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Chapter 7: Systems of Linear Equations

Chapter 8: Bivariate Data

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Foreword

Math Mammoth Grade 8 comprises a complete math curriculum for the eighth grade mathematics studies. The curriculum meets the Common Core standards.

In 8th grade, students spend the majority of the time with algebraic topics, such as linear equations, functions, and systems of equations. The other major topics are geometry and statistics.

The main areas of study in Math Mammoth Grade 8 are:

- Exponents laws and scientific notation
- Square roots, cube roots, and irrational numbers
- Geometry: congruent transformations, dilations, angle relationships, volume of certain solids, and the Pythagorean Theorem
- Solving and graphing linear equations;
- Introduction to functions;
- Systems of linear equations;
- Bivariate data.

This book, 8-B, covers the topic of graphing linear equations. The focus is on the concept of slope.

In chapter 6, our focus is on square roots, cube roots, the concept of irrational numbers, and the Pythagorean Theorem and its applications.

Next, in chapter 7, students solve systems of linear equations, using both graphing and algebraic techniques. There are also lots of word problems that are solved using a pair of linear equations.

The last chapter then delves into bivariate data. First, we study scatter plots, which are based on numerical data of two variables. Then we look at two-way tables, which are built from categorical bivariate data.

Part 8-A covers exponent laws, scientific notation, geometry, linear equations, and an introduction to functions.

I heartily recommend that you read the full user guide in the following pages.

I wish you success in teaching math!

Maria Miller, the author

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Chapter 5: Graphing Linear Equations
Introduction

This chapter focuses on how to graph linear equations, and in particular, on the concept of slope in that context.

We start by graphing and comparing proportional relationships, which have the equation of the form $y = mx$. Students are already familiar with these, and know that m is the constant of proportionality. In this chapter, they learn that m is also the slope of the line, which is a measure of its steepness.

Then we go on to study slope in detail, its definition as the ratio of the change in y-values and the change in x-values. Students learn that it doesn't matter which two points on a line you use to calculate the slope, and study a geometric proof of this fact. They practice drawing a line with a given slope and that goes through a given point, and determine if three given points fall on the same line.

Then it is time to study the slope-intercept equation of a line, and connect the idea of an initial value of a function (chapter 4) with the concept of y-intercept in the context of graphing. Students graph lines given in the slope-intercept form, and write equations of lines from their graphs.

Next, we study horizontal and vertical lines and their simple equations. The standard form of a linear equation follows next. The last major topic is how the slope reveals to us whether two lines are parallel or perpendicular to each other.

Pacing Suggestion for Chapter 5

This table does not include the chapter test as it is found in a different book (or file).
Please add one day to the pacing if you use the test.

The Lessons in Chapter 5	**page**	**span**	**suggested pacing**	**your pacing**
Graphing Proportional Relationships 1	13	*3 pages*	1 day	
Graphing Proportional Relationships 2	16	*3 pages*	1 day	
Comparing Proportional Relationships	19	*4 pages*	1 day	
Slope, Part 1	23	*4 pages*	1 day	
Slope, Part 2	27	*3 pages*	1 day	
Slope, Part 3	30	*5 pages*	2 days	
Slope-Intercept Equation 1	35	*4 pages*	1 day	
Slope-Intercept Equation 2	39	*3 pages*	1 day	
Write the Slope-Intercept Equation	42	*3 pages*	1 day	
Horizontal and Vertical Lines	45	*3 pages*	1 day	
The Standard Form	48	*3 pages*	1 day	
More Practice (optional)	51	*(2 pages)*	(1 day)	
Parallel and Perpendicular Lines	53	*3 pages*	1 day	
Mixed Review Chapter 5	56	*3 pages*	1 day	
Chapter 5 Review	59	*4 pages*	1 day	
Chapter 5 Test (optional)				
TOTALS		*48 pages*	15 days	
with optional content		*(50 pages)*	(16 days)	

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Helpful Resources on the Internet

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- **online practice** for concepts;
- online **games**, or occasionally, printable games;
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- **articles** that teach a math concept.

We heartily recommend you take a look! Many of our customers love using these resources to supplement the bookwork. You can use these resources as you see fit for extra practice, to illustrate a concept better and even just for some fun. Enjoy!

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Graphing Proportional Relationships 1

We will now review what it means when two variables are **in direct variation** or **in proportion**. The basic idea is that whenever one variable changes, the other varies (changes) proportionally or at the same rate.

Example 1. The wholesaler posted the following table for the price of potatoes:

weight (kg)	5	10	15	20	25	30
cost	$5.50	$11.00	$16.50	$22.00	$27.50	$33.00

Each pair of cost and weight forms a rate — and so does each pair of weight and cost. However, it is more common to look at the rate "cost over weight", such as $27.50/(25 kg), than vice versa.

If all of the rates in the table are equivalent, then the weight and the cost *are* proportional.

To check for that, we have several means. One is to calculate **the unit rate** (the rate for 1 kg) from each of these rates, and check whether you get the same unit rate.

In this case, that is so. The unit rate is $1.10/kg, no matter which rate from the table we'd use to calculate it.

One other way to check is, if one quantity doubles (or triples), will the other double (or triple) also?
This is especially useful for noticing if the quantities are *not* in direct variation.

Example 2. Here, when the weight doubles from 5 kg to 10 kg, the price also doubles. But what happens with the price when the weight doubles from 10 kg to 20 kg?

weight (kg)	5	10	15	20	25	30
cost	$6	$12	$18	$22	$26	$30

The price does not double! So, the quantities are not in proportion.
The seller is giving you some discount if you purchase higher quantities.

Also, if you calculate the unit rate from $6/(5 kg) and from $22/(20 kg), they are not equal. (Verify this.)

1. Are the quantities in a proportional relationship? If yes, list the unit rate.

a.

time (hr)	0	1	2	3	4	5
distance (km)	0	50	90	140	190	240

b.

time (hr)	0	1	2	3	4	5
distance (km)	0	45	90	135	180	225

c.

age (days)	0	1	2	3	4	5	6	7
height (in)	0	0	0	1	2	3	4	5

d.

length (m)	0	0.5	1	1.5	2	4	5	10
cost ($)	0	3	6	9	12	24	30	60

2. Now consider the tables of values in #1 as functions, where the variable listed on top is the independent variable. For the ones where the quantities were in proportion, calculate the rate of change.

   What is its relationship to the unit rate?

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When two quantities are in a proportional relationship, or in direct variation (the two are synonyms):

(1) Each rate formed by the quantities is equivalent to any other rate of the quantities.

(2) The equation relating the two quantities is of the form $y = mx$, where y and x are the variables, and m is a constant. The constant m is called the **constant of proportionality** and is also the unit rate.

(3) When plotted, the graph is a straight line that goes through the origin.

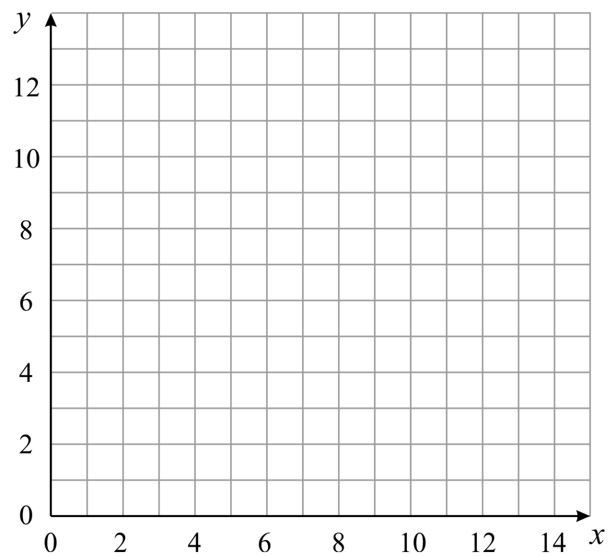
3. Choose an equation from below where the variables x and y are in direct variation (proportional):

$y = \dfrac{3}{x}$ $\qquad$ $y = 3x$

$xy = 3$ $\qquad$ $y = x^3$

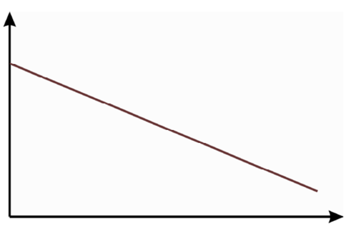
Then graph that equation in the grid.

Hint: The point (0, 0) is always included in direct variation. All you need to do is plot one other point, and then draw a line through the origin and that point.

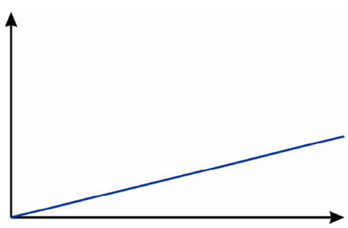


4. Choose the representations that show a proportional relationship.

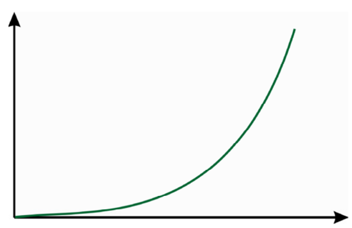
a.



b.



c.



d.

x	0	1	2	3	4	5
y	15	17	19	21	23	25

e. $y = 2x + 9$

f. $y = (3/4)x$

g.

x	0	4	8	12	16	20
y	0	3	6	9	12	15

5. Two of the above representations are the exact same relationship. Which ones?

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Example 2. In a direct variation, $y = 9$ when $x = 12$. Write an equation for the relationship.

Since this is direct variation (proportional relationship), the equation is of the form $y = mx$, where m is the constant of proportionality.

The constant of proportionality is the ratio **(dependent variable)/(independent variable)**, so in this case it is $y/x = 9/12$, or $3/4$. So, the equation is $y = (3/4)x$.

At this point, it is good to check that the point (12, 9) satisfies the equation, to check for errors: Is it true that $9 = (3/4) \cdot 12$? Yes, it is.

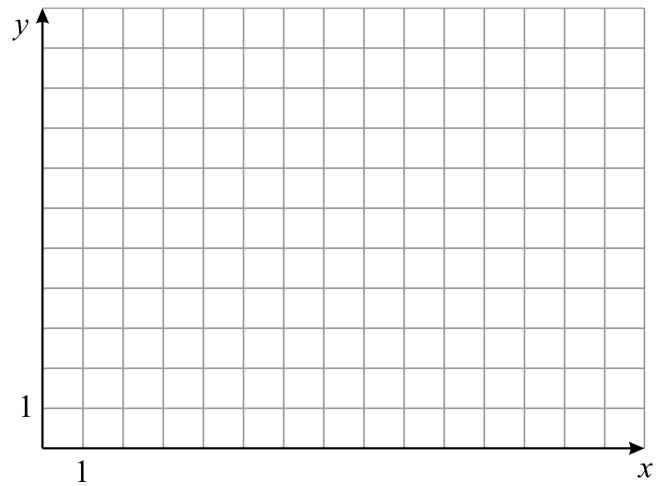
To graph the equation, we could simply plot the point (12, 9), and draw a line through it and the origin.

6. In a direct variation, when x is 14, y is 10.

a. Write an equation for this proportional relationship.

b. Graph a line for this relationship in the grid.

c. What is x when $y = 40$?

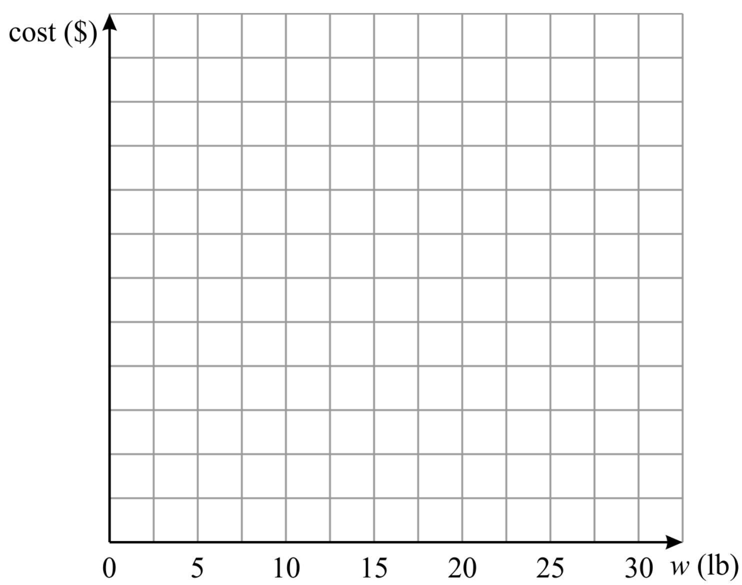


7. Organic rolled oats cost \$20 for 8 lb.

a. Write an equation for this proportional relationship, using the variables C for cost and w for the amount (weight) of oats.

b. Graph the equation in the grid. Design the scaling on the cost-axis so that the point corresponding to 30 pounds fits on the grid.

c. How much do 36 lb of the oats cost?



