and blow—hard!—into the horizontal top half. Your breath zips across the open bend, reducing the air pressure and causing water to rise up the straw.



When Daniel finishes medical school—with top grades—he expects to get a professor's job in Basel. He gets no help from his father. Daniel finally takes a position in Russia as a mathematician at the Imperial Academy of Sciences and Arts in St. Petersburg. His experiments and writings there soon make him widely known; and he becomes a quiet fan of Isaac Newton. He doesn't realize it, but his father is fuming.

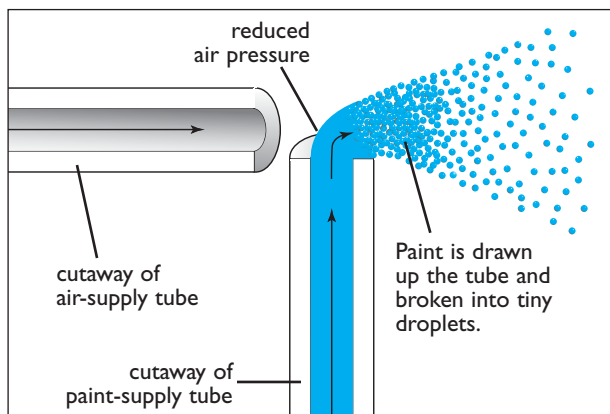
In 1734, both Daniel and his father, Johann, write papers for the Paris Academy of Sciences, which gives a big prize (much like today's Nobel Prize). That year, the top prize is split; it is awarded to the two Bernoullis—father and son. Daniel comes home to Basel. He thinks his father will be pleased. But Johann is furious. He says his son is trying to take over his position as Europe's top mathematician. Johann throws his son out of the house. Daniel never returns.

Now all of that is like gossip—interesting but not really important. What Daniel accomplishes becomes a landmark in science. Like so many achievements, it sounds simple, but no one else has figured it out.

Daniel Bernoulli comes up with a famous and very useful principle. It applies to moving fluids (liquids or gases) and the pressure put on them. (Don't confuse it with Boyle's Law, which is about gases that are compressed; Bernoulli's Principle relates to fluids that are flowing and incompressible.) Here it is: **When the speed of a moving fluid increases, pressure in the fluid decreases, and vice versa.**

If that theorem interests you, consider a career in

engineering. You can think about designing airplanes or ships or even bridges if you understand Bernoulli's Principle. If you want to build a carburetor or an atomizer (a sprayer, like on a perfume bottle), where air and liquid droplets are the moving fluid, you'll use Bernoulli's Principle. In an aspirator (a suction pump), it is water (or another liquid) that does what Bernoulli said it should do.



The principle, in simple language, is this: The faster a fluid (liquid or incompressible gas) is traveling, the lower its pressure. Here it is stated scientifically: **The pressure in a fluid varies inversely with its speed squared.**

That's what causes a house to lose its roof during a hurricane. The fast hurricane winds reduce the pressure on the top of the roof, and the air pressure *inside* the house pushes the roof off.

Engineers designing airplane wings know (thanks to Bernoulli) that the air flowing over the upper surface of an aircraft wing needs to travel at a faster rate than air flowing beneath the wing. Therefore, airplane wings are usually

Bernoulli's Principle doesn't describe the flow of fluids at or near the speed of sound. Supersonic fluids behave differently than those at slower velocities. Their density, temperature, and compressibility changes.

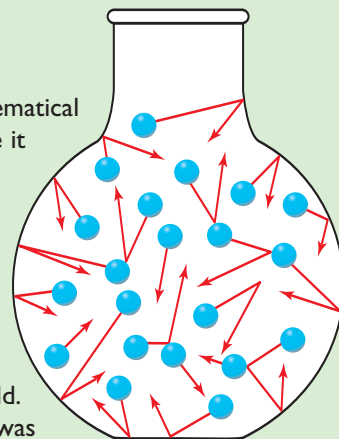
### A BIG, IMAGINATIVE KINETIC IDEA— BEFORE SCIENCE WAS READY FOR IT

Daniel Bernoulli made a creative conjecture (guess or speculation). He said that a container of gas is mostly empty space and that the atoms making up the gas are always in rapid, chaotic motion bumping against each other and against the walls of their container. That turned out to be true. It is called the Kinetic Theory of Gases (*kinetic* is from a Greek word that means "motion"). But it would be more than 100 years before his conjecture could actually be proved.

Bernoulli's ideas didn't have much luck. It took a long time before they were accepted. Maybe it was because Bernoulli's writings were

mostly in mathematical formulas. Maybe it was because he wasn't English, and for a while there was a rivalry between English science and that of the rest of the world.

But I believe it was because his ideas came before the technology to prove them.



### KINETIC THEORY: MOLECULES IN MOTION

Bernoulli realized that when you heat a gas, its pressure increases. Heating the gas makes its atoms and molecules move faster, and they wham the sides of the container with more vigor.

The Kinetic Theory states that all atoms and molecules—in gases, liquids, and solids—are in constant, vigorous motion. According to Brockhampton's *Dictionary of Science*, "At standard temperature and pressure... a litre

of air contains 30,000 million million million molecules, moving at an average speed of 450m/1,500 ft per second. Each molecule undergoes 5,000 million collisions every second."

Bernoulli was way ahead of his time in analyzing heat. Most scientists of his day thought of heat as an invisible substance; they had no idea it was a form of kinetic energy.



