Contents

Chapter 3: Linear Equations

Chapter 4: Introduction to Functions

Foreword

Math Mammoth Grade 8 comprises a complete math curriculum for the eighth grade mathematics studies. The curriculum meets the Common Core standards.

In 8th grade, students spend the majority of the time with algebraic topics, such as linear equations, functions, and systems of equations. The other major topics are geometry and statistics.

The main areas of study in Math Mammoth Grade 8 are:

- Exponents laws and scientific notation
- Square roots, cube roots, and irrational numbers
- Geometry: congruent transformations, dilations, angle relationships, volume of certain solids, and the Pythagorean Theorem
- Solving and graphing linear equations;
- Introduction to functions;
- Systems of linear equations;
- Scatter plots/bivariate data.

We start with a study of exponent laws, using both numerical and algebraic expressions. The first chapter also covers scientific notation (both with large and small numbers), significant digits, and calculations with numbers given in scientific notations.

In chapter 2, students learn about geometric transformations (translations, reflections, rotations, dilations), common angle relationships, and volume of prisms, cylinders, spheres, and cones.

Next, in chapter 3, our focus is on linear equations. Students both review and learn more about solving linear equations, including equations whose solutions require the usage of the distributive property and equations where the variable is on both sides.

Chapter 4 presents an introduction to functions. Students construct functions to model linear relationships, learn to use the rate of change and initial value of the function, and they describe functions qualitatively based on their graphs.

In part 8-B, students graph linear equations, learn about irrational numbers and the Pythagorean Theorem, solve systems of linear equations, and investigate patterns of association in bivariate data (scatter plots).

I heartily recommend that you read the full user guide in the following pages.

I wish you success in teaching math!

Maria Miller, the author

User Guide

Note: You can also find the information that follows online, at **https://www.mathmammoth.com/userguides/** .

Basic principles in using Math Mammoth Complete Curriculum

Math Mammoth is mastery-based, which means it concentrates on a few major topics at a time, in order to study them in depth. The two books (parts A and B) are like a "framework", but you still have some liberty in planning your student's studies. In eighth grade, chapters 2 (geometry), 3 (linear equations) and chapter 4 (functions) should be studied before chapter 5 (graphing linear equations). Also, chapters 3, 4, and 5 should be studied before chapter 7 (systems of linear equations) and before chapter 8 (statistics). However, you still have some flexibility in scheduling the various chapters.

Math Mammoth is not a scripted curriculum. In other words, it is not spelling out in exact detail what the teacher is to do or say. Instead, Math Mammoth gives you, the teacher, various tools for teaching:

- **The two student worktexts** (parts A and B) contain all the lesson material and exercises. They include the explanations of the concepts (the teaching part) in blue boxes. The worktexts also contain some advice for the teacher in the "Introduction" of each chapter.

  The teacher can read the teaching part of each lesson before the lesson, or read and study it together with the student in the lesson, or let the student read and study on his own. If you are a classroom teacher, you can copy the examples from the "blue teaching boxes" to the board and go through them on the board.

- Don't automatically assign all the exercises. Use your judgment, trying to assign just enough for your student's needs. You can use the skipped exercises later for review. For most students, I recommend to start out by assigning about half of the available exercises. Adjust as necessary.

- For each chapter, there is a **link list to various free online games** and activities. These games can be used to supplement the math lessons, for learning math facts, or just for some fun. Each chapter introduction (in the student worktext) contains a link to the list corresponding to that chapter.

- The student books contain some **mixed review lessons**, and the curriculum also provides you with additional **cumulative review lessons**.

- There is a **chapter test** for each chapter of the curriculum, and a comprehensive end-of-year test.

- You can use the free online exercises at https://www.mathmammoth.com/practice/
  This is an expanding section of the site, so check often to see what new topics we are adding to it!

- And of course there are answer keys to everything.

How to get started

Have ready the first lesson from the student worktext. Go over the first teaching part (within the blue boxes) together with your student. Go through a few of the first exercises together, and then assign some problems for the student to do on their own.

Repeat this if the lesson has other blue teaching boxes.

Many students can eventually study the lessons completely on their own — the curriculum becomes self-teaching. However, students definitely vary in how much they need someone to be there to actually teach them.

Pacing the curriculum

Each chapter introduction contains a suggested pacing guide for that chapter. You will see a summary on the right. (This summary does not include time for optional tests.)

Worktext 8-A	
Chapter 1	13 days
Chapter 2	27 days
Chapter 3	21 days
Chapter 4	14 days
TOTAL	**75 days**

Most lessons are 3 or 4 pages long, intended for one day. Some lessons are 5 pages and can be covered in two days.

It can also be helpful to calculate a general guideline as to how many pages per week the student should cover in order to go through the curriculum in one school year.

The table below lists how many pages there are for the student to finish in this particular grade level, and gives you a guideline for how many pages per day to finish, assuming a 180-day (36-week) school year. The page count in the table below *includes* the optional lessons.

Example:

Grade level	School days	Days for tests and reviews	Lesson pages	Days for the student book	Pages to study per day	Pages to study per week
8-A	90	8	214	82	2.6	13
8-B						
Grade 8 total						

The table below is for you to fill in. Allow several days for tests and additional review before tests — I suggest at least twice the number of chapters in the curriculum. Then, to get a count of "pages to study per day", **divide the number of lesson pages by the number of days for the student book**. Lastly, multiply this number by 5 to get the approximate page count to cover in a week.

Grade level	Number of school days	Days for tests and reviews	Lesson pages	Days for the student book	Pages to study per day	Pages to study per week
8-A			214			
8-B						
Grade 8 total						

Now, something important. Whenever the curriculum has lots of similar practice problems (a large set of problems), feel free to **only assign 1/2 or 2/3 of those problems**. If your student gets it with less amount of exercises, then that is perfect! If not, you can always assign the rest of the problems for some other day. In fact, you could even use these unassigned problems the next week or next month for some additional review.

In general, 8th graders might spend 45-75 minutes a day on math. If your student finds math enjoyable, they can of course spend more time with it! However, it is not good to drag out the lessons on a regular basis, because that can then affect the student's attitude towards math.

Using tests

For each chapter, there is a **chapter test**, which can be administered right after studying the chapter. **The tests are optional.** The main reason for the tests is for diagnostic purposes, and for record keeping. These tests are not aligned or matched to any standards.

In the digital version of the curriculum, the tests are provided both as PDF files and as html files. Normally, you would use the PDF files. The html files are included so you can edit them (in a word processor such as Word or LibreOffice), in case you want your student to take the test a second time. Remember to save the edited file under a different file name, or you will lose the original.

The end-of-year test is best administered as a diagnostic or assessment test, which will tell you how well the student remembers and has mastered the mathematics content of the entire grade level.

Using cumulative reviews and the worksheet maker

The student books contain mixed review lessons which review concepts from earlier chapters. The curriculum also comes with additional cumulative review lessons, which are just like the mixed review lessons in the student books, with a mix of problems covering various topics. These are found in their own folder in the digital version, and in the Tests & Cumulative Reviews book in the print version.

The cumulative reviews are optional; use them as needed. They are named indicating which chapters of the main curriculum the problems in the review come from. For example, "Cumulative Review, Chapter 4" includes problems that cover topics from chapters 1-4.

Both the mixed and cumulative reviews allow you to spot areas that the student has not grasped well or has forgotten. When you find such a topic or concept, you have several options:

1. Check for any online games and resources in the Introduction part of the particular chapter in which this topic or concept was taught.
2. If you have the digital version, you could simply reprint the lesson from the student worktext, and have the student restudy that.
3. Perhaps you only assigned 1/2 or 2/3 of the exercise sets in the student book at first, and can now use the remaining exercises.
4. Check if our online practice area at https://www.mathmammoth.com/practice/ has something for that topic.
5. Khan Academy has free online exercises, articles, and videos for most any math topic imaginable.

Concerning challenging word problems and puzzles

While this is not absolutely necessary, I heartily recommend supplementing Math Mammoth with challenging word problems and puzzles. You could do that once a month, for example, or more often if the student enjoys it.

The goal of challenging story problems and puzzles is to **develop the student's logical and abstract thinking and mental discipline**. I recommend starting these in fourth grade, at the latest. Then, students are able to read the problems on their own and have developed mathematical knowledge in many different areas. Of course I am not discouraging students from doing such in earlier grades, either.

Math Mammoth curriculum contains lots of word problems, and they are usually multi-step problems. Several of the lessons utilize a bar model for solving problems. Even so, the problems I have created are usually tied to a specific concept or concepts. I feel students can benefit from solving problems and puzzles that require them to think "out of the box" or are just different from the ones I have written.