8. If y is 120 when x is 400 in a direct variation, then what is y when x is 80?

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Graphing Proportional Relationships 2

Example 1. The graph for the cost of apples as a function of their weight is a line through the origin, which means the cost and the weight are proportional.

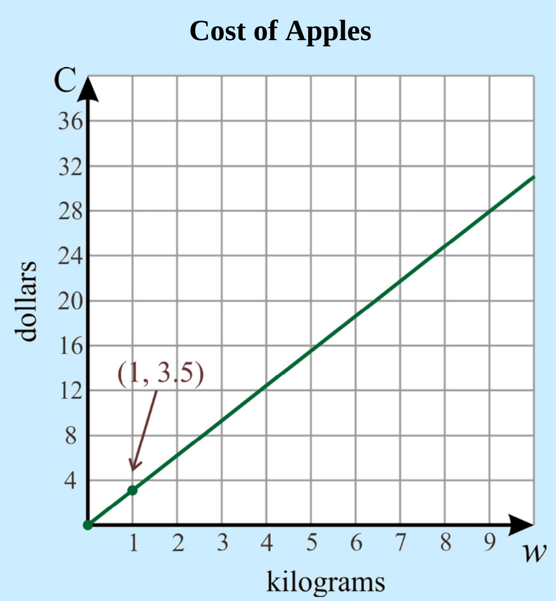
The equation for the graph is C = 3.5*w*. So, the constant of proportionality is 3.5, which is also the rate of change of this function.

The unit rate is $3.50/kg. This corresponds to the point (1, 3.5) on the graph.

We also talk about **the slope of the line**, which is a measure of the steepness of the line, or how quickly it rises upwards (or slopes downwards).

It is the same idea as the rate of change, but in the context of graphing. Here, for each 1-kg increase in weight, the cost increases by $3.50. This means the slope of this line is 3.5 — the same as the rate of change, and the unit rate.

We will look at slope in more detail in other lessons.



1. The graph shows the distance a caterpillar has crawled over time.

   a. Are the quantities *time* and *distance* proportional?

   How can you tell?

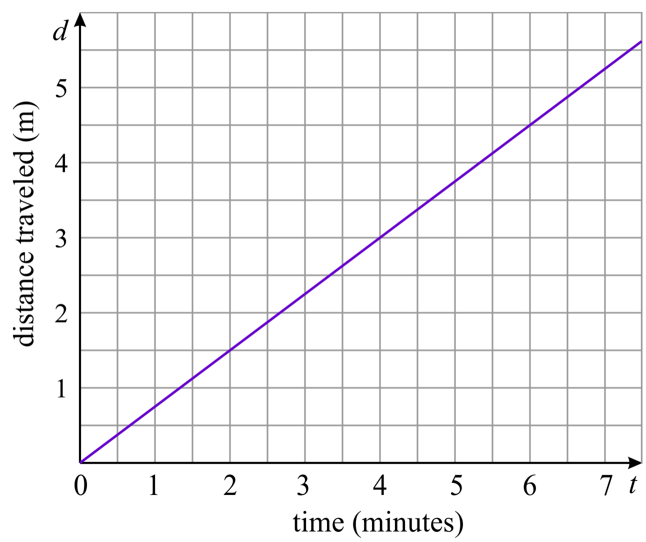
   b. What is the caterpillar's speed? Use the same units as in the graph.

   What is the unit rate?

   What is the slope of the line?

   c. Write an equation for the graph.

   d. Continuing with the same speed, how long will the caterpillar take to travel 9.5 meters?



2. Another caterpillar crawls 4.5 meters in 5 minutes. Draw another line in the grid for question #2, for the distance that this second caterpillar crawls over time, going with the same speed. Is this second caterpillar faster than the first?

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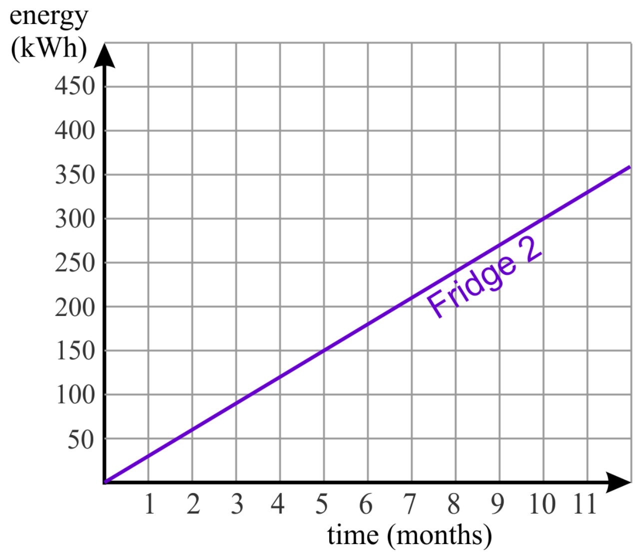
Chapter 5 Review

1. Refrigerator companies make estimates of how much energy their fridges consume in typical usage. The table shows how many kilowatt-hours (kWh) of energy fridge 1 consumed over time, and the graph shows the same for fridge 2.

Fridge 1

time (mo)	energy (kWh)
2	75
4	150
6	225
8	300
10	375
12	450

Fridge 2



a. Which fridge consumes more electricity in a month?

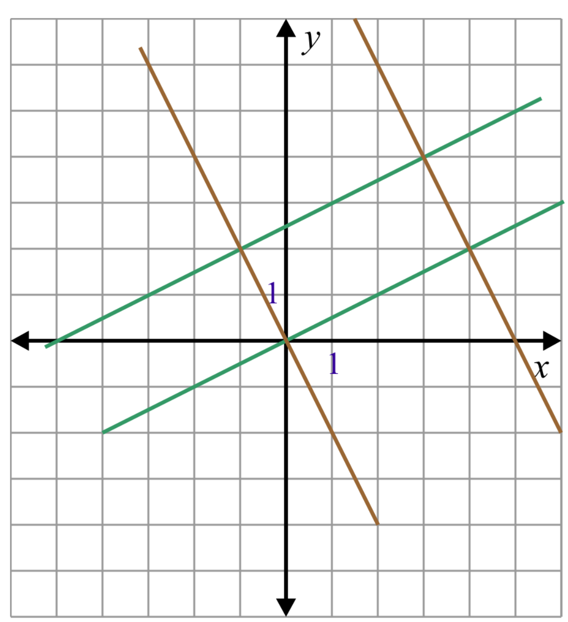
How much more?

b. Write an equation for each fridge, relating the energy (E, in kWh) and the time (t, in months).

c. Plot the equation for Fridge 1 in the grid.

d. Plot the point corresponding to the unit rate, for Fridge 1.

2. **a.** Find the equations of the four lines, in slope intercept form.



b. (optional) Find the area of the rectangle.

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3. Find the equation of each line, in slope-intercept form:

a. has slope 3/4 and passes through $(-2, 3)$

b. is horizontal and passes through $(9, -10)$

4. Find the slope of the lines.
Notice the scaling.

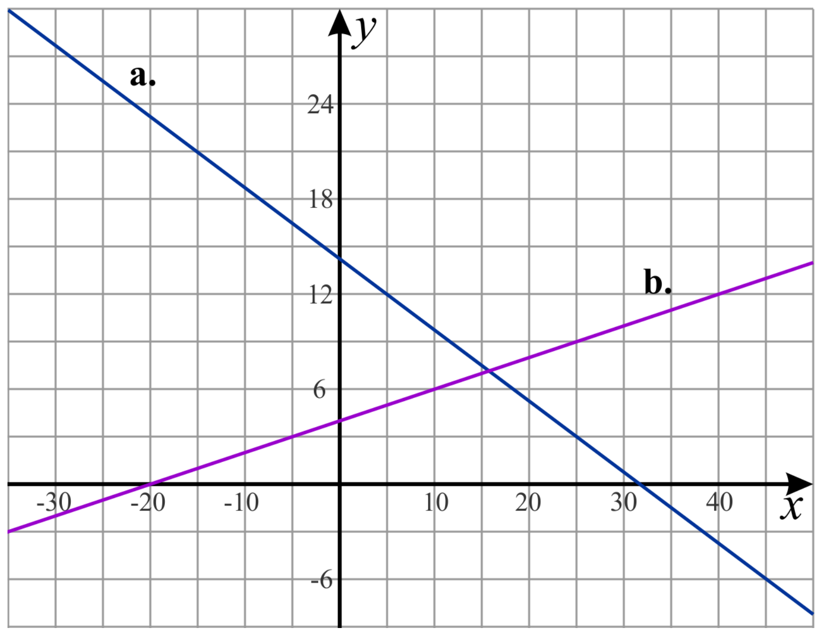
a.

b.

Now find the equations for the lines.

a.

b.



5. Do the three points fall on one line? Explain your reasoning.

$(-3, 1)$, $(-1, -4)$, $(1, -8)$

6. Find s so that the point $(s, 12)$ will fall on the same line as the points $(3, 9)$ and $(15, 18)$.

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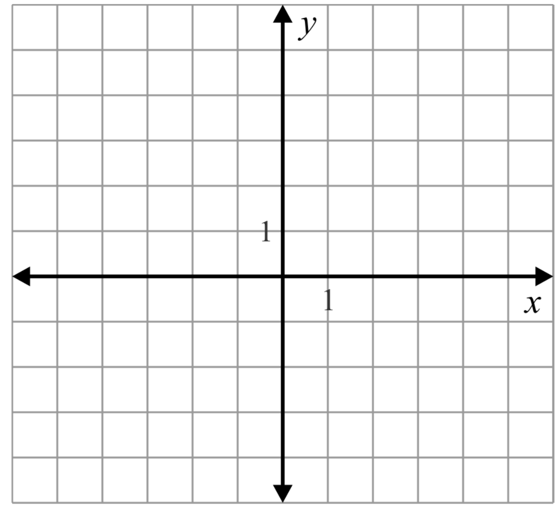
7. Line S passes through (−5, −2) and (0, 4). Line T is perpendicular to Line S, and passes through (1, 1).

a. Find the equation of line T, in slope-intercept form.

b. Write the equation also in the standard form.



8. Mr. Henson runs a garbage pick-up business, with 12 garbage trucks. To run one truck costs him $1,500 per month in maintenance costs, plus $110 a day for fuel.

Consider the cost of running one truck as a function of time, in days (during one month only). Is this a linear relationship, a proportional relationship, or neither?

Write an equation for it.

9. Match the descriptions and the equations.

$y = (-4/3)x - 7$	Is parallel to $x = 9$ and passes through (2, 7)
$3x - y = -21$	Has y-intercept −4 and is perpendicular to $y = -2x$.
$y = -4$	Passes through (−5, 6) and has slope 3.
$x - 2y = 8$	Passes through (−9, 5) and (−3, −3)
$x = 2$	Passes through (−3, 0) and (0, 9)
$y = 3x + 9$	Has y-intercept −4 and is parallel to $y = -2$.

