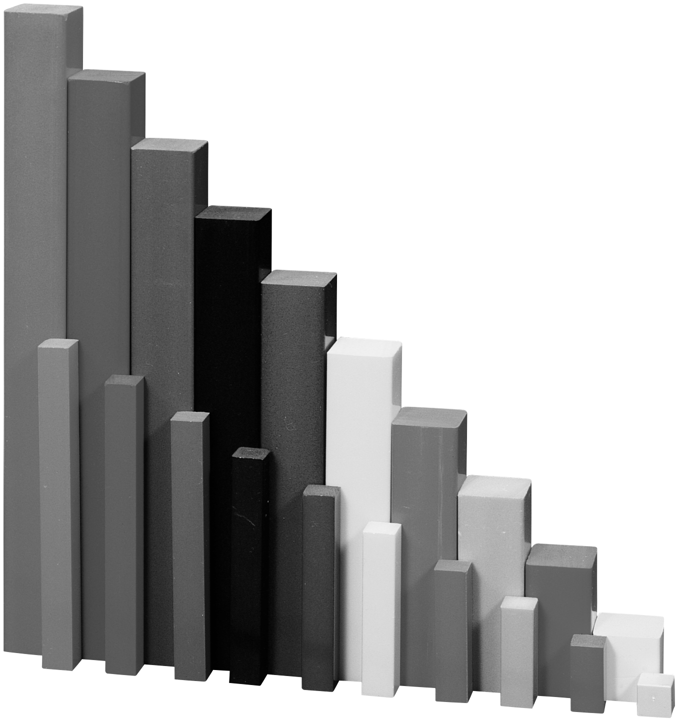
Learning About...®

Cuisenaire® Rods

A Guide to Teaching Strategies, Activities, and Ideas



ETA hand2mind®
hands-on learning for growing minds

INTRODUCTION

This *Learning About®… Cuisenaire® Rods* Activity Guide provides hands-on activities and ideas for leading students in an active exploration of the world of mathematics. The activities involve students in the process of exploring and investigating abstract math concepts through the use of manipulatives. Students are encouraged to think critically, plan strategies, and share conclusions.

The activities, described in detail, have proven successful in presenting and developing mathematical concepts at a wide variety of grade levels. The information included suggests ways both to deal with classroom organization and to introduce, investigate, and summarize the mathematics involved. These activities explore—

- whole numbers
- arithmetic operations
- fractions
- measurement
- ratio and proportion
- symmetry
- congruence
- three-dimensional geometry
- patterns and functions

We sincerely thank Alison Abrohms, Marilyn Burns, Patricia Davidson, Marji Freeman, Sandra Nye, Brenda Sue Pignata, Stephanie Sheffield, and Rosamond Welchman-Tischler for their contributions to this *Learning About… Cuisenaire Rods* Activity Guide. We hope that you find the lessons exciting and valuable and that they spark additional ideas in you and your students.

ABOUT CUISENAIRE RODS

Cuisenaire Rods help students relate abstract ideas about numbers and shapes to something students can see and touch. They make mathematical ideas easier to internalize and frequently allow students to understand and solve problems they might not otherwise be able to solve. Cuisenaire Rods is a collection of rectangular rods in 10 colors—white, red, green, purple, yellow, dark green, black, brown, blue, and orange—with each color corresponding to a specific length from 1 centimeter to 10 centimeters. The standard notation for Cuisenaire Rods is w (white), r (red), g (green), p (purple), y (yellow), d (dark green), k (black), n (brown), e (blue), σ (orange). Because black, brown, and blue all begin with *b*, their last letters are used. A "script" σ is used for orange to avoid confusion with zero. One set of 74 rods is sufficient for two to four students.

Cuisenaire Rods provide a continuous model of number, rather than a discrete one. Thus, they allow you to assign a value to one rod and then to determine the values of the remaining 9 rods by using the relationships between the rods. For example, when the white rod is given a value of 1, the orange rod, which is ten times as long, has a value of 10; if the white rod is given a value of 2, the orange rod is 20; if the orange rod is 1, the white is $\frac{1}{10}$.