I recommend you use the free Math Stars problem-solving newsletters as one of the main resources for puzzles and challenging problems:

Math Stars Problem Solving Newsletter (grades 1-8)
https://www.homeschoolmath.net/teaching/math-stars.php

I have also compiled a list of other resources for problem solving practice, which you can access at this link:

https://l.mathmammoth.com/challengingproblems

Another idea: you can find puzzles online by searching for "brain puzzles for kids," "logic puzzles for kids" or "brain teasers for kids."

Frequently asked questions and contacting us

If you have more questions, please first check the FAQ at https://www.mathmammoth.com/faq-lightblue

If the FAQ does not cover your question, you can then contact us using the contact form at the Math Mammoth.com website.

Chapter 1: Exponents and Scientific Notation
Introduction

The first chapter of Math Mammoth Grade 8 starts out with a study of basic exponent laws and scientific notation.

We begin with a review of the concept of an exponent and of the order of operations. The next lesson first reviews multiplication of integers, and then focuses on powers with negative bases, such as $(-5)^3$.

Then we get to the "meat" of the chapter: the various laws of exponents. The first lesson on that topic allows students to explore and to find for themselves the product law and the quotient law of exponents. After that, students find out the logical way to define negative and zero exponent by looking at patterns. They practice simplifying various expressions with exponents, both with numerical values and with variables.

The lesson "More on Negative Exponents" focuses on expressions with a negative exponent in the numerator, such as $7/(a^{-4})$. This is to prepare students for calculations that ask them to find how many times bigger one number is than another, when the numbers are written in scientific notation.

Next, in the lesson "Laws of Exponents, Part 2", students practice applying the power of a power law: $(a^n)^m = a^{nm}$.

Then the chapter has one more lesson on the laws of exponents ("Laws of Exponents, Part 3"), which summarizes the laws and gives more practice. This lesson is not absolutely essential if you're following Common Core Standards. It is presented here to give a summary, to give practice on all exponent laws, including the power of a quotient law which was not dealt with a lot in the previous lessons. This lesson also allows the book to meet the Florida B.E.S.T. standards for 8th grade.

Then we turn our attention to scientific notation, first learning how it is used with large numbers and then with small numbers. The lesson on significant digits follows, helping students to know how to round final answers in calculations with measurements.

The last topic of the chapter is calculations with numbers given in scientific notations. These calculations, naturally, involve many scientific topics such as the atomic world or astronomy.

Pacing Suggestion for Chapter 1

This table does not include the chapter test as it is found in a different book (or file).
Please add one day to the pacing if you use the test.

The Lessons in Chapter 1	page	span	suggested pacing	your pacing
Powers and the Order of Operations	13	*3 pages*	1 day	
Powers with Negative Bases	16	*3 pages*	1 day	
Laws of Exponents, Part 1	19	*3 pages*	1 day	
Zero and Negative Exponents	22	*3 pages*	1 day	
More on Negative Exponents	25	*2 pages*	1 day	
Laws of Exponents, Part 2	27	*3 pages*	1 day	
Laws of Exponents, Part 3	30	*2 pages*	1 day	

The Lessons in Chapter 1	page	span	suggested pacing	your pacing
Scientific Notation: Large Numbers	32	*3 pages*	1 day	
Scientific Notation: Small Numbers	35	*2 pages*	1 day	
Significant Digits	37	*3 pages*	1 day	
Using Scientific Notation in Calculations, Part 1	40	*3 pages*	1 day	
Using Scientific Notation in Calculations, Part 2	43	*3 pages*	1 day	
Chapter 1 Review	46	*2 pages*	1 day	
Chapter 1 Test (optional)				
TOTALS		*35 pages*	13 days	

Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this chapter, including pages that offer:

- **online practice** for concepts;
- online **games**, or occasionally, printable games;
- **animations** and interactive **illustrations** of math concepts;
- **articles** that teach a math concept.

We heartily recommend you take a look! Many of our customers love using these resources to supplement the bookwork. You can use these resources as you see fit for extra practice, to illustrate a concept better and even just for some fun. Enjoy!

https://l.mathmammoth.com/gr8ch1

Scan me

Powers and the Order of Operations

You will recall that we use **exponents** as a shorthand for writing repeated multiplications by the same number. For example, $7 \cdot 7 \cdot 7 \cdot 7 \cdot 7$ is written 7^5.

exponent

$$12^4 = 12 \cdot 12 \cdot 12 \cdot 12$$
$$= 20{,}736$$

base

The tiny raised number is called the **exponent**. It tells us how many times the **base** number is multiplied by itself.

The entire expression, 7^5, is a **power.** We read it as “seven to the fifth power,” “seven to the fifth,” or “seven raised to the fifth power.” Similarly, 0.5^8 is read as “five tenths to the eighth power” or “zero point five to the eighth.”

The “powers of 8” are the various expressions where 8 is raised to some power: for example, 8^3, 8^4, 8^{45}, and 8^{99} are powers of 8.

The expression 9^1 equals simply 9. In general, $a^1 = a$.

Powers of 2 are usually read as something “**squared.**” For example, 11^2 is read as “eleven squared.” That is because it gives us the area of a square with sides 11 units long.

Similarly, if the exponent is 3, the expression is usually read using the word “**cubed.**” For example, 1.5^3 is read as “one point five cubed” because it is the volume of a cube with edges 1.5 units long.

A calculator is not needed for the exercises of this lesson.

1. Evaluate.

a. four cubed

b. 2^4

c. 5^3

d. 0.2^3

e. 1^{60}

f. 100 squared

2. **a.** Which is more, 4^2 or 2^4?

b. Which is more, 2^5 or 5^2?

3. Complete the patterns.

a.	**b.**	**c.**
$10^1 =$	$2^1 =$	$0.1^1 =$
$10^2 =$	$2^2 =$	$0.1^2 =$
$10^3 =$	$2^3 =$	$0.1^3 =$
$10^4 =$	$2^4 =$	$0.1^4 =$
$10^5 =$	$2^5 =$	$0.1^5 =$
$10^6 =$	$2^6 =$	$0.1^6 =$
$10^7 =$	$2^7 =$	$0.1^7 =$

The order of operations dictates that powers (expressions with exponents) are solved before multiplication, division, addition, and subtraction.

Example 1. Find the value of $5 \cdot 0.1^3 + 0.2^2$.

First the powers: $0.1^3 = 0.1 \cdot 0.1 \cdot 0.1 = 0.001$, and $0.2^2 = 0.2 \cdot 0.2 = 0.04$.

The expression becomes
$5 \cdot 0.001 + 0.04 \;=\; 0.005 + 0.04 \;=\; \underline{0.045}$.

<u>The Order of Operations (PEMDAS)</u>
("*Please Excuse My Dear Aunt Sally*")

1) Solve what is within parentheses **(P)**.

2) Solve exponents **(E)**.

3) Solve multiplication **(M)** and division **(D)** from left to right.

4) Solve addition **(A)** and subtraction **(S)** from left to right.

4. Find the value of the expressions.

a. $4 \cdot 10^3 - 5 \cdot 10^2$	**b.** $4(5^2 - 2^3)$	**c.** $\dfrac{3}{1^8} + \dfrac{5}{3^2}$
d. $7 \cdot 10^3 - 5(800 - 10^2)$	**e.** $500 - \dfrac{3 \cdot 8}{2^3} + 2 \cdot 8^2$	**f.** $\dfrac{2 \cdot 17 + 2^4}{7 \cdot 7 - 3^2} + 20$

5. Find the value of the expressions.

a. $0.5^2 - 0.2^2 - 0.1^2$	**b.** $3(0.1^2 - 0.2^3)$	**c.** $0.6^2 + 2(1 - 0.3^2)$

6. The table on the right shows a list of powers of 4.

a. Find the value of 4^7 using the value for 4^6. (Do not use a calculator.)

b. Which power of 4 is equal to 65,536? Use estimation and the table, not a calculator.

c. Use the table to check whether $4^2 + 4^3 = 4^5$.

d. Use the table to check whether $4^2 \cdot 4^3 = 4^5$.

$4^1 = 4$
$4^2 = 16$
$4^3 = 64$
$4^4 = 256$
$4^5 = 1{,}024$
$4^6 = 4{,}096$

7. **a.** Find a power of 3 that is greater than seven squared.

b. Find a power of 5 that is greater than ten cubed.

c. Find a power of 1 that is greater than three squared.

8. **a.** If $3^6 = 729$, find the value of 3^8.

b. If $2^8 = 256$, find the value of 2^{11}.