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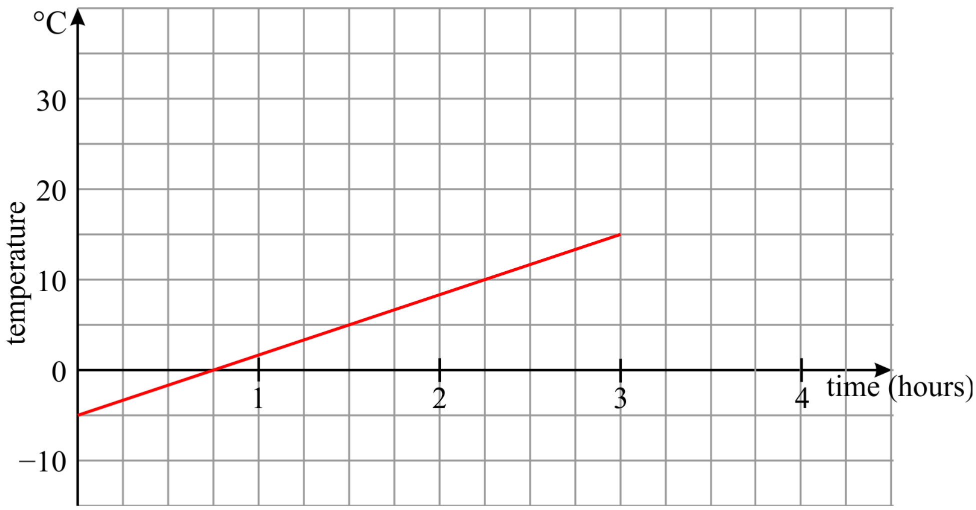
10. Transform each equation of a line to the standard form, and then list its *x* and *y*-intercepts.

a. $y - 6 = 2(x + 2)$	**b.** $-\frac{1}{3}x - \frac{3}{2}y = 1$

11. A heater was turned on at 10 AM in a cold, uninhabited house, to prepare it for people later that day. The graph shows the temperature of the house. The count of hours starts at 10 AM.



a. Write an equation for the line.

b. If the temperature continues to rise in the same fashion, what will the temperature be at 2:30 PM?

c. When will the temperature reach 22°C?

d. Let's say the heater is turned off at 1:45. What is the temperature at that time?

e. If the house had started out at a temperature of −12°C instead, and the heating process worked in the same fashion (the temperature rose at the same rate), at what time would the house reach a temperature of 22°C?

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Chapter 6: Irrational Numbers and the Pythagorean Theorem

Introduction

We start out this chapter by studying the concept of a square root, as the opposite operation to squaring a number. In the next lesson, on irrational numbers, students find values of square roots by hand. They make a guess and then square the guess, and based on how close the square of their guess is to the radicand, they refine their guess until desired accuracy is reached. This will help solidify the concept of a square root, while also showing how most square roots are nonending decimal numbers, and how in real life, we need to use approximations of them to do calculations. Students also practice placing irrational numbers on the number line, using mental math to find their approximate location.

Next, the chapter has a review lesson on how to convert fractions to decimals. The following lesson has to do with writing decimals as fractions, and teaches a method for converting repeating decimals to fractions.

Then it is time to learn to solve simple equations that involve taking a square or cube root, over the course of two lessons. After learning to solve such equations, students are now fully ready to study the Pythagorean Theorem and apply it.

The Pythagorean Theorem is introduced in the lesson by that name. Students learn to verify that a triangle is a right triangle by checking whether it fulfills the Pythagorean Theorem. They apply their knowledge about square roots and solving equations to solve for an unknown side in a right triangle when two of the sides are given.

Next, students solve a variety of geometric and real-life problems that require the Pythagorean Theorem. This theorem is extremely important in many practical situations. Students should show their work for these word problems to include the equation that results from applying the Pythagorean Theorem to the problem and its solution.

There are literally hundreds of proofs for the Pythagorean Theorem. In this book, we present one easy proof based on geometry (not algebra). As an exercise, students are asked to supply the steps of reasoning to another geometric proof of the theorem. Students also study a proof for the converse of the theorem, which says that if the sides of a triangle fulfill the equation $a^2 + b^2 = c^2$ then the triangle is a right triangle.

Our last topic is distance between points in the coordinate grid, as this is another simple application of the Pythagorean Theorem.

Pacing Suggestion for Chapter 6

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Please add one day to the pacing if you use the test.

The Lessons in Chapter 6	page	span	suggested pacing	your pacing
Square Roots	65	*4 pages*	1 day	
Irrational Numbers	69	*4 pages*	1 day	
Cube Roots and Approximations of Irrational Numbers ...	73	*4 pages*	1 day	
Fractions to Decimals (optional)..	77	*(2 pages)*	(1 day)	
Decimals to Fractions	79	*3 pages*	1 day	
Square and Cube Roots as Solutions to Equations	82	*3 pages*	1 day	
More Equations that Involve Roots	85	*3 pages*	1 day	
The Pythagorean Theorem	88	*5 pages*	2 days	
Applications of the Pythagorean Theorem 1	93	*3 pages*	1 day	

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A Proof of the Pythagorean Theorem and of Its Converse	96	*4 pages*	1-2 days
Applications of the Pythagorean Theorem 2	100	*4 pages*	1 day
Distance Between Points	104	*3 pages*	1 day
Mixed Review Chapter 6	107	*3 pages*	1 day
Chapter 6 Review	110	*6 pages*	2 days
Chapter 6 Test (optional)			
TOTALS		*49 pages*	15-16 days
with optional content		*(51 pages)*	(16-17 days)

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Cube Roots and Approximations of Irrational Numbers

Similarly to the square root, we can take a **cube root** of a number.

Recall that the **cube of a number** is that number multiplied by itself three times. For example, two cubed = $2^3 = 2 \cdot 2 \cdot 2 = 8$. This gives us the volume of a cube with edges 2 units long.

The cube root of 8 is 2. We write it as $\sqrt[3]{8} = 2$. Notice the little "3" that is added to the radical sign to signify a cube root.

Example 1. Since $(-3)(-3)(-3) = -27$, then $\sqrt[3]{-27} = -3$.

Like square roots, most cube roots are irrational numbers. When it comes to integers, only the cube roots of perfect cubes are rational; the rest are irrational.

1. Find the cube roots without a calculator.

a. $\sqrt[3]{27}$	**b.** $\sqrt[3]{125}$	**c.** $\sqrt[3]{64}$	**d.** $\sqrt[3]{1{,}000}$
e. $\sqrt[3]{1}$	**f.** $\sqrt[3]{216}$	**g.** $\sqrt[3]{27{,}000}$	**h.** $\sqrt[3]{-8}$
i. $\sqrt[3]{-1}$	**j.** $\sqrt[3]{-125}$	**k.** $\sqrt[3]{0}$	**l.** $\sqrt[3]{-8{,}000}$

2. **a.** The volume of a cube is 216 cm^3. How long is its edge?

b. What is $(\sqrt[3]{4})^3$?

c. If the edge of a cube measures 50 cm, find its volume.

d. If the volume of a cube is 729 in^3, find its surface area.

3. (optional) Find the cube roots of these fractions and decimals, without a calculator.

a. $\sqrt[3]{0.008}$	**b.** $\sqrt[3]{0.125}$	**c.** $\sqrt[3]{-0.027}$
d. $\sqrt[3]{\frac{8}{125}}$	**e.** $\sqrt[3]{\frac{64}{27}}$	**f.** $\sqrt[3]{-\frac{1}{8}}$

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Example 2. We can know that $\sqrt{98}$ lies between 9 and 10, because $9 = \sqrt{81} < \sqrt{98} < \sqrt{100} = 10$.

We can even tell it is much closer to 10 than to 9, since 98 is much closer to 100 than to 81.

From that, we can estimate that $2\sqrt{98}$ is slightly less than 20, and that $\sqrt{98} + 4$ is slightly less than 24.

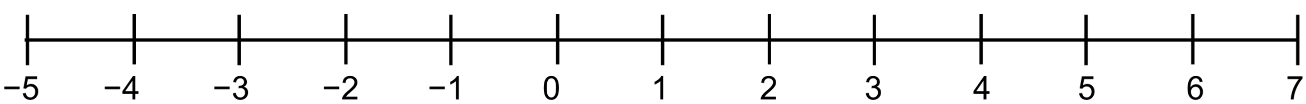
Example 3. The opposite of $\sqrt{2}$ is $-\sqrt{2}$. Since $\sqrt{2}$ is approximately 1.41, then $-\sqrt{2} \approx -1.41$.

4. Find between which two whole numbers the root lies. Notice some of them are cube roots.

a. _____ $< \sqrt{31} <$ ______	**b.** _____ $< \sqrt{65} <$ ______	**c.** _____ $< \sqrt{87} <$ ______
d. _____ $< -\sqrt{5} <$ ______	**e.** _____ $< -\sqrt{44} <$ ______	**f.** _____ $< -\sqrt{50} <$ ______
g. _____ $< \sqrt[3]{7} <$ ______	**h.** _____ $< \sqrt[3]{37} <$ ______	**i.** _____ $< \sqrt[3]{101} <$ ______

5. Plot the following numbers *approximately* on the number line. Do not use a calculator, but think about between which two whole numbers the root lies, and whether it is close to one of those whole numbers.

$\sqrt{15}$ $\quad$ $\sqrt{47}/2$ $\quad$ $\sqrt[3]{9}$ $\quad$ $-\sqrt[3]{27}$ $\quad$ $-\sqrt{10}$ $\quad$ $\sqrt{66}/2$ $\quad$ π $\quad$ $\sqrt{18} + 1$



6. Compare, writing >, <, or = between the numbers. Think between which two whole numbers the root lies, using mental math.

a. $5 \;\square\; \sqrt{27}$	**b.** $\sqrt{48} \;\square\; 7$	**c.** $\sqrt{18} \;\square\; 4$	**d.** $\sqrt[3]{9} \;\square\; 2$
e. $2 \;\square\; \sqrt{2} + 1$	**f.** $\sqrt{32} + 1 \;\square\; 6$	**g.** $\sqrt{43} + 5 \;\square\; 10$	**h.** $\sqrt{88} - 3 \;\square\; 7$

7. **a.** Between which two whole numbers does $\sqrt{30}$ lie? And $\sqrt{60}$?

b. Use your answers to (a) to determine whether $2\sqrt{30}$ is equal to $\sqrt{2 \cdot 30}$.

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8. Is $\dfrac{\sqrt{50}}{2}$ equal to $\sqrt{\dfrac{50}{2}}$? Explain your reasoning.

9. Use the decimal approximations of common irrational numbers on the right to estimate the value of the expressions below, to one decimal digit. Use mental math and paper-and-pencil calculations, not a calculator.

$\pi \approx 3.14$

$\sqrt{2} \approx 1.41$

$\sqrt{5} \approx 2.24$

a. $5\sqrt{2}$ **b.** π^2 **c.** $\sqrt{5} - \sqrt{2}$ **d.** $2\sqrt{5} - 5\sqrt{2}$

10. **a.** Find an approximation to $\sqrt{11}$ to one decimal digit, without using the square root function of a calculator.

b. Use the approximation you found to estimate the values of $\sqrt{11} - \sqrt{2}$ and $3\sqrt{11}$.

11. Sarah has used the method of squaring her guesses to find out that $\sqrt{45}$ is between 6.7 and 6.8. How can she continue from this point to get a better approximation? Do it for her, to two decimal digits.

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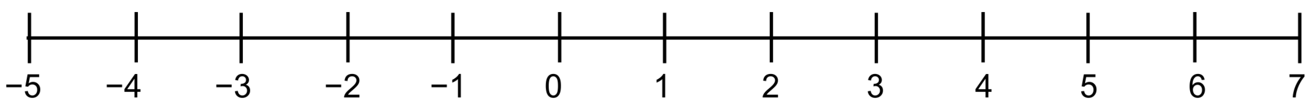
Use these exercises for additional practice.

12. Order the numbers from smallest to greatest. Estimate the value of the roots, thinking between which two whole numbers each square root lies, using mental math.