GETTING TO KNOW CUISENAIRE RODS

Students should be given ample time to freely examine, explore, and become acquainted with the rods before beginning guided activities. Students are likely to make designs, create shapes and pictures, and build three-dimensional structures. They will begin to notice rod attributes and relationships—for instance, that all blue rods are the same size, that two red rods equal a purple rod, or that a purple rod is one white rod less than a yellow rod.

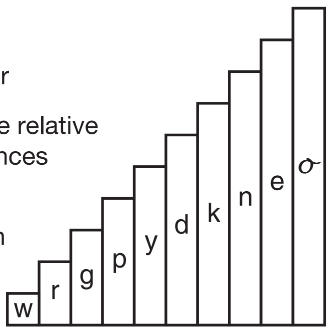
ROD STAIRCASES

Group Size: Teams of two students

Additional Materials Needed Per Group: Ruler

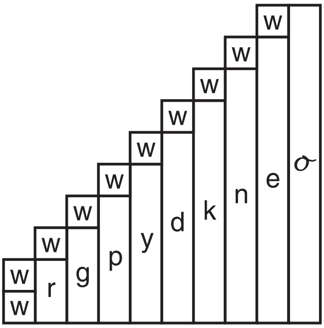
Objective: Use Cuisenaire Rods to compare the relative lengths of the various rods and to find equivalences in rod lengths

Procedure: Ask students to lay one rod of each color flat on their desks to make a staircase. You might provide rulers to students to enable them to align the rods so that they are in a straight line.



Ask—

- *What do you notice about the length of each consecutive rod?* [Each rod is one white rod longer than the previous rod.]
- *How could you prove that each consecutive rod is one white rod longer than the subsequent rod?* [Sample response: Start at the top of the staircase and add one white rod to each subsequent rod after the top step.]



Ask—

- *How much is each rod worth if the white rod is assigned a value of one?* [Sample responses: The red rod is worth two because it is twice as long as the white rod. It takes two white rods to make a red rod, and 1 + 1 = 2.]

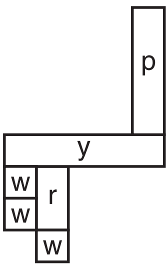
To reinforce finding a value for each rod, assign the white rod a value of two. When students have found the values, again ask them to explain their reasoning. In this way, students clarify their thinking and are more likely to remember the relationships they discover.

BUILD WHAT I HAVE

Group Size: Teams of two students

Objective: Use Cuisenaire Rods to increase students' familiarity with the rods, to introduce or reinforce mathematical vocabulary, and to improve students' communication and listening skills

Procedure: This activity is appropriate for students in grades 2–8. One student secretly arranges rods in a particular way and then gives verbal instructions so that the other student reproduces the arrangement. To model this activity for the class, choose 6 to 10 rods of a variety of colors (such as 1 yellow, 1 purple, 1 red, and 3 white). Have each student select the same set of rods from their supply. Then, using some or all of your rods, build a structure, keeping it hidden from the class.



Explain that students will try to reproduce a structure that you make by following the directions you give. Ask students to set up barriers, such as books, that will keep their workspaces private. Once students are ready, begin your directions, describing your structure in detail, using color, direction, shape, placement, etc. For example, you might say, “Place a yellow rod flat on your desk so that it is horizontal. Now put a purple rod to the right of it so that the rods are perpendicular. Make the two rods touch so that they look like a capital L on its side.” Allow students to ask questions at any time. Use correct mathematical language to enable students to learn the correct vocabulary in context.

After all the directions have been given and everyone has finished, ask students to remove the barriers and compare their structures with one another. Give students time to discuss among themselves any differences they see and then report back to the class.

Ask—

- *Why are some of the structures that you created different from other students’ structures?* [Sample responses: I didn’t understand some of the words you used (such as perpendicular or vertical). Some of the words you used had several meanings (such as on top, over, or below).]
- *Which words were helpful, and which were confusing?*

Finally, reveal your structure for comparison.