9. Find the missing exponents.

a. $10^4 = 100^{\square}$	**b.** $2^6 = 4^{\square}$	**c.** $9^2 = 3^{\square}$	**d.** $0 = 0^{\square}$
e. $0.1^{\square} = 0.0001$	**f.** $0.2^{\square} = 0.00032$	**g.** $625 = 5^{\square}$	**h.** $128 = 2^{\square}$

10. Find the value of these powers.

a. $\left(\frac{1}{6}\right)^2 =$	**b.** $\left(\frac{3}{10}\right)^3 =$	**c.** $\left(\frac{2}{3}\right)^4 =$	**d.** $\left(\frac{3}{4}\right)^3 =$

Example 2. Simplify $3 \cdot s \cdot s \cdot s \cdot 3 \cdot t \cdot s \cdot t \cdot t$.

We can multiply in any order, so let's reorganize the expression as $3 \cdot 3 \cdot s \cdot s \cdot s \cdot s \cdot t \cdot t \cdot t$.

The variable s is multiplied by itself four times, and t three times. Naturally, $3 \cdot 3$ is 9.

So, we get $3 \cdot 3 \cdot s \cdot s \cdot s \cdot s \cdot t \cdot t \cdot t \;=\; \mathbf{9s^4t^3}$.

11. Simplify.

a. $2 \cdot x \cdot x \cdot x \cdot x \cdot x \cdot 7$	**b.** $4 \cdot x \cdot x \cdot x \cdot y \cdot y \cdot 9 \cdot x \cdot y \cdot x$
c. $5 \cdot a \cdot b \cdot b \cdot a \cdot a \cdot 2 \cdot b \cdot 6$	**d.** $0.3 \cdot p \cdot r \cdot p \cdot r \cdot r \cdot 0.2 \cdot r \cdot 10$

12. **a.** Find the value of the expression $10a^4b^2$ when $a = 2$ and $b = 3$.

b. Find the value of the expression $14x^3y^5$ when $x = 2$ and $y = 0$.

13. When you fold a sheet of paper in half, its area is naturally now only 1/2 of the area of the original paper. Let's say you repeat this process, and fold that paper again in half, and again, and again. How many times do you need to fold a sheet of paper in order for the area of the folded piece to be 1/64 of the area of the original?

Puzzle Corner What is the simple value of $\frac{9^6}{9^5}$? There is no need for actual calculations!

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Using Scientific Notation in Calculations, Part 2

1. Some students get confused with the rules of exponents when *adding* numbers in scientific notation. Compare the problems carefully, and solve. Give your answers in decimal notation. Do not use a calculator.

a. $(2 \cdot 10^6) \cdot (3 \cdot 10^4)$

b. $2 \cdot 10^6 + 3 \cdot 10^4$

c. $8 \cdot 10^3 + 7 \cdot 10^5$

d. $(8 \cdot 10^3) \cdot (7 \cdot 10^5)$

It is easy to add or subtract numbers in scientific notation IF they have the same power of ten: you can simply add or subtract their decimal parts.

Example 1. Add $2.81 \cdot 10^{13} + 5.2 \cdot 10^{12}$.

We will write $2.81 \cdot 10^{13}$ with 10^{12} instead of 10^{13}: $2.81 \cdot 10^{13} = 28.1 \cdot 10^{12}$. Now, the problem becomes $28.1 \cdot 10^{12} + 5.2 \cdot 10^{12}$. We can simply add $28.1 + 5.2 = 33.3$. The final sum is $33.3 \cdot 10^{12}$.

Another possibility is to simply use decimal notation to add the numbers, like you probably did in question #1. This works if the absolute values of the exponents are not very large.

2. Solve. Give your answer in scientific notation.

a. $4.8 \cdot 10^8 + 5 \cdot 10^7$	**b.** $9.3 \cdot 10^6 + 8 \cdot 10^7$	**c.** $5 \cdot 10^7 - 7 \cdot 10^5$	**d.** $8.4 \cdot 10^9 - 4.7 \cdot 10^8$

3. Jeremy and Mia were working on the problem $5 \cdot 10^{-3} + 2 \cdot 10^{-4}$. One of them got the answer $7 \cdot 10^{-7}$ and the other got $5.2 \cdot 10^{-4}$. Is either answer correct? If not, find the correct answer (without a calculator).

4. Solve.

a. $8 \cdot 10^{-2} + 6 \cdot 10^{-3}$	**b.** $3 \cdot 10^{-6} + 5 \cdot 10^{-5}$
c. $2 \cdot 10^{-4} - 7 \cdot 10^{-6}$	**d.** $5.4 \cdot 10^{-3} - 7 \cdot 10^{-4}$

5. Compare the volumes of different planets, the moon, and the sun that are given in the table, in cubic kilometers.

Celestial Body	Volume
Earth	$1.0832 \cdot 10^{12}$ km^3
Moon	$2.1968 \cdot 10^{10}$ km^3
Mars	$1.6318 \cdot 10^{11}$ km^3
Jupiter	$1.4313 \cdot 10^{15}$ km^3
Sun	$1.4093 \cdot 10^{18}$ km^3

a. How many Jupiters would "fit" in the sun?

b. How much bigger is the volume of the earth than of Mars?

c. What is the combined volume of the earth and the moon?

6. Count how many breaths you take in a minute (at rest), and from that, estimate how many breaths in total you would take in a 70-year lifespan. Then choose the closest estimate from the options below.

The number of breaths a person takes in a lifetime is about: **a.** $5 \cdot 10^6$ **b.** $5 \cdot 10^8$ **c.** $5 \cdot 10^{10}$ **d.** $5 \cdot 10^{12}$

7. A 70-kg male body contains approximately $7 \cdot 10^{27}$ atoms. Of these, approximately $4.22 \cdot 10^{27}$ are hydrogen atoms, $1.61 \cdot 10^{27}$ are oxygen atoms, $8.03 \cdot 10^{26}$ are carbon atoms, and $3.9 \cdot 10^{25}$ are nitrogen atoms.

a. About what percentage of the atoms in a human body are hydrogen atoms?

b. About how many more times oxygen atoms does the human body have than nitrogen atoms?

c. About how many more oxygen atoms does the human body have than carbon atoms?

8. It is estimated that there are about 10^{15} ants on this planet, and that the average mass of each ant is about 1 mg.

a. Find the total mass of the ants living on this planet. Give your answer in a sensible unit.

b. Now find the total mass of humans living in Asia, in kilograms. Use 60 kg for the average weight of humans in Asia, and 4,800 million for the population of Asia (or check the current population at https://www.worldometers.info/world-population/asia-population/).

c. Which have a larger mass, all the ants on the planet, or the people living in Asia?

9. The water volume in Lake Victoria is approximately 2,750 km^3.

a. Convert this to cubic meters and write the resulting number in scientific notation.
Hint: the unit "km^3" means "1000 meters, cubed", or $(1000\text{ m})^3$.

b. Now calculate how many bucketfuls of water there are in Lake Victoria. Use 20 liters for the volume of one bucket (consider it accurate to two significant digits). (1 m^3 = 1000 liters.)

Chapter 2: Geometry
Introduction

The second chapter of Math Mammoth Grade 8 covers geometric transformations, angle relationships, and the volume of prisms, cylinders, pyramids, cones, and spheres.

The chapter starts out with the basics of congruent transformations: translations, reflections, rotations. Students use transparent paper to perform several of these transformations hands-on, so as to gain an understanding of the attributes that are preserved in these transformations.

Next we practice these same transformations in the coordinate grid. Students learn how the coordinates of the points change when a figure is translated or reflected in the *x* or *y*-axis. They also explore rotating figures in the coordinate grid; here we limit the rotations to 90°, 180°, or 270° degrees.

Then it is time to study sequences of transformations, which enable us to describe more complex transformations. The key idea here is to understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of transformations.

All of this work has related to congruent transformations, which means the size of the figure has not changed. Now we turn our attention to dilations. In a dilation, the figure is transformed so that its size changes but its shape does not. Such figures are called similar figures. Yet another term describing the same process is scaling a figure.

Next, we study angle relationships. The first lesson in this section reviews certain angle relationships from 7th grade (complementary, supplementary, and vertical angles). Then students learn about angles formed when a transversal crosses two parallel lines: corresponding angles, alternate interior angles, and alternate exterior angles. They also investigate angle relationships related to triangles and learn how these relationships allow us to deduce angle measurements of other angles.

In all of this work, students are guided to reason using mathematical facts they have learned, and to justify their reasoning, thus becoming familiar with the process of mathematical proof.

The last major topic of the chapter is volume of various three-dimensional figures. Students solve a variety of real-world and mathematical problems involving multiple three-dimensional shapes.

Pacing Suggestion for Chapter 2

This table does not include the chapter test as it is found in a different book (or file).
Please add one day to the pacing if you use the test.