$\sqrt{5} - 1$ $\sqrt[3]{1}$ $\sqrt{19}/2$ $\sqrt[3]{100}$ $\sqrt[3]{8}$ $\sqrt{13}$ $\sqrt{9}$ 2π $\sqrt{22} + 1$

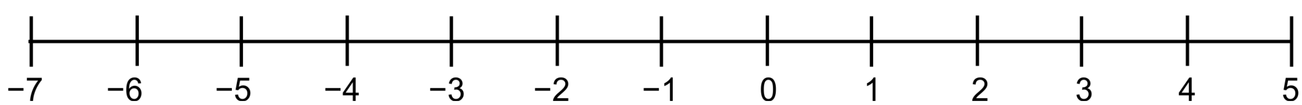
13. Plot the following numbers *approximately* on the number line. Do not use a calculator, but think about between which two whole numbers the root lies, and whether it is close to one of those whole numbers.

a. $-2\sqrt{2}$ **b.** $\sqrt{80}/3$ **c.** $\sqrt{27} - 1$ **d.** $-\sqrt{5} + 7$



14. Plot the following numbers *approximately* on the number line.

a. $-\sqrt{2} - 3$ **b.** $-\pi$ **c.** $-\sqrt[3]{9}$ **d.** $-\sqrt{36} + 9$ **e.** $-\sqrt{26}/2$



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Fractions to Decimals

(This lesson is review, and optional.)

Each fraction is a rational number (by definition!). Each fraction can be written as a decimal. It will either be a terminating decimal, or a non-terminating repeating decimal.

It is easy to rewrite a fraction as a decimal when the denominator is a power of ten. However, when it is not (which is most of the time), simply treat the fraction as a division and divide. You will get either a **terminating decimal** or a non-terminating **repeating decimal**. See the examples below.

1. The denominator is a power of ten. In this case, writing the fraction as a decimal is straightforward. Simply write out the numerator. Then add the decimal point based on the fact that the number of zeros in the power of ten tells you the number of decimal digits.

Examples 1. $\frac{7809}{100} = 78.09$ $\frac{1458}{1000} = 1.458$ $\frac{506}{100{,}000} = 0.00506$

2. The denominator is a factor of a power of ten. Convert the fraction into one with a denominator that is a power of ten. Then do as in case (1) above.

Examples 2. $\frac{9}{20} = \frac{45}{100} = 0.45$ $\frac{2}{125} = \frac{16}{1000} = 0.016$ $\frac{33}{30} = \frac{11}{10} = 1.1$

3. Use division (long division or with a calculator). This method works in all cases, even if the denominator happens to be a power of ten or a factor of a power of ten.

Example 3. Write $\frac{31}{40}$ as a decimal.

$$\begin{array}{r}
0\,0.7\,7\,5 \\
40\overline{)\,3\,1.0\,0\,0\,0} \\
\underline{-\,2\,8\,0} \\
3\,0\,0 \\
\underline{-\;\;2\,8\,0} \\
2\,0\,0 \\
\underline{-\,2\,0\,0} \\
0
\end{array}$$

This division terminates (comes out even) after just three decimal digits.

We get $\frac{31}{40} = 0.775$. This is a **terminating decimal**.

(The fact the division was even means that the denominator 40 is a factor of some power of ten, and so we could have used method 2 from above. In this case, $1000 = 40 \cdot 25$.)

Example 4. Write $\frac{18}{11}$ as a decimal.

$$\begin{array}{r}
0\,1.6\,3\,6\,3 \\
11\overline{)\,1\,8.0\,0\,0\,0} \\
\underline{-1\,1} \\
7\,0 \\
\underline{-\,6\,6} \\
4\,0 \\
\underline{-\,3\,3} \\
7\,0 \\
\underline{-\,6\,6} \\
4\,0 \\
\underline{-\,3\,3} \\
7
\end{array}$$

We write 18 as 18.0000 in the long division “corner” and divide by 11. Notice how the digits “63” in the quotient, and the remainders 40 and 70, start repeating.

So $\frac{18}{11} = 1.\overline{63}$.

The fraction 18/11 equals $1.\overline{63}$, which is a **repeating decimal**.

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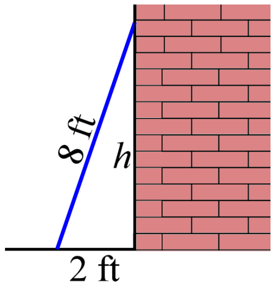
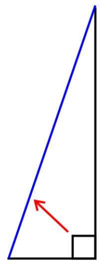
Applications of the Pythagorean Theorem 1

Example 1. An eight-foot ladder is placed against a wall so that the base of the ladder is 2 ft away from the wall. What height does the top of the ladder reach?



Since the ladder, the wall, and the ground form a right triangle, this problem is easily solved by using the Pythagorean Theorem. Let h be the unknown height. From the Pythagorean Theorem, we get:

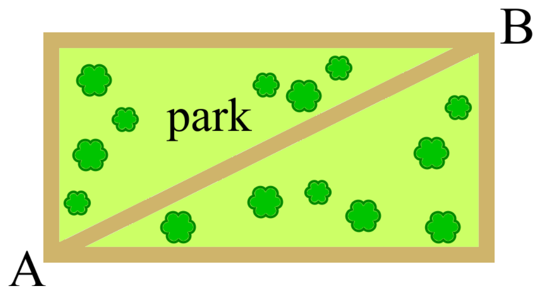
$$\begin{aligned} 2^2 + h^2 &= 8^2 \\ 4 + h^2 &= 64 \\ h^2 &= 60 \\ h &= \sqrt{60} \\ h &\approx 7.75 \end{aligned}$$



Our answer, 7.75, is in feet. This means the ladder reaches to about 7 3/4 ft = 7 ft 9 in. high.

1. The area of a square is 100 m^2. How long is the diagonal of the square?

2. A park is in the shape of a rectangle and measures 48 m by 30 m. How much longer is it to walk from A to B around the park than to walk through the park along the diagonal path?



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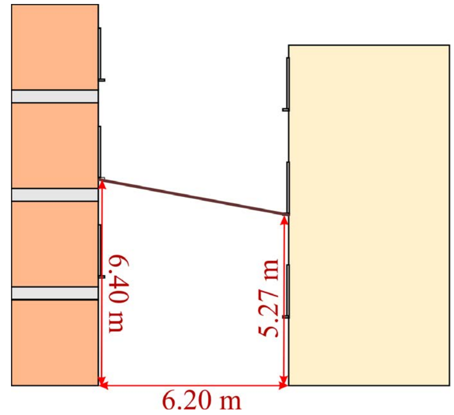
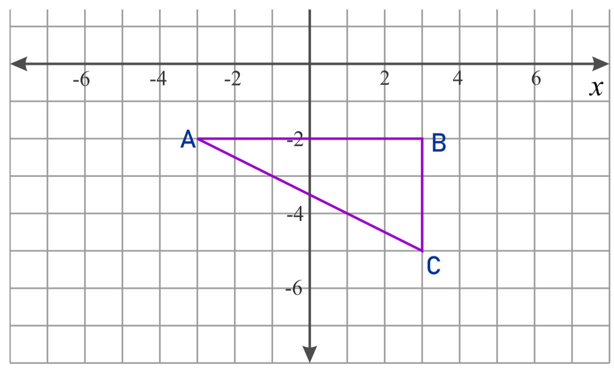
3. A clothesline is suspended between two apartment buildings. Calculate its length, assuming it is straight and doesn't sag any.



4. Find the perimeter of triangle ABC to the nearest tenth of a unit.



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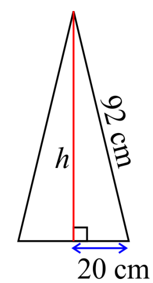
Example 2. Find the area of an isosceles triangle with sides 92 cm, 92 cm, and 40 cm.

Solution: To calculate the area of any triangle, we need to know its altitude.
When we draw the altitude, we get a right triangle:



The next step is to apply the Pythagorean Theorem to solve for the altitude *h*,
and after that calculate the actual area.

5. Calculate the area of the isosceles triangle in the example above to the nearest ten square centimeters.

6. Calculate the area of an equilateral triangle with 24-cm sides to the nearest square centimeter.
Don’t forget to draw a sketch.

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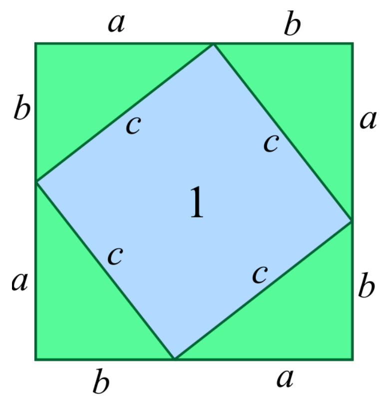
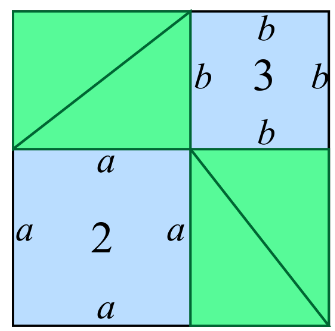
A Proof of the Pythagorean Theorem and of Its Converse

There exist hundreds of different proofs for the Pythagorean Theorem. In this lesson, we will look at two geometric proofs.

Proof.



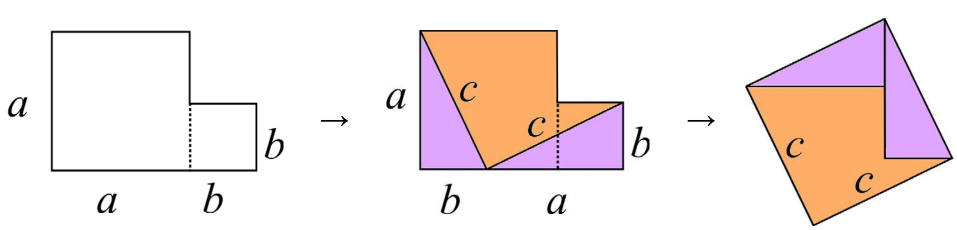
The figure above has four right triangles, each with sides *a*, *b* and *c*. The sides of the outside square are *a* + *b*. The triangles enclose a square with sides *c* units long.



Here the sides of the large square are still *a* + *b*, but the four right triangles have been rearranged to form two smaller squares, with sides *a* and *b*.

Since the areas of both large squares are equal, and the areas of the four right triangles are equal, it follows that the remaining (blue) areas are also equal. In other words, the area of square 1, which is c^2, equals the area of square 2 (which is a^2) plus the area of square 3 (which is b^2). In symbols, $c^2 = a^2 + b^2$. 🙂

1\. Figure out how this proof of the Pythagorean Theorem works.



2\. Study one of the proofs enough so you can explain it to someone else. Then do so.

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Chapter 7: Systems of Linear Equations
Introduction

This chapter covers how to analyze and solve pairs of simultaneous linear equations. (The equations studied contain only two variables.)

The first lesson, *Equations with Two Variables*, is optional. It reinforces the idea that a point on a line satisfies the equation of the line, and thus prepares the way for the main topics in the book.

First, students learn to solve systems of linear equations by graphing. Since each equation is an equation of a line, this is a simple technique, but it has its limitations, thus, algebraic solution methods will be taught also, in later lessons.

In the next lesson, we look at the number of solutions that a system of two linear equations can have. The three possible situations are easy to see based on the graphs of the equations: either one solution (the lines intersect in one point), no solutions (lines are parallel), or an infinite number of solutions (the lines are the same).

In the next lesson, students learn the algebraic method of solving systems of equations by substitution. This is a straightforward technique that many students will grasp easily. However, one has to be careful not to make simple mistakes. The lesson has some practice problems where students practice finding errors in solutions. Instruct the student(s) to check their solutions each time, as that is the best way to catch errors.

As for me (Maria, the author), as I wrote the answer key, I immediately checked my solution for each system of equations, and several times found an error that way. (The funniest errors were when I had switched from x to y in the middle of the solution!) So, checking the solution is important. To save space the answer key does not include the checks, but the student should always do that, whether with mental math or with a calculator.

The following lesson, *Applications, Part 1*, has a variety of word problems that students can now solve using a system of equations.

After that, students learn another algebraic method for solving systems of equations: the addition or elimination method. This is useful when the coefficients of the variables are such that you can easily find their least common multiple. Students also practice solving more complex systems, where the equations first have to be transformed and simplified, or include fractions.

Then it is time for more word problems, in the lesson *Applications, Part 2*. One lesson is devoted to problems about speed, time, and distance, and another for mixtures and comparisons. Making a chart is very helpful in these situations.