Now, teams of two students play Build What I Have. Once they’ve selected the rods they want to use, are positive each person has an identical set, and have put up barriers, one student creates a structure and describes it for his or her partner to build. Remind students that the object is not to trick each other, but rather for each person to end up with the same structure. When they’re ready, students remove the barriers and compare their results, discussing the directions that were given. Have them focus on which directions were helpful, which directions may have led to differences, and alternative directions that might have been given. Have students reverse roles and repeat the exercise.

After students have had an opportunity to do this activity several times, ask—

- *In what ways was this activity easy? In what ways was it difficult?*
- *Was it easier to be the person creating the building or the person giving the directions? Why?*
- *Was it helpful to be able to ask questions as you built your structure? Why?*
- *What mathematical words or phrases did you find most useful?*

FINDING ALL THE TRAINS

Group Size: Teams of two students

Objective: Use Cuisenaire Rods to investigate addends, patterns, and permutations

Procedure: Ask students to place one green rod in front of themselves and to line up all the rods, like cars on a train, that equal it in length. [one green, three whites, one white and one red, and one red and one white]. Explain that trains having the same rods in a different order are considered to be different trains.

Ask groups to build and record all the trains equal to one purple rod. Next, ask groups to build and record all the trains equal to one yellow rod and then to one dark green rod. (There are 8 combinations for purple, 16 for yellow, and 32 for dark green.)

Ask students to look for patterns that can help determine how many trains are possible for any rod. Younger students can record by coloring their solutions on one-centimeter grid paper. Older students can create their own recording systems and share them in a follow-up class discussion. Some students may sketch each solution; others may use grid paper; others may make lists, using abbreviations of their own or the standard ones. Some students may even use number sentences.

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Ask—

- *What strategies did you use to solve these problems?*
- *How did you record your findings?*
- *What patterns did you notice as you solved these problems?* [Sample response: There were twice as many possible trains for the purple rod as for the green rod.]

- *If you noticed that there were twice as many possible trains for the purple rod as for the green rod, did you then assume that the yellow rod had twice as many possible trains as the purple rod?*
- *Did those of you who suspected that doubling might be the pattern test to see if this pattern holds true for rods shorter than green?*

After the discussion, show students the recording scheme that mathematicians often use when investigating patterns. Point out that listing the rods in order of increasing size makes patterns easier to discern.

CONNECTING NUMBERS TO THE RODS

Group Size: Teams of two students

Objective: Use Cuisenaire Rods to represent rod trains, using addition notation

Procedure: Select one of the trains they recorded in the previous activity for the green rod, such as the red-white train. Write 2 + 1 = 3 on the board or overhead.

Ask—

- *Why is this a way to describe the red-white train?* [The white rod is worth 1 and the red rod is worth 2, so the sum of the white rod and the red rod is the green rod, which is worth 3.]

Select another train students wrote for the green rod and ask students to write the number sentence or equation that describes it. Continue until students have written a number sentence or equation for each train: 2 + 1 = 3; 1 + 2 = 3; and 1 + 1 + 1 = 3.

Next, have students write number sentences or equations for each train they recorded for the purple rod and yellow rod. For students in the primary grades, this activity is a way of relating the concrete rod trains to the appropriate addition notation. For students in all grades, it is a way to connect the commutative property to something concrete—just as the order of the rods does not affect the length of the train, the order of the addends does not affect the sum.

TRAINS OF ONE COLOR

Group Size: Teams of two students

Objective: Use Cuisenaire Rods to investigate multiples, prime numbers, and multiplication

Procedure: In this activity, students investigate to find different ways in which one rod can be replicated by using multiple rods of only one color. Give each pair one set of Cuisenaire Rods. Model the activity with the orange rod.

Ask—

- *How many white rods does it take to equal the length of one orange rod?* [10]
- *How many red rods does it take to equal the length of one orange rod?* [5]
- *How many yellow rods does it take to equal the length of one orange rod?* [2]
- *Can you find multiples of any other color rods that equal the length of one orange rod?* [no] *Why not?* [Sample responses: Only groups of the white, red, and yellow rods can be used to equal the length of the orange rod. Ten is a multiple of only 2 and 5.]