The Lessons in Chapter 2	page	span	suggested pacing	your pacing
Geometric Transformations and Congruence, Part 1	51	*4 pages*	1 day	
Geometric Transformations and Congruence, Part 2	55	*3 pages*	1 day	
Translations in the Coordinate Grid	58	*3 pages*	1 day	
Reflections in the Coordinate Grid	61	*3 pages*	1 day	
Translations and Reflections	64	*3 pages*	1 day	
Rotations in the Coordinate Grid	67	*4 pages*	1 day	
Sequences of Transformations	71	*3 pages*	1 day	
Sequences of Transformations, Part 2	74	*2 pages*	1 day	
Dilations	76	*3 pages*	1 day	
Dilations in the Coordinate Grid	79	*3 pages*	1 day	

The Lessons in Chapter 2	page	span	suggested pacing	your pacing
Similar Figures, Part 1	82	*3 pages*	1 day	
Similar Figures, Part 2	85	*2 pages*	1 day	
Similar Figures: More Practice	87	*3 pages*	1 day	
Review: Angle Relationships	90	*3 pages*	1 day	
Corresponding Angles	93	*2 pages*	1 day	
More Angle Relationships with Parallel Lines	95	*2 pages*	1 day	
The Angle Sum of a Triangle	97	*3 pages*	1 day	
Exterior Angles of a Triangle	100	*3 pages*	1 day	
Angles in Similar Triangles, Part 1	103	*2 pages*	1 day	
Angles in Similar Triangles, Part 2	105	*2 pages*	1 day	
Volume of Prisms and Cylinders	107	*2 pages*	1 day	
Volume of Pyramids and Cones	109	*3 pages*	1 day	
Volume of Spheres	112	*2 pages*	1 day	
Volume Problems	114	*2 pages*	1 day	
Chapter 2 Mixed Review	116	*2 pages*	1 day	
Chapter 2 Review	118	*5 pages*	2 days	
Chapter 2 Test (optional)				
TOTALS		*72 pages*	27 days	

Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this chapter, including pages that offer:

- **online practice** for concepts;
- online **games**, or occasionally, printable games;
- **animations** and interactive **illustrations** of math concepts;
- **articles** that teach a math concept.

We heartily recommend you take a look! Many of our customers love using these resources to supplement the bookwork. You can use these resources as you see fit for extra practice, to illustrate a concept better and even just for some fun. Enjoy!

https://l.mathmammoth.com/gr8ch2

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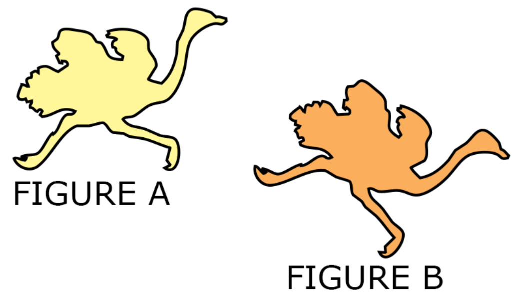
Geometric Transformations and Congruence, Part 1

Two figures are congruent when they are, you might say, identical in the sense that they have the same shape and size (but may be of different color). We can define congruency as follows:

Two figures are **congruent** if they perfectly match, when one is placed on top of the other.

The figures don't have to be in the same position or orientation. For example, these two figures are congruent — if you rotate and move figure A, you can place it exactly on top of figure B.

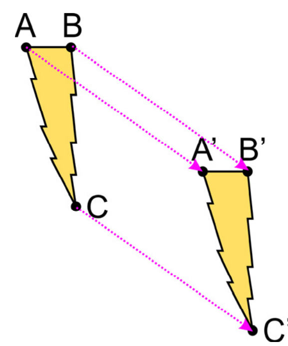


We will now study three geometric **transformations**, or basic ways to move a point, or by extension, a figure, since a figure can be considered to consist of many points.

1. A **translation** of a figure means sliding or moving it a certain distance in a certain direction, without turning or rotating it. The arrows show how three individual points of the figure were moved.

   We say the translation maps point A onto point A' (read "A prime"), point B onto point B', and point C onto point C'.

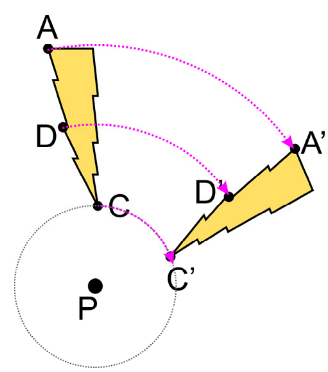
   We also say that point A' is the image of point A under the translation.



2. A **rotation** means turning a figure around a certain point.

   Here, the lightning figure is rotated around point P. Each point of the figure moves in a **circular arc around point P**.

   A rotation is measured in degrees, just like angles are. In this example, the lightning figure was rotated 67 degrees clockwise around point P.

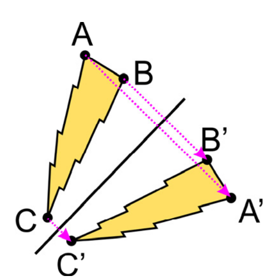


3. A **reflection** across a line means mirroring the figure in that line. You could also say the figure was "flipped".

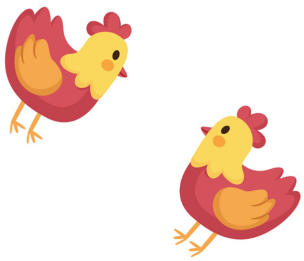
   In a reflection, the distance from each point to the reflection line and the distance of its image to the line are equal (measured along a line segment that is perpendicular to the line).

   For example, the distance from point C to the line equals the distance from point C' to the line.

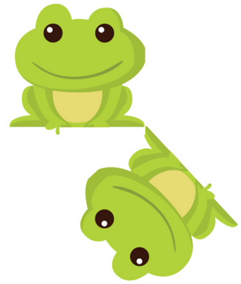
   A reflected figure is congruent to the original.



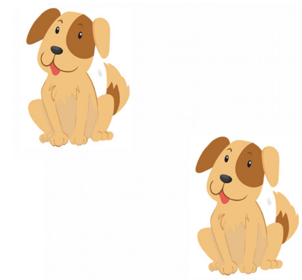
1. Name the transformation that was used to transform the figure on the left to the figure on the right.



a.



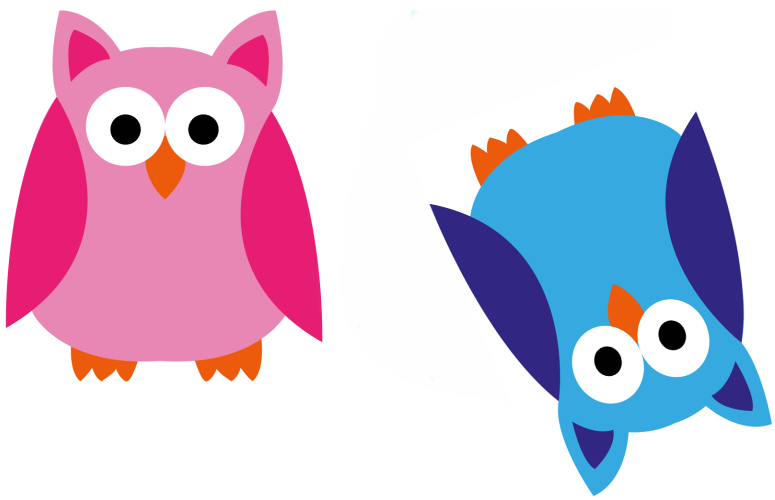
b.



c.

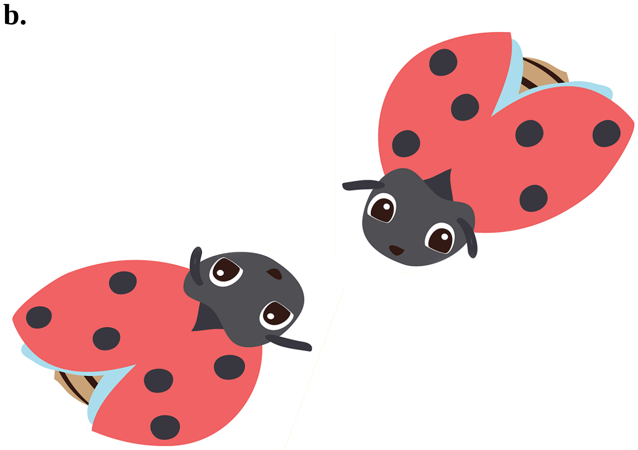
In continuation, we will explore geometric transformations and how they relate to congruence with the help of tracing paper (patty paper) or a transparency.