Pacing Suggestion for Chapter 7

This table does not include the chapter test as it is found in a different book (or file).
Please add one day to the pacing if you use the test.

The Lessons in Chapter 7	page	span	suggested pacing	your pacing
Equations with Two Variables	119	*4 pages*	1 day	
Solving Systems of Equations by Graphing	123	*5 pages*	1-2 days	
Number of Solutions	128	*4 pages*	1 day	
Solving Systems of Equations by Substitution	132	*7 pages*	2 days	
Applications, Part 1	139	*4 pages*	1 day	
The Addition Method, Part 1	143	*5 pages*	1 day	
The Addition Method, Part 2	148	*5 pages*	1 day	
More Practice	153	*4 pages*	1 day	

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Applications, Part 2	157	*3 pages*	1 day
Speed, Time, and Distance Problems	160	*6 pages*	1-2 days
Mixtures and Comparisons	166	*5 pages*	1 day
Mixed Review Chapter 7	171	*4 pages*	1 day
Chapter 7 Review	175	*5 pages*	2 days
Chapter 7 Test (optional)			
TOTALS		*61 pages*	15-17 days

Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this chapter, including pages that offer:

- **online practice** for concepts;
- online **games**, or occasionally, printable games;
- **animations** and interactive **illustrations** of math concepts;
- **articles** that teach a math concept.

We heartily recommend you take a look! Many of our customers love using these resources to supplement the bookwork. You can use these resources as you see fit for extra practice, to illustrate a concept better and even just for some fun. Enjoy!

https://l.mathmammoth.com/gr8ch7

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Equations with Two Variables

(This lesson is optional.)

The equation $2x + 3y = 16$ has **two variables**, x and y. One solution to the equation is $x = 2$ and $y = 4$, because when we substitute those values to the equation, it checks, or is a true equation:

$$2(2) + 3(4) = 16$$

But it also has the solution $x = 0.5$ and $y = 5$:

$$2(0.5) + 3(5) = 16$$

In fact, we can choose any number we like for the value of x, and then *calculate* the value of y, and thus find another solution to the equation.

For example, if we choose $x = -1$, then we get

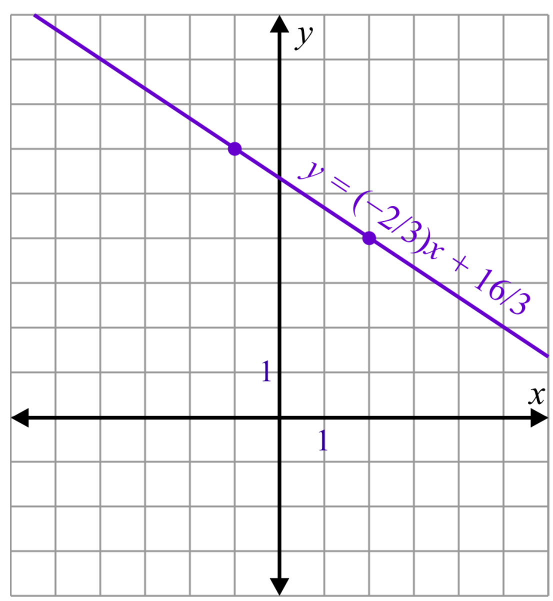
$$2(-1) + 3y = 16$$

from which $y = (16 + 2)/3 = 6$. So, $x = -1$, $y = 6$ is yet another solution.

All of these solutions, having both x and y values, are **number pairs**, and can be considered as **points on the coordinate plane.**

We can make a table of some of the possible (x, y) values (solutions):

x	y
−1	6
0	16/3
0.5	5
2	4



...and there are many more. When plotted, **these points fall on a line** — and you can probably guess, the equation of that line is $2x + 3y = 16$!
(Or, in slope-intercept form, $y = (-2/3)x + 16/3$.)

A line in the coordinate plane represents all the solutions to the equation that is the equation of the line. In other words, **each point on the line is a solution to the equation.**

1. Find three solutions to the equation $5x + 2y = 32$.

2. Find three solutions to the equation $-4x + y = -6$.

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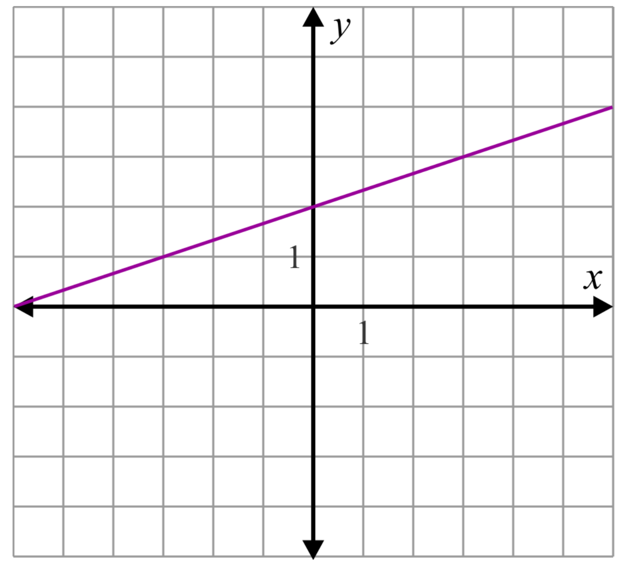
3. **a.** What is the equation if its solution set is represented by this line?



b. List two distinct integer number pairs that are solutions to the equation.

4. A certain linear equation with two variables has as solutions (0, −5), (2, 3) and (4, 11). Find the equation.

5. A certain linear equation with two variables has as solutions (−1, −5) and (2, 8). Find the equation.

6. Party hats cost \$2 apiece and party whistles cost \$3 apiece. Randy buys *x* hats and *y* whistles.

a. Write an expression depicting his total cost (C).

b. Now write an equation stating that his total cost is \$48.

How many hats and how many whistles could Randy have bought?

c. Find two other solutions to your equation.

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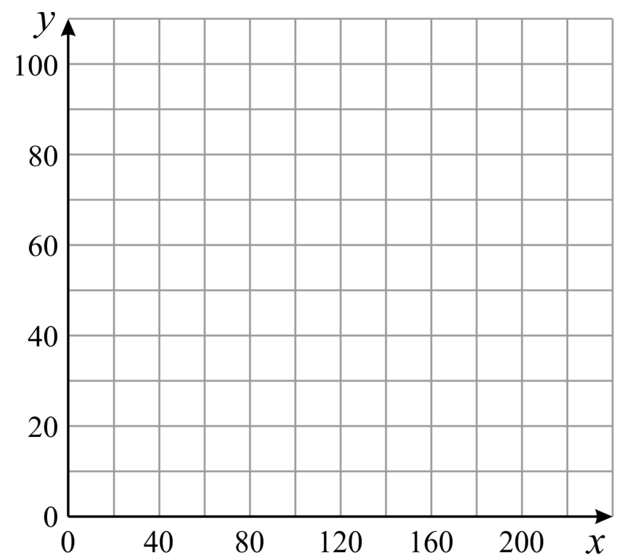
7. Recall the formula tying together distance (*d*), constant speed (*v*), and time (*t*): $\boldsymbol{d = vt}$.
Sarah jogs at the speed of 6 miles per hour, and she rides her bicycle at the speed of 12 miles per hour.

a. Convert these speeds to miles per minute.

b. Write an expression for the total distance (*d*) Sarah covers in *x* minutes of jogging plus *y* minutes of bicycling.

c. What distance does Sarah cover if she jogs for 20 minutes and bicycles for 10 minutes?

d. Let's say the distance Sarah covers, jogging and bicycling, is 20 miles. Write an equation stating this. How many minutes could she have jogged/bicycled? Find three possible solutions.



e. Write the equation in slope-intercept form and plot it.

8. General admission to a gardening seminar was $15 but seniors paid only $10. If the total of the admission fees was $900, give three possibilities as to how many non-seniors and how many seniors could have attended.

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9. A mystery basket contains a mixture of adult cats and kittens (it could even contain zero adults or zero kittens).
Each cat weighs 4 kg and each kitten weighs 0.5 kg.
The total weight of the cats and kittens is 20 kg.

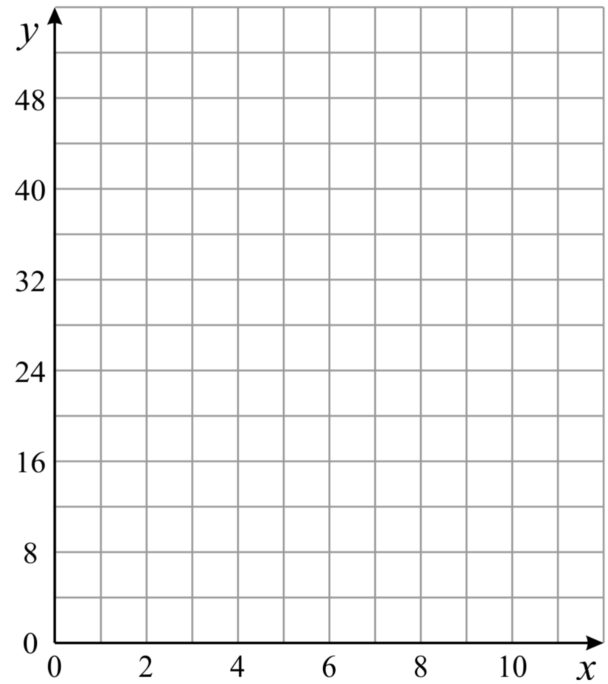
a. If there are x cats and y kittens, write an equation to match the situation.

b. How many adult cats and how many kittens could there be? Find at least three different solutions.

c. Plot your equation from (a).

d. If $x = 1.5$, what is y?
Why is this not a valid solution?

Plot the individual points on the graph that *are* valid solutions.



10. Ava and her family went to stay in a resort for a few nights. Each night cost \$120 (for the whole family). The resort offered horse rides for \$20 per person.

a. If the family stayed for x nights and did y horse rides in total, write an expression for the total cost of these two things.

b. In total, Ava's family spent \$760 on the horse rides plus the nights they stayed.
How many nights and how many horse rides could they have paid for?

11. The equation $2x^2 - 6x - y = 5$ is a quadratic equation because the variable x is squared. If $x = 0$, then $y = -5$, so $(0, -5)$ is one solution to the equation. Find two other solutions to it.

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Solving Systems of Equations by Graphing

A system of equations consists of several equations that have the same variables.

A **solution** to a system of equations is a list of values of the variables that satisfy *all* the equations in the system. For two equations, this is an ordered pair.

Example 1. This system of equations consists of two equations. We signify the system with a bracket.

$$\begin{cases} 5x + 4y = 12 \\ y = -x + 2 \end{cases}$$

The solution to the above system is the ordered pair (4, −2), because those values make both equations true: 5(4) + 4(−2) does equal 12, and −2 does equal −4 + 2.

Example 2. The equation $y = (3/2)x - 4$ has an infinite number of solutions, and we can represent those solutions with a line drawn in the coordinate plane.

Similarly, the equation $y = -2x + 3$ has infinitely many solutions.

Here is a system of equations consisting of both:

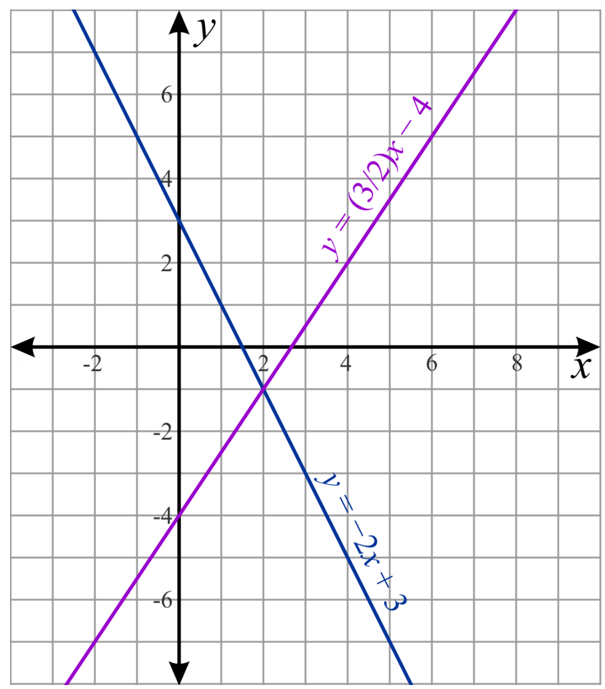
$$\begin{cases} y = (3/2)x - 4 \\ y = -2x + 3 \end{cases}$$

Since the solutions to the first equation form a line, and the solutions to the second also form a line, what would the point of intersection (2, −1) signify?