To record their findings, have students write the name of the rod they were replicating (orange) and list below it all the ways they found to replicate the rod with rods of one color. Approaching problems systematically not only helps children keep track of where they are but also helps them identify patterns.

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Alternatively, students can set up a chart, as shown below.

Rod Color	Number
white	10
red	5
yellow	2

Have pairs of students continue this activity by finding all the ways to make trains of one color that are equal in length to each of the nine remaining rods and then recording their solutions. When students have completed the activity, discuss their findings.

Ask—

- *Why can every other rod be made with two rods of one color?* [These rods are equal to an even number of units, representing the numbers 2, 4, 6, 8, and 10. They are divisible by two.]

- *Which rods could be made only with white rods?* [white, red, green, yellow, and black] *Why?* [These rods represent 1 and the prime numbers 2, 3, 5, and 7. Prime numbers only have two factors—one and themselves. Therefore, only white rods, which represent 1, can be used to build them.]
- *What other patterns did you identify when you replicated the different rods?* [Sample responses: Only two of the rods can be made by using three rods of the same color. There are more ways to make the brown rod, the dark green rod, and the orange rod than to make any other rod.]

Note: Students in grade 4 and above can investigate replicating numbers larger than 10 by making trains consisting of two or more rods. For example, anorange-white train is equal to 11; an orange-red train is equal to 12; and an orange-orange-white train is worth 21.

ROD PAIRS—A FRACTION INVESTIGATION

Group Size: Teams of two students

Objective: Use Cuisenaire Rods to investigate fractions

Procedure: In this activity, partners explore part-whole relationships between pairs of rods.

Ask—

- *Which rod is exactly half the length of one orange rod?* [1 yellow rod]

Show students that this fact can be recorded in either of two ways:
$o = 2y$ or $y = \frac{1}{2}\ o$.

Ask—

- *Why can the relationship between one yellow rod and one orange rod be expressed in these two different ways?* [Sample responses: They both mean the same thing. The first expression means that it takes two yellow rods to equal an orange rod. The second expression means that a yellow rod is one-half the size of an orange rod.]

Next, have students find all other pairs of rods that share the same part-whole relationship of $\frac{1}{2}$. Then ask them to find all the pairs of rods in which one rod is $\frac{1}{3}$ of the other, $\frac{1}{4}$ of the other, and so on, up to $\frac{1}{10}$ of the other. Remind students to record all the relationships they find.

When students have completed the activity, have them share their findings. List students' responses on the board or overhead.

Ask—

- *What other rod pairs did you find in which one rod was one-half of another rod?* [white and red; red and purple; purple and brown; green and dark green]
- *How do you know that you found all possible pairs?* [Sample response: I tried to match each color rod with a larger rod to see if the smaller rod equaled one-half of the larger rod.]
- *Why is the yellow rod the longest rod you found that is half of another rod?* [The longest rod, orange, is worth 10. Half of 10 is 5, the value of the yellow rod.]
- *Is it possible to make a rod train for which one dark green rod would be half of the rod train? Give one example.* [Yes; Sample response: orange and red rod train]
- *For which color rods could you find no smaller rod to represent one half of the larger rod?* [white, green, yellow, black, and blue]
- *Could you make rod trains for which no rod could be half?* [Yes; Sample responses: orange and white train, red and black train, or purple and blue train]

List and discuss the remaining part-whole relationships that students found for $\frac{1}{3}$, $\frac{1}{4}$, etc. It is important that they verbalize the idea that if it takes three of one color rod to make another rod, the first is one-third of the second; that if it takes four of one color rod to make another, the first is one-fourth of the second, and so on. In discussing the fractional notation of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, etc., make sure that students note that the bottom number, or denominator, of the fraction indicates how many parts are required to make the whole.