2. Use tracing paper to determine whether the two figures are congruent. You may move, turn, and/or flip the tracing paper. First, copy the outline of **one** figure to the tracing paper.
(Note: when checking for congruency, we ignore the colors.)

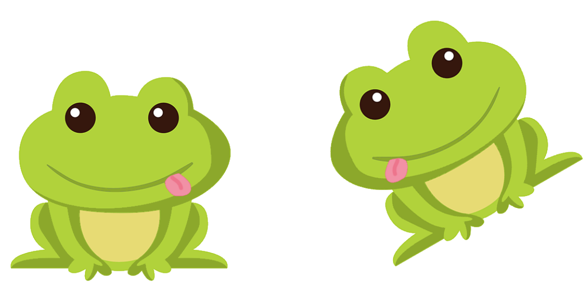


a.

b.



c.



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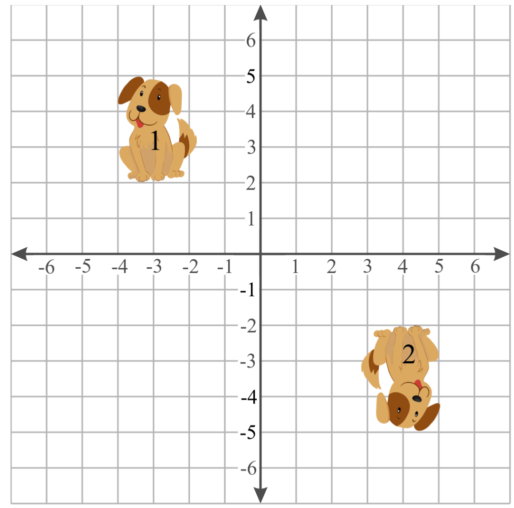
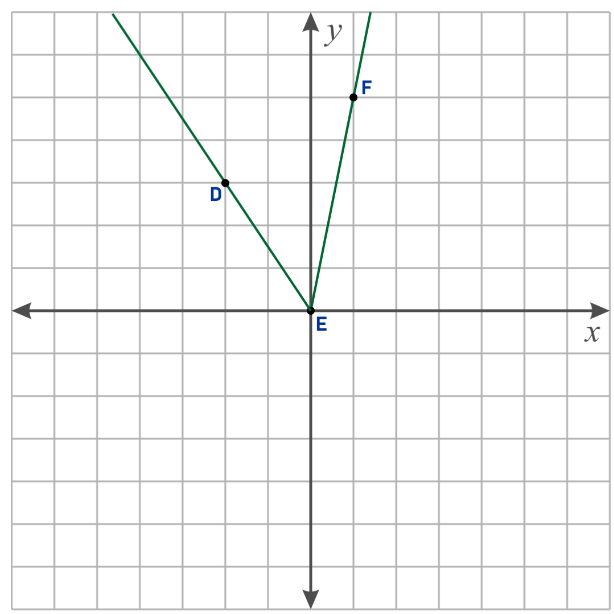
Chapter 2 Review

1. Describe a sequence of transformations that can map figure 1 to figure 2.

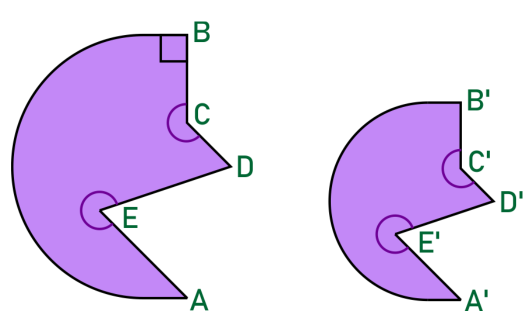


2. Rotate angle DEF both 90 degrees and also 180 degrees clockwise around the origin.



3. A quadrilateral was first reflected in the *y*-axis, and then rotated around the origin clockwise 90 degrees. Its vertices are now at points (3, −5), (5, −2), (4, −1), and (1, −4). What were the coordinates of its vertices before these transformations?

4. Figure A'B'C'D'E' is a dilation of figure ABCDE with scale factor 3/4. Angle B is a right angle. Check all the statements that are true.



a. Angle B' is a right angle.

b. The measure of ∠CDE is 3/4 of the measure of ∠C'D'E'.

c. ∠E is equal to ∠E'.

d. If CD = 1 inch, then C'D' = 3/4 inch.

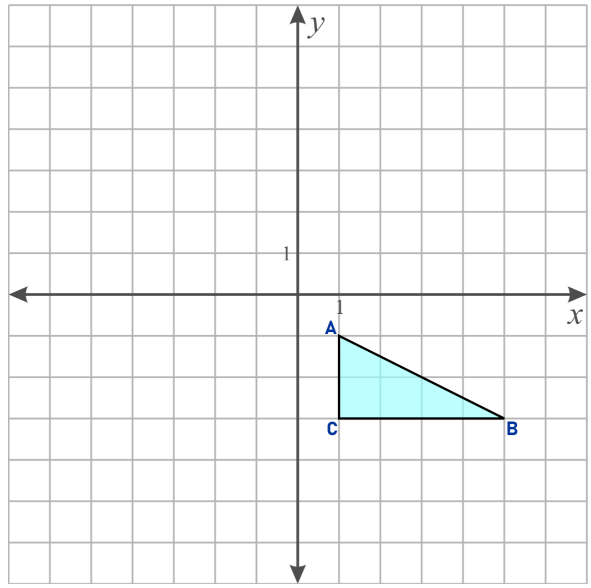
e. ∠D is equal to ∠E'.

f. If the perimeter of figure ABCDE is 20 inches, then the perimeter of A'B'C'D'E is 12 inches.

5. **a.** Perform the following sequence of transformations to triangle ABC:

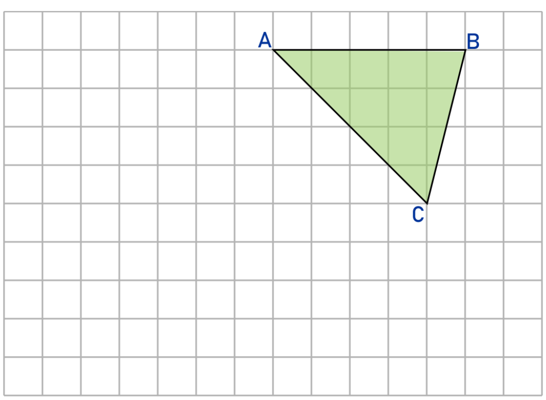
First rotate it counterclockwise around point C 90 degrees.
Then reflect it in the vertical line at $x = -1$.
Lastly, translate it two units to the right and three down.

b. Find another, different sequence of transformations that does the same as the sequence in (a), and starts with a reflection.

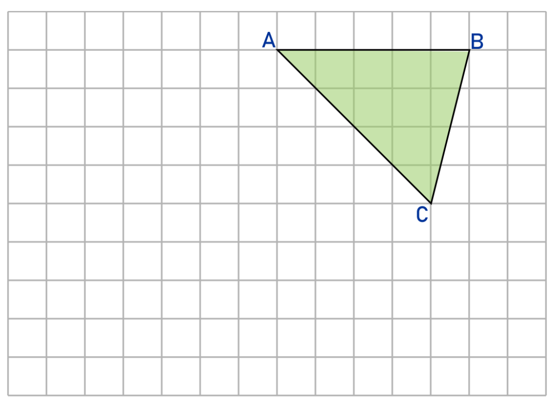


6. Draw a dilation of triangle ABC...

a. from point C and scale factor 1/2.



b. from point B and scale factor 2.