____

____

(The answer is found at the end of the lesson.)



1. Solve each system of equations using the image. The lines are already plotted in it.

a. $\begin{cases} y = -7x - 23 \\ y = (1/3)x - 1 \end{cases}$

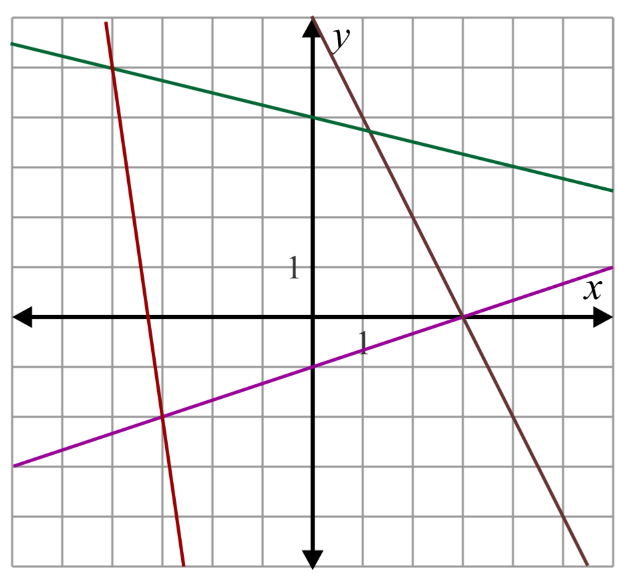
Solution: (_____, _____)

b. $\begin{cases} y = -(1/4)x + 4 \\ y = -2x + 6 \end{cases}$

Solution: (_____, _____)

c. $\begin{cases} -(1/3)x + y = -1 \\ 2x + y = 6 \end{cases}$

Solution: (_____, _____)



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Speed, Time, and Distance Problems

There are many jokes about algebra word problems where a train leaves a station at a certain hour. You can now solve these types of problems with your knowledge of systems of equations. One of the most effective ways to do so is to first build a chart.

Example 1. A train leaves a station at 9:00 AM and travels with a constant speed of 90 km/h. Another train leaves the same station 10 minutes later, traveling to the same direction at the speed of 100 km/h. At what time will the second train reach the first?

We will be using the formula $d = vt$ extensively in these problems. Let's build a chart. The goal is to have TWO, not three or more, variables present in the chart. The formula $d = vt$ has three variables, and since the speed, distance, and time can be different for each train, theoretically we could have six variables. However, invariably, the problem gives information for one or some of these variables, and something about the situation means that the distance or the time or the speed is the same for both trains.

To get started, we gather some information in the chart. The distance that train 1 and train 2 travel until they meet is the same, so that is why we use the same variable, *d*, for it.

	distance	***velocity***	***time***
Train 1	d	90 km/h	t_1
Train 2	d	100 km/h	t_2

The times (t_1 and t_2) are different, but we do know that they differ by 10 minutes, so, actually we will get by using only *one* variable for time, like this:

	distance	***velocity***	***time***
Train 1	d	90 km/h	t
Train 2	d	100 km/h	$t - 10$

The chart now contains only two variables. However, we have one more thing to change. The speed is in km/h, whereas the 10 has to do with minutes. For our equation to work, the time units need to be the same, so we will change the 10 to 1/6 (in hours).

	distance	***velocity***	***time***
Train 1	d	90 km/h	t
Train 2	d	100 km/h	$t - 1/6$

The equations always follow the same formula: $d = vt$, and we use that same formula for both Train 1 and Train 2. So, the two equations we get are:

$$\begin{cases} d = 90t \\ d = 100(t - 1/6) \end{cases}$$

The quickest way to solve this system is to set $90t$ equal to $100(t - 1/6)$ and solve for t.

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1. Solve the system of equations from example 1 and answer the question: At what time will the second train reach the first? Is the answer surprising?

$$\begin{cases} d = 90t \\ d = 100(t - 1/6) \end{cases}$$

2. Your friend starts walking at a speed of 6 km/h from your home to his. Exactly 15 minutes later, you decide you want to join him so you take your bicycle and start after him, with a speed of 18 km/h. How far are you when you reach your friend?

	distance	***velocity***	***time***
Your friend			
You			

3. A tortoise and hare race a distance of 100 m. The hare gives the tortoise a 10-minute lead time. Then he quickly runs the 100 meters and wins the race. After the hare has finished, the tortoise takes an *additional* 6 minutes to reach the finish line. If the speed of the hare is 15 m/s, find the time the tortoise takes to finish the race and the tortoise's speed.

Hint: since the speed is in meters per second, and the distance is in meters, the time unit will be seconds.

	distance	***velocity***	***time***
Tortoise			
Hare			

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Example 2. A train leaves Turin (Italy), heading for Milan (Italy), a distance of 125 km, at 1 PM and travels with a constant speed of 90 km/h. Another train leaves Milan, heading for Turin and traveling at a constant speed, at the same time. They meet 45 minutes later. (We hope they don't crash!) What is the speed of the second train? What distance has the second train traveled by that time?

We fill our chart again. The time, 45 minutes, is 3/4 hour. The speed of the second train, v_2, is unknown.

There is a relationship between the two distances, because $d_1 + d_2 = 125$ km. So, we can get by with just one variable for the distance:

	distance	***velocity***	***time***
Train 1	d	90 km/h	3/4
Train 2	$125 - d$	v_2	3/4

Our equations are: $\begin{cases} d = 90(3/4) \\ 125 - d = (3/4)v_2 \end{cases}$

Here, it is handy to use the substitution method, since we have an expression for d. So, we substitute 90(3/4), which equals 67.5, in place of d in the second equation:

$$\begin{aligned} \textbf{(2)}\quad 125 - \mathbf{67.5} &= (3/4)v_2 \\ 57.5 &= (3/4)v_2 \quad \Big|\cdot 4 \\ 230 &= 3v_2 \\ v_2 &= 76.\overline{6} \end{aligned}$$

So, the speed of the second train is $76.\overline{6}$ km/h. The problem also asked what distance the second train has traveled by that time. To find that, we use the formula $d = vt$: $d = 76.\overline{6}$ km/h · (3/4 h) = 57.5 km.

4. Two trains leave the same station at the same time, one traveling due east and the other traveling due west. Train 1 travels at a speed of 120 km/h and Train 2 at the speed of 100 km/h. When are the trains 50 km apart from each other?

	distance	***velocity***	***time***
Train 1			
Train 2			

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5. Train 1 leaves the station heading due south and Train 2 leaves the same station at the same time, heading due north. Train 1 travels at the speed of 70 mph. After 30 minutes, the trains are 75 miles apart. How fast is Train 2 traveling?

	distance	***velocity***	***time***
Train 1			
Train 2			

6. **a.** Two horses, Ranger and Chip, start racing at the same time. Ranger runs at a steady speed of 16 m/s. After 100 seconds, they are 600 m apart from each other, Ranger leading. How fast is Chip running?

	distance	***velocity***	***time***
Ranger			
Chip			

b. What would Chip's speed need to be, so that after 100 seconds, he would only be 50 m behind Ranger?

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Example 3. A motorboat travels downstream on the river a distance of 5 miles, in 20 minutes. Doing the same trip upstream takes it 4 minutes longer. How fast is the river flowing? What is the speed of the boat in still water?

Our chart method will still work. We are dealing with two speeds: that of the boat (v_b) (in still water), and that of the water (v_w) in the river. Going downstream, the boat's actual speed is its own speed PLUS the speed of the water. Going upstream, it has to fight the current and its actual speed is $v_b - v_w$.

This is what the chart looks like. Using these quantities, the speeds will end up being in miles per hour.

	distance	***velocity***	***time***
Downstream	5 mi	$v_b + v_w$	20 min
Upstream	5 mi	$v_b - v_w$	24 min

Our equations are: $$\begin{cases} 5 = 20(v_b + v_w) \\ 5 = 24(v_b - v_w) \end{cases}$$

7. Solve the problem in example 3.

8. An airplane travels from City A to City B in 2 hours 30 minutes, flying with the wind, and in 2 hours 45 minutes flying against the wind. If the speed of the airplane in still air is 900 km/h, find the distance between the cities to the nearest 10 km.

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9. With a tailwind, an airplane can fly from City 1 to City 2, a distance of 650 km, in 40 minutes. If the speed of the wind is 30 km/h, find the time the plane takes to fly the same distance against the wind.

10. You swim downstream, from a dock to a certain rock in the middle of the river, in 43 seconds. Swimming back (upstream) takes you 10 seconds longer. If your swimming speed in still water is 1.0 mph, what is the speed of the water in the river?

Puzzle Corner

The following are “trick” problems. Have fun!

(1) Train 1 leaves Jackson at 1:30 PM, traveling at 95 km/h towards Atlanta, a distance of 560 km, and Train 2 leaves Atlanta, heading towards Jackson at the same time, traveling at 105 km/h. When they meet, which train is closer to Atlanta?

(2) At 6:30 PM, you board a train in Dallas, heading south, and 10 minutes later, your friend boards a train at Kansas City, heading north. If both trains travel at 70 mph, when do you pass each other?

(3) Two trains leave a station at the same time, one heading east, the other heading west. After 15 minutes, they are 40 miles apart. Which train is traveling faster?

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Chapter 8: Bivariate Data
Introduction

The last chapter of grade 8 covers statistical topics that have to do with bivariate data, or data involving two variables.

The first lesson introduces scatter plots. Students analyze the data in a variety of scatter plots, and determine visually whether there is an association between the variables. Next, they learn about basic patterns we often see in scatter plots, such as positive and negative association, linear association, clusters, and outliers. They also make scatter plots from given data, describe any special features in the plot, and answer a variety of questions related to the data.

In the following lesson, students fit a line (informally) to the data points displayed in a scatter plot. Mathematicians have developed several algorithms for finding a line of best fit, such as linear regression, but we are not using those here. Students use the basic idea of trying to leave close to an equal number of points on each side of the line, and also judging the fit by the closeness of the points to the line. This resembles the thought behind the linear regression algorithm, which finds the line of best fit by minimizing the squares of the distances of the data points to the line.

The last topic relating to scatter plots is the equation of the trend line. Students use the equation of the trend line to solve problems in the context of the data, interpreting the slope and intercept of the equation.

Then we turn our attention to categorical bivariate data, that is, data involving two variables that may or may not be numerical, but is divided into categories. Students learn that bivariate categorical data can be summarized in a two-way table, and if there is a pattern of association between the variables, it can be seen in the table.

Students construct and interpret two-way tables summarizing data on two categorical variables. In the last lesson, they calculate relative frequencies for rows or columns, and use those to describe the possible association between the two variables.

Pacing Suggestion for Chapter 8

This table does not include the chapter test as it is found in a different book (or file).
Please add one day to the pacing if you use the test.

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Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this chapter, including pages that offer:

- **online practice** for concepts;
- online **games**, or occasionally, printable games;
- **animations** and interactive **illustrations** of math concepts;
- **articles** that teach a math concept.