ROD PAIRS—A FRACTION INVESTIGATION EXTENSION

Group Size: Teams of two students

Objective: Use Cuisenaire Rods to investigate fractions with numerators other than 1

Procedure: In this activity, students are asked to find the number of single units in each rod in the pair to determine the relationship of the rods to each other. Begin by asking—

- *What is the part-whole relationship between one red rod and one green rod? Explain your reasoning.* [Sample responses: The red rod is $\frac{2}{3}$ of the green rod, because it takes three white rods to equal the green rod, and the red rod is equal to two of the white rods. A white rod is $\frac{1}{3}$ of the green rod, and the red rod is equal to two white rods. So the red rod is equal to $\frac{2}{3}$ of the green rod.]

FINDING EQUIVALENT FRACTIONS

Group Size: Teams of two students

Objective: Use Cuisenaire Rods to investigate and find equivalent fractions

Procedure: In this activity, students choose a rod, build all the one-color trains they can for that rod, and then find all the names for the rods they use. This enables students to learn that different fractional names can represent the same amount of the whole. The exploration becomes more challenging when students, taking each rod in turn, give it a value of one and determine all the fractional names for the other rods.

Ask—

- *How many one-color trains that equal the orange rod can you find?* [three: a train of 2 yellow rods, a train of 5 red rods, and a train of 10 white rods]

Assign a value of 1 to the orange rod. Ask students to build all the one-color trains that equal one orange rod and then to find the fractional names to represent the part-whole relationship of one of the smaller rods to the orange rod. You may need to remind some students that they will need to use white rods to substitute for the odd number rods (green, yellow, black, and blue) to find the numerator, or top number, for these fractions.

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When students have completed this task, have a class discussion to give students an opportunity to share their findings and strategies.

Ask—

- *What fractional part of one orange rod is one white rod?* [$\frac{1}{10}$]
- *One red rod?* [$\frac{2}{10}$ or $\frac{1}{5}$]
- *One light green rod?* [$\frac{3}{10}$]
- *One purple rod?* [$\frac{4}{10}$ or $\frac{2}{5}$]
- *One yellow rod?* [$\frac{5}{10}$ or $\frac{1}{2}$]
- *One dark green rod?* [$\frac{6}{10}$ or $\frac{3}{5}$]
- *One black rod?* [$\frac{7}{10}$]
- *One brown rod?* [$\frac{8}{10}$ or $\frac{4}{5}$]
- *One blue rod?* [$\frac{9}{10}$]

Some students may not realize that the red rod has two equivalent names—$\frac{1}{5}$ and $\frac{2}{10}$. Draw their attention to the fact that two white rods equal one red rod. Since one white rod is $\frac{1}{10}$ and two white rods $\frac{2}{10}$, the red rod is also $\frac{2}{10}$.

Assign a value of 1 to the blue rod. Ask students to build all the one-color trains that equal one blue rod and to find the fractional names to represent the part-whole relationship of one of the smaller rods to the blue rod. If time permits, have students build all the one-color trains that equal one black rod and find the fractional names to represent the part-whole relationship of each rod to the black rod.

When students have completed this activity, have groups share their results for a particular set of rods and explain the reasoning they used in naming each rod.

Ask—

- *Why were you able to find only one fraction name for the black, yellow, green, and red rods?* [These rods represent prime numbers that can only be built with white rods.]

FINDING ALL THE NAMES

Group Size: Teams of two students

Objective: Use Cuisenaire Rods to investigate equivalent fractions, improper fractions, and mixed numbers

Procedure: Model the activity with the brown rod having a value of 1. Begin with the white rod, as other rods may need to be compared with it before their values can be determined.

Ask—

- *What fractional part of one brown rod is one white rod?* [$\frac{1}{8}$]
- *What fractional part of one brown rod is one red rod?* [$\frac{2}{8}$ or $\frac{1}{4}$]
- *What fractional part of one brown rod is one green rod?* [$\frac{3}{8}$]

As students did in the rod pairs activity, they will need to use the white rod to show that one green rod is $\frac{3}{8}$ of one brown rod. Continue, finding and discussing all the values for each rod and recording the values on the board or overhead

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Ask—

- *What fractional part of one brown rod is one blue rod?* [$\frac{9}{8}$ or $1\frac{1}{8}$]

Not all students will realize that the blue rod's relationship to the brown rod has two fraction names—$\frac{9}{8}$ since it is as long as 9 white rods, and $1\frac{1}{8}$ since it is as long as 1 brown rod and 1 white rod. This is a good time to talk about improper fractions and mixed numbers.