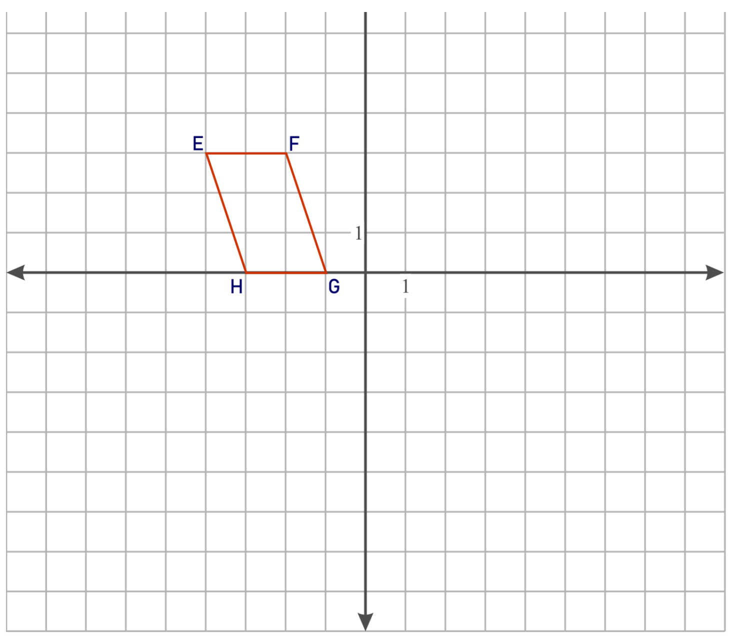
7. Parallelogram EFGH underwent the following transformations :

1. Reflection in the vertical line at $x = -0.5$.

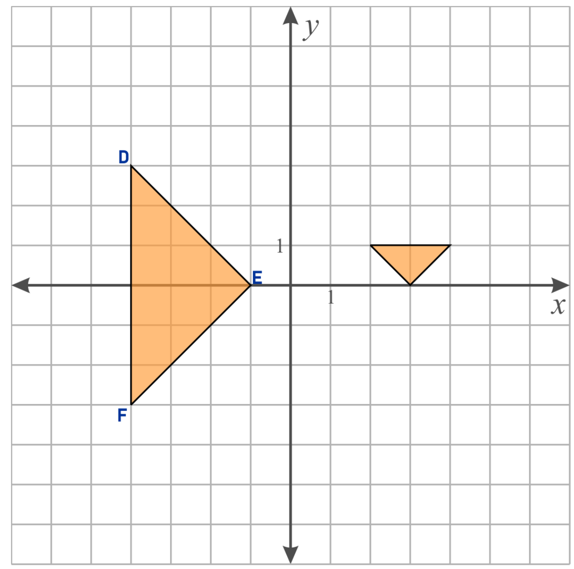
2. Translation 3 units to the left and 4 units down.

3. Dilation from point E" with scale factor 2.

What are the coordinates of the image of point F after all these transformations?

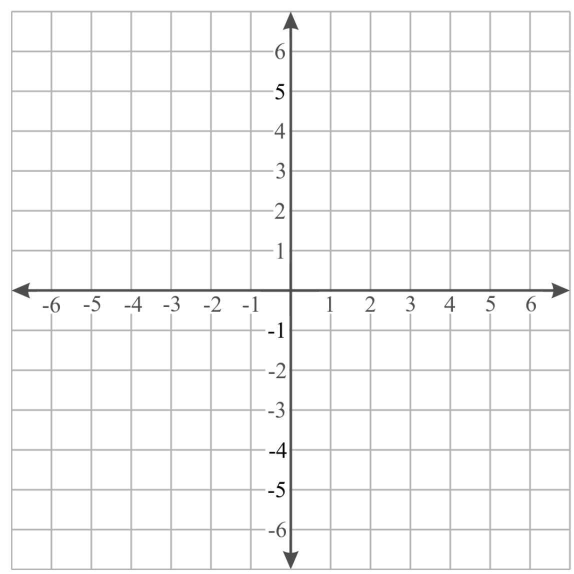


8. Show that the two triangles are similar by describing a sequence of transformations that could map △DEF to the smaller triangle



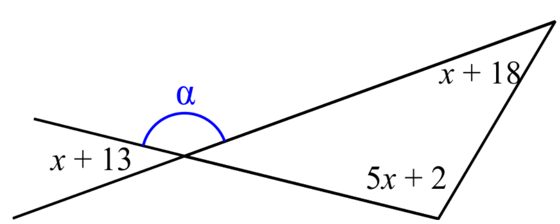
9. Figure PQRS underwent a dilation, then a rotation. Study the coordinates to find out the details about each transformation, then fill in the missing coordinates.

Original figure	**Dilation**	**Rotation**
P(−5, 3)	P'(−6, 5)	P"(___ , ___)
Q(0, 3)	Q'(4, 5)	Q"(___ , ___)
R(−1, 1)	R'(___ , ___)	R"(−4 , −5)
S(−4, 1)	S'(−4, 1)	S"(−4, 1)

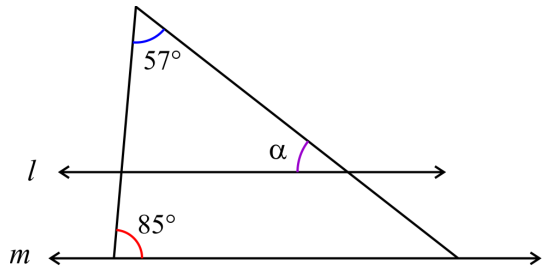


10. **a.** Find the value of x.

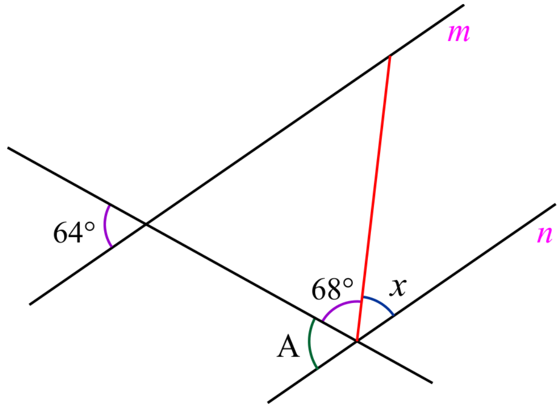
b. Find the value of α.



11. Lines l and m are parallel. Figure out the measure of the angle α. (You may need to mark more angles in the diagram.)



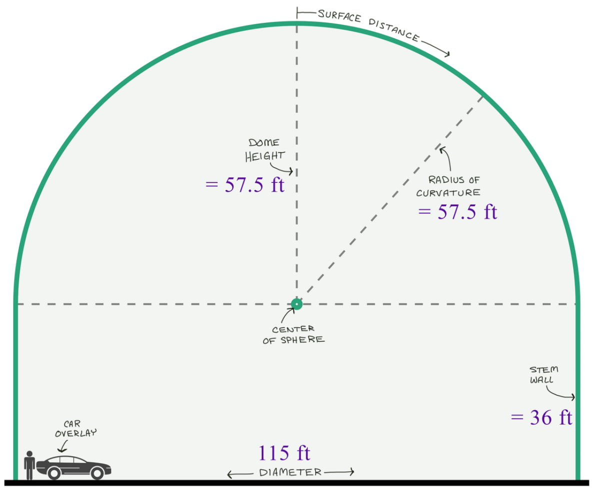
12. Lines m and n are parallel. Find the measure of angle x, and prove why it is what you find it to be. In other words, explain and justify your reasoning. You may need to mark more angles in the diagram.

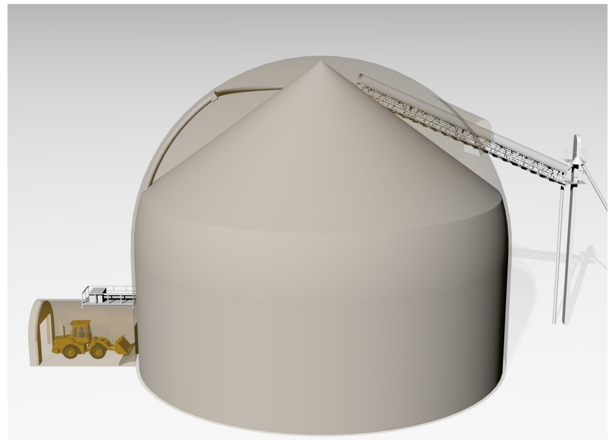


13. A shampoo bottle is in the shape of a circular cylinder. It says it contains 473 ml of shampoo. Its inner diameter is 6.0 cm and its height is 17 cm. What percent of the bottle does the shampoo take up?

14. Compare a sphere with radius 5 cm with a cone with the same radius. What is the height of the cone, given the two have the same volume?

15. Elkhart Ammonium Nitrate Storage in Elkhart, Texas, is a large building consisting of a half sphere on top of a circular cylinder. The stem wall is 36 ft high, and the diameter of the circle (which is also the diameter of the sphere) is 115 ft. When the storage is filled with ammonium nitrate, the top part of it (the part inside the half-sphere) forms a cone.





Images courtesy of Monolithic Dome Institute, www.monolithic.com

Find the volume of the ammonium nitrate mound when the cone reaches the top of the structure, to the nearest thousand cubic feet.

Chapter 3: Linear Equations
Introduction

The third chapter of Math Mammoth Grade 8 focuses both on the mechanics of solving linear equations and on problem solving.

The chapter starts with a vocabulary reference sheet. The first actual lesson is a review of integer addition and subtraction, which you can omit at your discretion. The next several lessons after that review simple equations of the form $px + q = r$ and $p(x + q) = r$ and and the distributive property from 7th grade.