We heartily recommend you take a look! Many of our customers love using these resources to supplement the bookwork. You can use these resources as you see fit for extra practice, to illustrate a concept better and even just for some fun. Enjoy!

https://l.mathmammoth.com/gr8ch8

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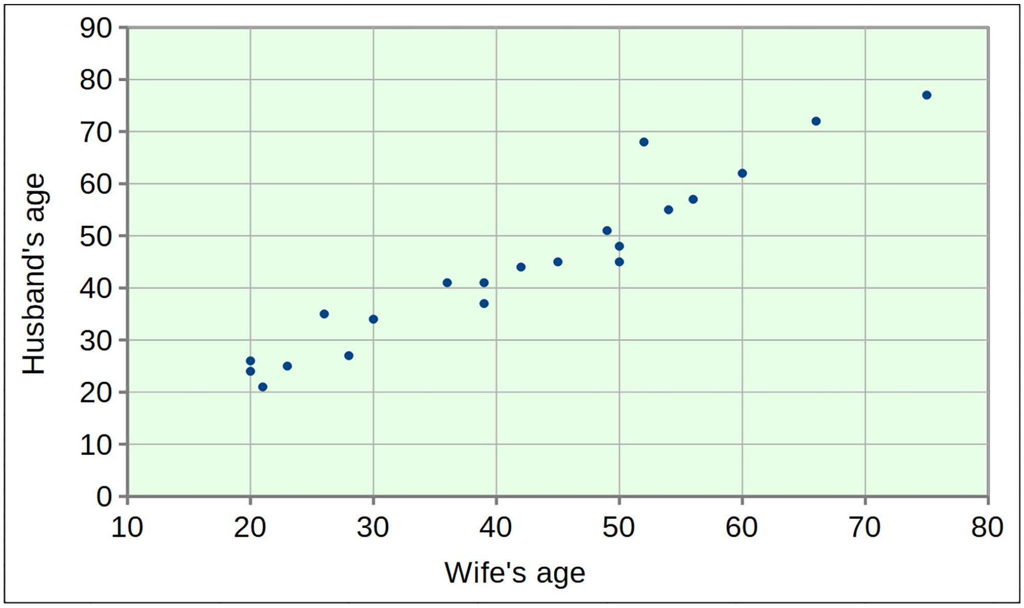
Scatter Plots

A **scatter plot** depicts **bivariate data**, meaning that the data involves **two variables**. In the scatter plot below, the variables are the husband's age and the wife's age. Each dot in this scatter plot represents a husband-wife couple. In other words, the coordinates of the dot give us the ages of the husband and the wife.



1. Refer to the scatter plot above.

a. Locate the dot with coordinates (36, 41). What does it signify?

b. Find two couples where the wife is the same age in both cases. Estimate the ages of their husbands.

c. Find the couple with the third oldest husband in this data set. How old is his wife?

d. Is it true that the youngest wife is married to the youngest husband? Explain.

e. Is it true that the oldest wife is married to the oldest husband? Explain.

f. Do you notice a relationship between the two variables? Explain what you see.

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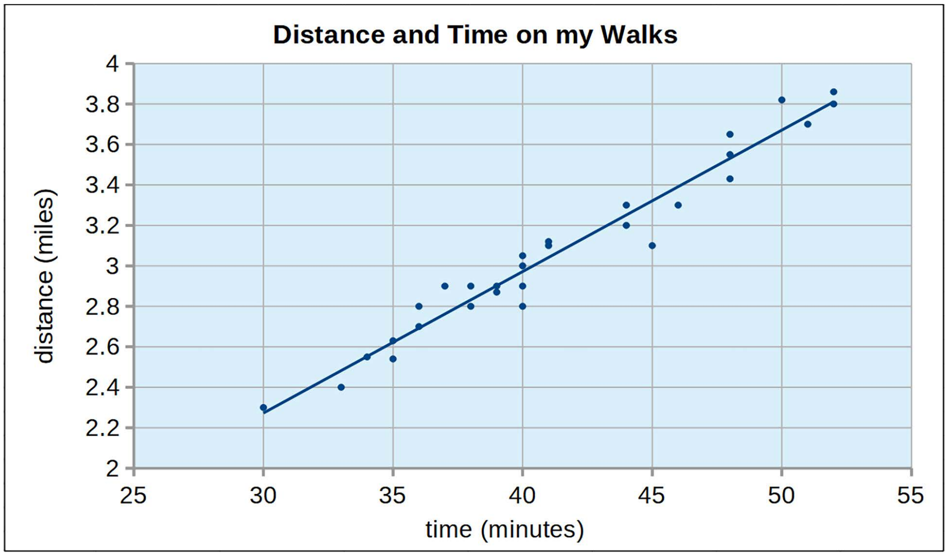
Equation of the Trend Line

Going back to the Sofia's walks, the graph shows a line that is fitted to the data points. Its equation is $d = 0.07t + 0.175$, where d is the distance in miles and t is the time in minutes. The equation was calculated by a spreadsheet program. The line we drew in the previous lesson by using the ellipse method is remarkably close to this line calculated with an algorithm.



The slope of the line, 0.07, tells us that for each one-minute increment, the distance increases by 0.07 miles. In other words, Sofia tends to walk at a speed of about 0.07 miles per minute.

The d-intercept, 0.175 miles, means that the equation predicts that at zero minutes, Sofia would have walked 0.175 miles. This doesn't make sense. Keep in mind that the equation is calculated using data that was limited to between 30 and 52 minutes. Estimating values outside of that range is called extrapolating, and we need to be careful with that. The equation is a model, and it may not be valid outside the original range of values.

1. Use the equation, $d = 0.07t + 0.175$ as a model for Sofia's walks. Choose all the correct statements.

- Each 1-minute increment in time is associated with a 0.175-mile increment in distance.
- Each 1-minute increment in time is associated with a 0.07-mile increment in distance.
- The average distance she walks is 0.175 miles.
- The equation predicts a distance of 0.07 miles when the time is 0.175 minutes.

2. Concerning Sofia's walks again, if we tell the spreadsheet program to force the line to go through (0, 0), the program calculates the equation as $d = 0.074t$. This line is a slightly worse fit considering the data points, but it matches reality in the sense that when $t = 0$, the distance is zero also.

a. Using this equation, predict how many miles Sofia would walk in 32 minutes.

b. How much time would you expect Sofia to take for a 3.3-mile walk?

c. Use the equation $d = 0.074t$ and fill in. Each five minutes added to the walk time is associated with

a ______-mile increase in her walking distance.

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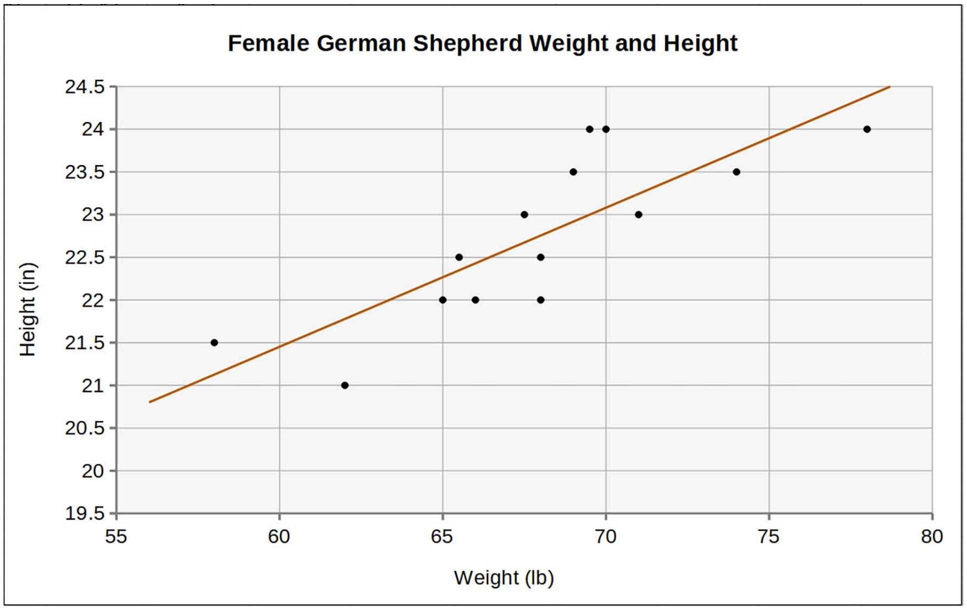
3. The scatter plot below shows the weight and height of various adult female German shepherds. (It does not have to do with weight gain/loss of an individual dog — each dot signifies a different dog.) The equation for the trend line is $h = 0.16w + 11.68$, where h is the height in inches and w is the weight in pounds.



a. Which statements are correct?

- Each 0.16-lb increase in weight is associated with a 1-inch increase in height.

- Each 1-lb increase in weight is associated with a 0.16-inch increase in height.

- Heavier dogs tend to be taller; and for each 5-lb increase in the weight, the dogs tend to be 0.8 inches taller.

- The model predicts a height of 11.68 inches for a dog weighing zero pounds.

- The model predicts a weight of 11.68 lb for a dog that is zero inches tall.

- We should be careful in using this model to extrapolate the heights of dogs less than 55 pounds.

b. Use the equation to predict the weight of a dog that is 22.5 inches tall, to the nearest pound.

c. Use the equation to predict how tall a 63-lb dog would be.

d. Would a dog that weighs 60 lb and is 21 inches tall be considered an outlier?

e. What is the difference between the predicted height of a 75-lb dog and its real height, if in reality it is 24 1/4 inches tall?

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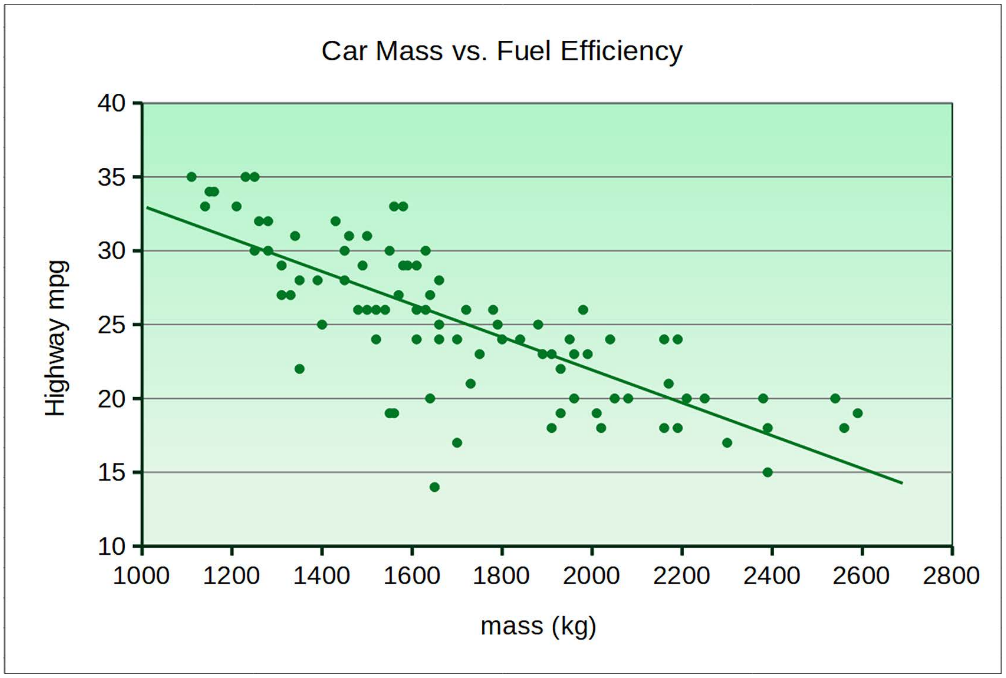
4. The line fitted to the data about the mass (m) and fuel efficiency (MPG) of cars has the equation

$$\text{MPG} = -0.011115m + 44.156$$



a. What does the slope of −0.011115 signify in this context?

b. For each 100-kg increase in a car's mass, how much would you expect the fuel efficiency to change?

c. Predict the fuel efficiency of a 2500-kg car.

d. If a car gets 27.5 mpg in highway driving, what would you expect its mass to be?

e. Find the dot in the graph for Lamborghini Murcielago A-S6, which weighs 1650 kg and gets only 14 mpg in highway driving. Based on the equation of trend line, what would you expect this car's fuel efficiency to be?

f. Let's say you want to buy a car that gets at least 27 mpg in highway driving. Based on the scatter plot, how much would you expect your car to weigh, at a maximum?