## WHY DO PLANES STAY IN THE SKY?

What explains flight? Well, it happens that physicists argue about that. Some cite Bernoulli's Principle, and that does have something to do with it. Air travels faster over the top of the curved wing than under the flat bottom. The slower moving air below the wing exerts more pressure

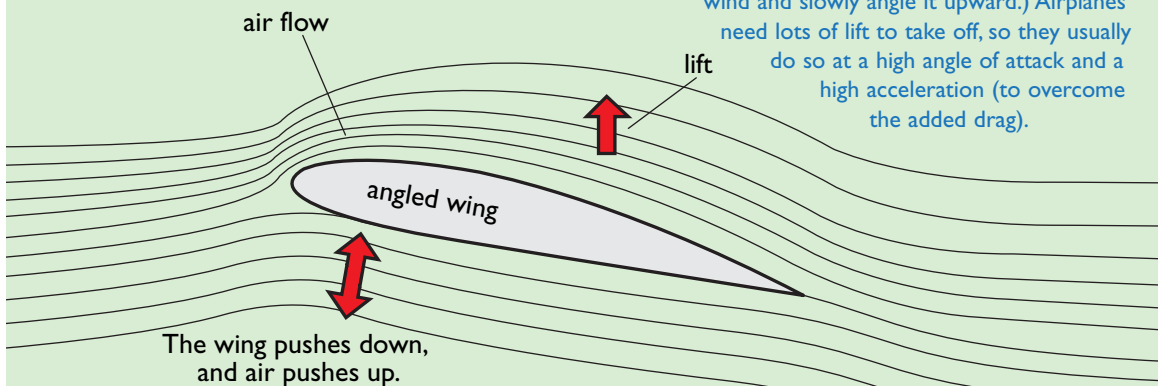
than the faster moving air on top, creating lift.

Airplane designers create curved wings to take advantage of this boost. Yet even a plane with flat wings or a plane that's upside down (with the curved side of the wings facing the ground) can still fly. The reason is Isaac Newton's third law: For every action there's an equal and opposite reaction. The wing is angled, and so its downward force on the air creates an equal and opposite upward force. This makes more sense if you never forget that air has mass. Think of an airplane riding on a massive cushion of air. If it dives, nose first, it slices through the air quickly, but then, as it flattens out, more of the airplane's surface pushes down on the air (there's more air resistance), and the plane bounces back up with more force.

The faster a plane is moving, the more force it exerts on the air (and vice versa). That's why, when an airplane slows down too much, it can stall out—stop and tumble from the sky.



Here's the simple reason airplanes can fly: Airplane wings push down on air, which pushes back. This plane just skimmed over some fog. Its downwash (downward motion of air) is recorded as a trough; the upwash is apparent in the rising, curled edges.



The angle of attack is the angle at which a wing slices through the air. Even a small  $5^\circ$  or  $10^\circ$  tilt adds enough air resistance (drag) under the wing to push it up. The greater the angle of attack, the greater both the drag and the lift. (To feel this for yourself, hold a cookie sheet edge-first into a strong wind and slowly angle it upward.) Airplanes need lots of lift to take off, so they usually do so at a high angle of attack and a high acceleration (to overcome the added drag).

designed so that the upper surface of the wing is curved, which means air passing across it will move faster than the air flowing along the straight bottom. That makes the pressure on top of the wing lower than that below. And that helps the airplane to lift. (There are other factors involved with lift; see page 228.)

Bernoulli's Principle also led to a conservation law. Here it is: **The total energy in a fluid stays the same no matter what shape the fluid takes.** If liquid goes from a big bottle into a smaller container, the speed and pressure of its atoms will change but its total energy will not.

Daniel Bernoulli goes even further than Robert Boyle in anticipating atoms. He says that gases are made up of a vast number of tiny particles, and it is the random, constant motion of those atoms hitting the walls of a container that explains pressure in a gas. (Later this will be called the Kinetic Theory of Matter.) It is a remarkable deduction, since no one in his time can be sure of atoms. The atomic idea, which goes back to the ancient Greeks, has mostly been discarded. But Bernoulli believes atoms exist. (Like Newton and Boyle, he thinks that atoms are hard, solid, impenetrable bits of matter—which they aren't.)

Believing in atoms and figuring out that they are in constant motion—well, that is an astonishing achievement. It will be several generations before there are tools to prove it true. Writing about atoms—getting the idea down on paper as Boyle and Bernoulli do (even if they called them “corpuscles”)—means others will consider them.

This is happening at the same time that some people are saying science is a closed field. They think that Newton has said everything that needs to be said on the subject of science. They are wrong (understatement)! Energy is a wide-open field. So is the chemical world of elements. Atoms are going to be verified, and that will change the whole way we look at matter. Physics is just gearing up, and mathematics will provide its key tool.

Daniel Bernoulli seems to sense those possibilities (and probabilities). He isn't an Isaac Newton, but he is impressive. His thoughts and his principle will percolate in other heads and help bring changes in science and lift to technology.

**In physics, CONSERVATION basically means that nothing is lost or gained. More on that idea coming up.**

### OH, THOSE MATHEMATICIANS!

*In Mathematicks he was  
greater  
Than Tycho Brahe, or  
Erra Pater:  
For he by Geometrick  
scale  
Could take the size of  
Pots of Ale;  
Resolve by Signes and  
Tangents straight,  
If Bread or Butter wanted  
weight;  
And wisely tell what hour  
o'th day  
The Clock does strike,  
by Algebra.*  
—Samuel Butler  
(1612–1680), English  
poet, *Hudibras*, Part One

Hudibras is the comic hero of the poem, which makes fun of the serious Puritans. He is a buffoon, but the poem gives you an idea of the esteem in which mathematicians were held in his time. (This Samuel Butler is not to be confused with a more famous one who lived in the nineteenth century.)