The next step towards solving more complex equations is the lesson *Combining Like Terms*. Students add and subtract like terms, including with decimal or fractional coefficients, and solve equations where like terms need combined first.

Having learned this, students then tackle some typical algebraic word problems in the following lesson.

Then it is time to learn to solve equations where the variable is on both sides. There are often several possible solution pathways. Students also learn about the common error of adding or subtracting "across the sides."

The lesson *Simplifying Linear Expressions* focuses on how to remove parentheses after a minus sign, such as in the expression $2(3 + 2y) - 7(3 - 5y)$. After that, it is time for more practice and word problems, including age and coin word problems.

Then we turn our attention to equations with fractions, and the student learns to multiply both sides of the equation by a common multiple of the denominators. In the lessons on formulas, the student both solves various formulas for a variable in it, and uses formulas to solve a variety of word problems.

The lesson *More on Equations* deals with equations that have an infinite number of solutions (identities) or no solutions.

The chapter ends with two more lessons on word problems (percent word problems and miscellaneous problems).

Pacing Suggestion for Chapter 3

This table does not include the chapter test as it is found in a different book (or file).
Please add one day to the pacing if you use the test.

The Lessons in Chapter 3	**page**	**span**	**suggested pacing**	**your pacing**
Algebra Terms (For Reference)	125	*(1 page)*		
Review: Integer Addition and Subtraction	126	*3 pages*	1 day	
Equations Review, Part 1	129	*4 pages*	1 day	
The Distributive Property	133	*3 pages*	1 day	
Equations Review, Part 2	136	*4 pages*	1 day	
Equations Review, Part 3	140	*4 pages*	1 day	
Combining Like Terms	144	*3 pages*	1 day	
Word Problems	147	*4 pages*	1 day	
A Variable on Both Sides	151	*4 pages*	1 day	

The Lessons in Chapter 3	page	span	suggested pacing	your pacing
Word Problems and More Practice	155	*3 pages*	1 day	
Simplifying Linear Expressions	158	*3 pages*	1 day	
More Practice	161	*3 pages*	1 day	
Age and Coin Word Problems	164	*3 pages*	1 day	
Equations with Fractions 1	167	*2 pages*	1 day	
Equations with Fractions 2	170	*3 pages*	1 day	
Formulas, Part 1	173	*2 pages*	1 day	
Formulas, Part 2	175	*2 pages*	1 day	
More on Equations	177	*3 pages*	1 day	
Percent Word Problems	180	*2 pages*	1 day	
Miscellaneous Problems	182	*2 pages*	1 day	
Chapter 3 Mixed Review	184	*3 pages*	1 day	
Chapter 3 Review	186	*3 pages*	1 day	
Chapter 3 Test (optional)				
TOTALS		*63 pages*	21 days	

Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this chapter, including pages that offer:

- **online practice** for concepts;
- online **games**, or occasionally, printable games;
- **animations** and interactive **illustrations** of math concepts;
- **articles** that teach a math concept.

We heartily recommend you take a look! Many of our customers love using these resources to supplement the bookwork. You can use these resources as you see fit for extra practice, to illustrate a concept better and even just for some fun. Enjoy!

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Algebra Terms
For Reference

Expressions in mathematics consist of:

- numbers;
- mathematical operations (+ , − , · , ÷ , exponents);
- and letter variables, such as x, y, a, T, and so on.

Note: Expressions do *not* have an equals sign!

Examples of expressions:

$\frac{3}{5}x^2 - 3x + 5$ $\quad 5 \quad$ $\left(\frac{3x}{y^2}\right)^2$

$T - 29$ $\quad$ $2^x - 5^y$

An **equation** has two expressions separated by an equals sign:

(expression 1) = **(expression 2)**

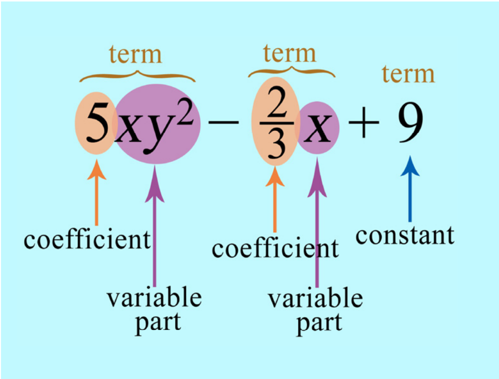
Examples of equations:

$0 = 0$ $\quad$ $2(z - 9) = -z^2$

$9 = -8$ (a false equation) $\quad$ $\frac{x + 3}{2} = -1.5$

A **term** is an expression that consists of numbers and/or variables that are *multiplied*. For example, $7x$ is a term and so is $0.6mn^2$.

A single number or a single variable is also a term. If the term is a single number, such as 4.5 or ¾, we call it a **constant**.



In the expression on the right, we have three terms: $5xy^2$, $\frac{2}{3}x$, and 9, that are separated by subtraction and addition.

If a term is not a single number, then it has a **variable part** and a **coefficient**.

- The coefficient is the single number by which the variable or variables are multiplied.
- The variable part consists of the variables and their exponents.

For example, in $4.3ab$, 4.3 is the coefficient, and ab is the variable part.

Note: a term that consists of variables only still has a coefficient: it is one. For example, the coefficient of the term x^3 is one, because you can write x^3 as $1 \cdot x^3$.

Example. Is $s - 5$ a term? No, it is not since it contains subtraction. Instead, $s - 5$ is an expression consisting of two terms, s and 5, separated by subtraction.

1. Write the expression based on the clues.

- It has four terms.
- The constant term is the square of the third smallest prime.
- The variable parts of the variable terms are ab, a^2, and a, respectively.
- The coefficients of the variable terms are the three consecutive integers with a sum of 21.
- The first two terms are separated by subtraction, the rest by addition.

Review: Integer Addition and Subtraction

Integers consist of the counting numbers (1, 2, 3, 4, ...), zero, and the negative counterparts of the counting numbers (−1, −2, −3, −4, ...). So, the set of integers is {..., −4, −3, −2, −1, 0, 1, 2, 3, 4, ...}.

An **absolute value** of an integer is its distance from zero, and is marked with two vertical lines. For example, $|2| = 2$ and $|-18| = 18$.

We obtain the **opposite** or **negation** of an integer by changing its sign from positive to negative, or vice versa. For example, the opposite or negation of 17 is −17. The opposite of −4 is 4.
We can use the negative sign "–" to signify this: −(−5) means the opposite of −5, which is 5.

To **add several negative integers**, simply add their absolute values and write the answer as negative.	**Example 1.** To find the sum −8 + (−3) + (−7) + (−11), add 8 + 3 + 7 + 11 = 29. The value of the original sum is −29.

To **add a negative and a positive integer**, find the difference in their absolute values. The integer with the bigger absolute value determines the sign of the final answer.

Example 2. In the sum −9 + 11, the absolute values of the two integers are 9 and 11. Their difference is 11 − 9 = <u>2</u>. This means the answer is either 2 or −2. To determine which, check the sign of the integer with the larger absolute value. In our case it is 11 (which is positive), so the answer is 2 (and not −2).

Example 3. In the sum 7 + (−12), the absolute values of the two integers are 7 and 12. Their difference is 12 − 7 = <u>5</u>. This means the answer is either 5 or −5. To determine which, check the sign of the integer with the larger absolute value. Here it is −12 which is negative, so the answer is −5 (and not 5).

So, this is the mechanical rule, but you don't have to use it if you have learned other methods, such as visualizing a number line.

To **add several integers** where some are negative, some positive, first calculate the partial sums of all the negative integers and of all the positive ones. Lastly add those sums.	**Example 4.** −8 + 12 + (−9) + (−1) + 5 + (−6) = ? <u>Positives:</u> 12 + 5 = 17 <u>Negatives:</u> −8 + (−9) + (−1) + (−6) = −24 <u>Total:</u> 17 + (−24) = <u>**−7**</u>

1. Add.

a. (−4) + 8 =	**b.** 15 + (−25) =	**c.** −12 + 6 =	**d.** −11 + (−32) =

e. −12 + (−2) + (−5) =	**f.** 6 + (−1) + (−5) + 2 =	**g.** −7 + 10 + (−6) + 1 =

h. −11 + (−2) + 7 + (−5) + 4 + (−3) =	**i.** −6 + (−5) + 8 + (−12) + 24 + 1 =

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Word Problems

Example 1. The width of a rectangle is three times its length, and its perimeter is 28 m.
What are the width and the length of the rectangle?

First, check *what quantity* is unknown, and choose a variable for it.

We don't know the length nor the width of this rectangle. However, since we know that the width is three times the length, we can write the width as an expression of the length. In other words, we can let x be the length, and then naturally, the width is $3x$. We don't need another variable for the width.