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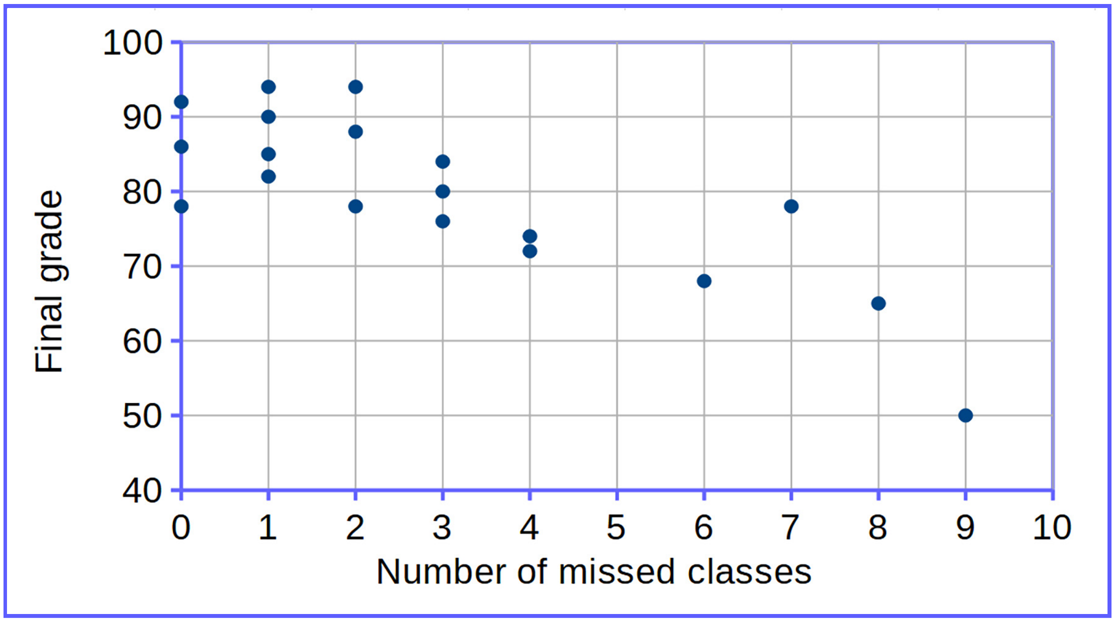
5. **a.** Draw a line to fit the trend in the scatter plot below.



b. Find the (approximate) equation of your line.
Hint: find the y-intercept and the slope.

c. What does the slope of your equation signify in this context?

d. What does the *y*-intercept of your equation signify in this context?

e. Using your equation, predict what the final grade would be for a student that missed five classes.

f. Tanya calculated the equation of the trend line as $G = 3m + 89$, where G is the grade and *m* is the number of the missed classes. Explain why this cannot possibly fit the data.

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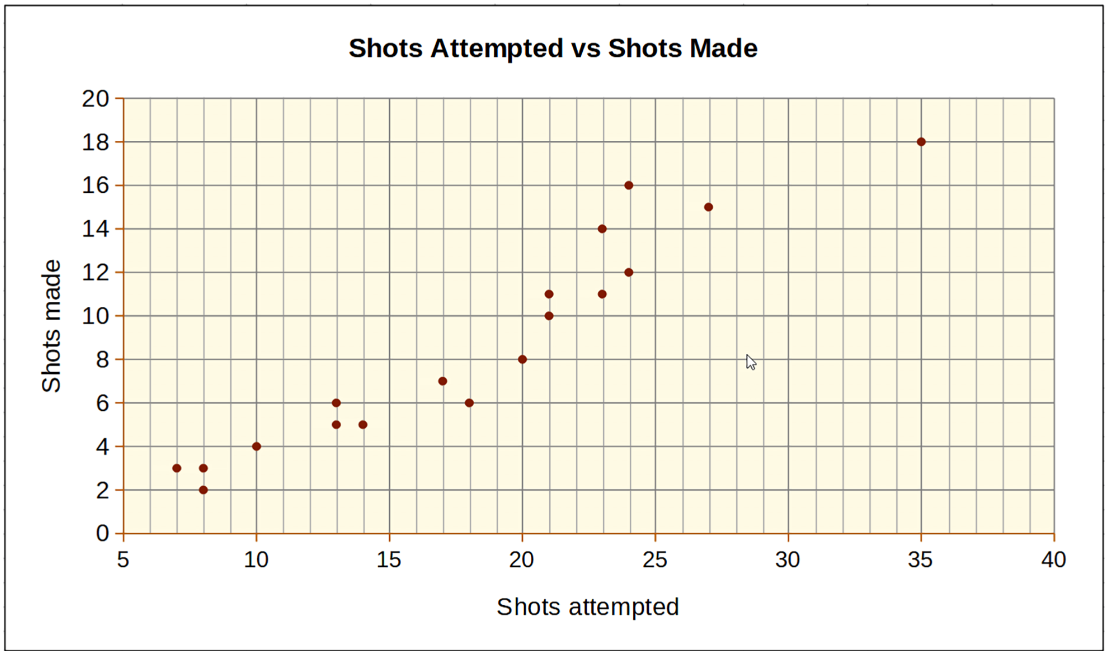
6. The data shows shots the attempted shots and the shots made of 18 basketball players.



a. Draw a line that fits the trend in the data.

b. Find the equation of your line.

c. What does the slope in the equation signify in this context?

d. And the *y*-intercept?

e. If a player attempts 30 shots, how many shots does your model predict they would make?

f. What prediction does your model make about the number of shots attempted by a player if they made 8 shots?

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Two-Way Tables

We have been studying bivariate data, in other words, data that involves two variables. So far, that data has all been numerical: we have had two numerical values for each data item, and therefore, we have been able to plot the data as points in the coordinate plane, the two coordinates being the values of the two variables.

Now we will look at bivariate data that is organized into categories, and the values may or may not be numerical.

The **two-way table** on the right shows how many students in each grade of an elementary school can swim, and how many cannot. It also shows the row and column totals.

It is called a two-way table because it records the information from *two* variables. In this case, the two variables are the student's grade level and whether the student can swim or not.

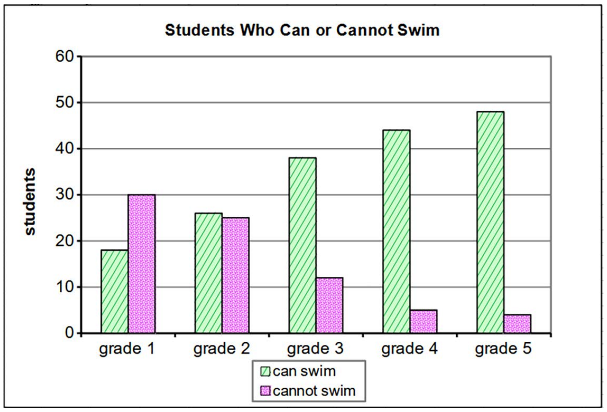
The first variable takes numerical values from 1 to 5. The table has a row corresponding for each of those values. The second takes the values "Yes" and "No", and those correspond to the two columns labeled "can swim" and "cannot swim".

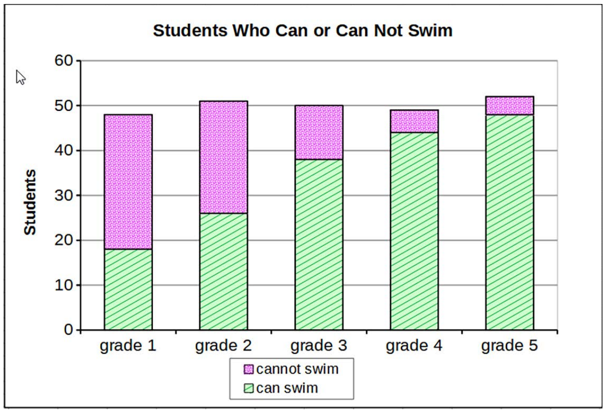
Grade	can swim	cannot swim	Total
1	18	30	48
2	26	25	51
3	38	12	50
4	44	5	49
5	48	4	52
TOTAL	**174**	**76**	**250**

The original data may look like this: (1, no), (1, yes), (2, no), (2, no), (3, no), (5, yes), etcetera, each pair of data depicting one student. But we cannot analyze or summarize the data when it is in that format. A two-way table allows us to **categorize and tally up the data**, and then also to analyze it to see if there is any **association between the two variables**.

In this case, we can see that with advancing grade level, there are many more students who can swim than who cannot. In 5th grade, nearly all students can swim. So, there *is* an association between the variables.

The data from a two-way table can be presented as a double-bar graph (left), or a stacked bar graph (right):





A stacked bar graph is more common. From the graphs, it is easy to see that as the grade level advances, more and more students are able to swim.

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1. A community college tracked how many of their students in a given year completed a certain course that was offered both as an in-person course and as an online version.

	completed the course	did not complete the course	Total
In person	57	8	65
Online	23	25	48
TOTAL	80	33	113

a. Looking only at those who took the online course, what percentage of them completed the course? What percentage did not?

b. Looking only at those who took the in-person course, what percentage of them completed the course? What percentage did not?

c. Is there a relationship or association between the variables? Explain.

2. Jordan asked a bunch of people at a local gym as to their opinion on increasing the membership price in return for some improved facilities. He categorized the people as 20-30 year olds and as 31+ year olds. Here are some highlights of Jordan's research:

- Of the seventy-seven 20-30 year olds, 21 were in favor of the plan.
- In total, there were 79 people for and 70 people against this plan.

a. Fill in the two-way table below from this data. Use the two age groups as rows. Fill in all the numbers.

	for	against	Total
20-30 years old			
31+ years old			
TOTAL			

b. Overall, are most of the 20-30 year olds for or against this plan?

What about of the 31+ year olds?

c. Now look at the column totals. Are there more people in general that are for the price increase or that are against it?

d. Do you feel there is an association between the variables? Explain.

e. Why do you think so many of the younger people are against this plan?

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3. At a family reunion, Ashley asked all her relatives whether at the next year's reunion they should pay a fee and gather at the local amusement park grounds. Here are the answers she got.

- Adults: (18) no, no, no, yes, no, no, yes, no, no, yes, yes, no, no, no, no, no, yes, yes
- Teens: (8) yes, yes, yes, yes, yes, yes, no, no
- Children (16): yes, yes, yes, yes, yes, yes, yes, yes, yes, yes, yes, yes, yes, yes, yes, yes

a. Create a two-way table from the results.

b. Based on the results, are there more people in favor or against gathering at the amusement park next year?

c. Does the answer change if children are ignored?

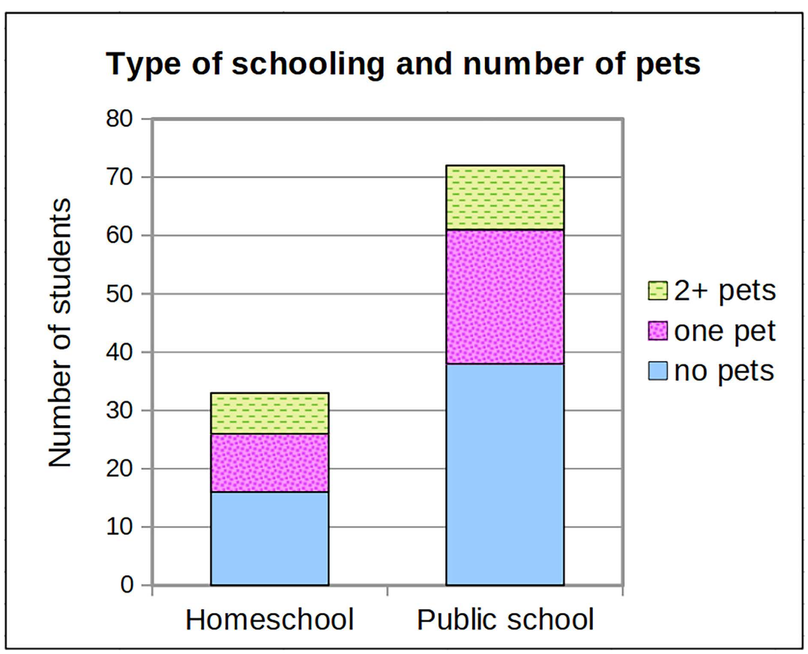
d. What percentage of the adults are in favor of this?

What percentage of the teens? Of the children? Of everyone?

e. The two variables are: age group (adult/teen/child) and opinion (yes/no). Is there a relationship or an association between the variables? What kind?

Puzzle Corner

Is there an association between the variables? Explain.



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