Once students understand how to proceed, ask them to find all the fraction names for each rod when the green rod is assigned the value of 1. Have them record their findings and be prepared to explain them. Continue with the black rod, the orange rod, etc.

EXPLORING RATIO—HOW MANY YELLOWS? (GRADES K–3)

Group Size: Teams of two students

Objective: Use Cuisenaire Rods to investigate the meaning of ratio and proportion

Procedure: In this activity, students use the orange rod measurement of an object and the relationship between the length of the orange rod and other rods to calculate, without measuring, the length of the object in a color rod other than the orange rod. To model this activity, choose an object that measures between 9 and 12 orange rods in length—a window ledge, bookcase, etc.

Ask—

- *How many orange rods do you think it would take to equal the length of the object?*

Have students discuss their prediction with a partner before sharing with the class. List their predictions on the board; then find the length of the object in orange rods.

Next, ask—

- *How many yellow rods would it take to equal the length of the same object?*

Give each pair of students one orange rod and one yellow rod and tell them that they need to solve the problem without actually measuring the object with the yellow rod. When they have an answer, they should record it and explain in writing why they think their answer is correct.

If students are having difficulty, ask them what they notice about the comparative length of the orange and yellow rods. [Two yellow rods equal one orange rod.] If necessary, have them measure the orange rod with the yellow rod. Have students write their answer and a short explanation that describes the strategies used to find the answer. When students have completed this activity, have them take turns reading their explanations aloud to the class.

EXPLORING RATIO—HOW MANY YELLOWS? (GRADES 4–8)

Group Size: Teams of two students

Objective: Use Cuisenaire Rods to investigate the meaning of ratio and proportion

Procedure: Ask students to help you locate an object that is about 12 orange rods long, measuring each object as it is suggested.

Next, ask—

- *How many yellow rods would it take to equal the length of the same object?* [Sample responses: Two yellow rods fit on one orange, so we multiplied 2 times 12 and got 24. The object is 12 orange rods long, and 2 yellow rods are equal to one orange rod, so we counted by twos. We got an answer of 24.]

After they have shared their thinking, assign partners the task of figuring out the length of the object in white rods, in red rods, and then in dark green rods, without measuring it. Each pair of students needs one set of rods. Have them record their explanations in writing. Ask students who finish early to find the length of the object in blue rods, black rods, etc. When students have completed this activity, have them share their findings.

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Cuisenaire Rods Alphabet Book (Grades K–4)
Idea Book for Cuisenaire Rods, Primary (Grades PreK–2)
Idea Book for Cuisenaire Rods, Intermediate (Grades 3–5)
Addition & Subtraction with Cuisenaire Rods (Grades K–2)
Everything's Coming Up Fractions with Cuisenaire Rods (Grades 4–6)
Rod-Clue Puzzles (Grades 5 and up).

THE SUPER SOURCE® CUISENAIRE RODS

- A powerful resource full of classroom-tested problem-solving activities to help you explore the rich potential of this classic, colorful manipulative! Students investigate topics such as counting, measurement, fractions, and spatial visualization. Each grade-level specific book contains 18 activities that include teaching notes about the underlying mathematics. Each kit contains a Super Source Teacher Resource Book; 12 trays of 74 wood or plastic Cuisenaire Rods; Cuisenaire Rods for the Overhead; and a Cuisenaire Rods Template. Kit includes enough materials for an entire class working in small groups. Grades K–6.

K–2 The Super Source Cuisenaire Rods Book
3–4 The Super Source Cuisenaire Rods Book
5–6 The Super Source Cuisenaire Rods Book
K–2 The Super Source Plastic Cuisenaire Rods Kit
3–4 The Super Source Plastic Cuisenaire Rods Kit
5–6 The Super Source Plastic Cuisenaire Rods Kit
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