In your solution, **you need to specify what x denotes.** You can write: "Let x be the length of the rectangle." After that, write something to the effect: "Then, $3x$ is the width." This is to ensure that a reader understands your thought process.

Continuing on with the solution, we know the perimeter is the sum of all four sides, and equals 28 m. From that we can write the equation: $x + 3x + x + 3x = 28$. Solving that will be easy!

1. Solve the equation in example 1. Lastly, answer the actual question (specify what the width and the height of the rectangle actually are).

2. The length of a certain rectangle is 12 cm more than its width. The perimeter is 136 cm. How much do the sides of the rectangle measure? Write an equation.

   Reminder: Note clearly what your chosen variable denotes in the problem.

Example 2. Susan and Henry divide a task of washing 170 windows in a large building in a ratio of 3:2. How many windows will Henry wash?

You have learned to solve this type of problem using a bar model. We can also use an equation (which will exactly match the basic idea in the bar model).

Let $3x$ be the amount of windows that Susan washes, and $2x$ the amount that Henry washes. This way, the windows they washed are in the ratio of 3:2. Since we know they washed a total of 170 windows, we can write the equation $3x + 2x = 170$.

3. Solve the equation in example 2. How many windows will Henry wash?

4. An inheritance of $354,000 was divided between three heirs in the ratio of 1:6:5. How much did each heir get?

5. The two sides of a rectangle are in a ratio of 3:5 and its perimeter is 416 cm. What are the dimensions of the rectangle?

Example 3. Another type of problem you have solved previously using a bar model is where the total is known, and the parts making up the total differ by a known amount.

Eric and Jeremy worked last week for a total of 99 hours, and Eric worked 5 more hours than Jeremy. How many hours did Eric work?

Let x be Jeremy's working hours. Then, Eric worked for $x + 5$ hours. Knowing the total, we can easily write an equation for this situation.

6. Write an equation for Example 3, and solve it. Lastly, answer the actual question that was asked.

7. Anna has a flock of chickens and a flock of ducks. She has 17 more chickens than ducks, and in total she has 135 birds. How many chickens does she have?

8. Hans has to cross a bridge and pay a $6 bridge toll when going to work (coming back, he doesn't have to pay it). Some days he carpools with two other people, and they share the toll fee equally.

   a. Hans noticed that in the last two weeks (which had 10 workdays), he had paid a total of $36 in bridge tolls. How many days of those 10 did he carpool?

   b. In a month (which had 22 work days), he had paid a total of $96 in bridge tolls. How many days of those did he *not* carpool?

9. The pet store had three different leashes for sale. The price of one was $5.40 more than the price of the cheapest, and the price of another was $11.60 more than the cheapest. If you bought all three, your bill came to $62.60. How much did the most expensive leash cost?

10. The sum of three consecutive whole numbers is 360. What are the numbers?

Hint: Let *x* be the first of the three consecutive whole numbers. What are the other two, in terms of *x*?

11. The sum of three consecutive odd numbers is 1,971. What are the numbers?

Hint: Let *x* be the first of the three consecutive odd numbers. What is the next odd number, in terms of *x*?

12. The sum of four consecutive multiples of 5 is 1,570. What are the numbers?

Puzzle Corner

It is possible to write a set of equations for the following problems, but you haven't studied those types of equations (quadratic) yet. So, use **guess and check**. It can work equally efficiently!

a. The sum of two numbers is 35, and their product is 300. What are the numbers?

b. The sum of two numbers is 220, and their product is 9,600. What are the numbers?

A Variable on Both Sides

Example 1. Solve $2x + 8 = -5x$.

Notice that the unknown appears on both sides of the equation. This is not a problem; we can still use the principle of doing the same operation to both sides in order to isolate the unknown on one side. In this case, we can either subtract $2x$ from both sides or add $5x$ to both sides. See both options below.

First subtract 2*x*:

$$\begin{aligned} 2x + 8 &= -5x & &\mid -2x \\ 8 &= -7x & &\text{(Switch sides.)} \\ -7x &= 8 & &\mid \div -7 \\ x &= -8/7 \end{aligned}$$

First add 5x:

$$\begin{aligned} 2x + 8 &= -5x & &\mid +5x \\ 7x + 8 &= 0 & &\mid -8 \\ 7x &= -8 & &\mid \div 7 \\ x &= -8/7 \end{aligned}$$

Check:

$$\begin{aligned} 2 \cdot (-8/7) + 8 &\stackrel{?}{=} -5 \cdot (-8/7) \\ -16/7 + 8 &\stackrel{?}{=} 40/7 \\ -2\ 2/7 + 8 &\stackrel{?}{=} 5\ 5/7 \\ 5\ 5/7 &= 5\ 5/7 \end{aligned}$$ ✓

1. Solve the equation in two ways, as instructed.

First add 2*s*:

$10 - 2s = 4s + 9 \quad \mid +2s$

First subtract 4*s*:

$10 - 2s = 4s + 9 \quad \mid -4s$

2. Solve. Check your solutions (as always!).

a. $3x + 2 = 2x - 7$

b. $9y - 2 = 7y + 5$

3. A common student error is to add or subtract "across the sides," instead of carefully adding or subtracting the same quantity to/from both sides.

   Here is an example of it: the student added $7w$ and $2w$, and wrote $9w$ on the next line. Correct the error and solve the equation.

$$7w + 8 = 2w - 5$$
$$9w + 8 = -5$$

4. Solve. Check your solutions (as always!).

a. $-2y - 6 = 20 + 6y$	**b.** $8x - 12 = -1 - 3x$	**c.** $6z - 5 = 9 - 2z$

5. Fred is contemplating two different job offers. In one, he gets paid \$19.50 per hour plus he will receive a bonus based on the sales he brings in, which he estimates to be about \$150 per week. In another job, he will earn \$21 per hour (no bonuses).

   a. Write an expression for the weekly earnings in each job, for *m* hours of work.

   Job 1: Job 2:

   b. In which job would he earn more, if he worked 20 hours per week?

   c. For what amount of work hours would both jobs provide him the same wages?

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Chapter 4: Introduction to Functions
Introduction

The fourth chapter of Math Mammoth Grade 8 covers various introductory topics from the theory of functions. These topics prepare students for studying functions in great detail in high school math, and even include preparatory ideas for calculus (rate of change).

The first lesson focuses on the basic definition of a function, as a relationship between two sets that assigns exactly one output for each input. It also briefly explains the range and domain of a function, even though those terms are not required in the CCS.

Next, we study the rate of change in the context of linear functions. Students calculate the rate of change from functions given as a table of values or from their graphs. They also encounter nonlinear functions and calculate the rate of change for those in specific intervals.

Then, students learn about the initial value of a function (its value when the input is zero), and learn that the equation $y = mx + b$ defines a linear function. They write and plot equations of that form to model linear relationships. We also spend one lesson looking at linear versus nonlinear relationships.

The following major topic is describing functions. Students analyze a graph and tell whether a function is increasing, decreasing, or constant; linear or nonlinear. They sketch a graph matching a given verbal description, and interpret given graphs of nonlinear functions in a variety of the real-life contexts.

Lastly, students compare properties of two functions represented in different ways (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, distance as a function of time is given as an equation for one airplane, and as a graph for another, and students answer questions concerning the speed and distance of the two airplanes.

Pacing Suggestion for Chapter 4

This table does not include the chapter test as it is found in a different book (or file).
Please add one day to the pacing if you use the test.

The Lessons in Chapter 4	**page**	**span**	**suggested pacing**	**your pacing**
Functions	191	*4 pages*	1 day	
Linear Functions and the Rate of Change 1	195	*4 pages*	1 day	
Linear Functions and the Rate of Change 2	199	*3 pages*	1 day	
Linear Functions as Equations	202	*3 pages*	1 day	
Linear versus Nonlinear Functions	205	*3 pages*	1 day	
Modeling Linear Relationships	208	*4 pages*	1 day	
Describing Functions 1	212	*3 pages*	1 day	
Describing Functions 2	215	*3 pages*	1 day	
Describing Functions 3	218	*4 pages*	1 day	
Comparing Functions 1	222	*3 pages*	1 day	
Comparing Functions 2	225	*2 pages*	1 day	
Chapter 4 Mixed Review	227	*3 pages*	1 day	
Chapter 4 Review	230	*4 pages*	2 days	
Chapter 4 Test (optional)				
TOTALS		*43 pages*	14 days	

Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this chapter, including pages that offer:

- **online practice** for concepts;
- online **games**, or occasionally, printable games;
- **animations** and interactive **illustrations** of math concepts;
- **articles** that teach a math concept.

We heartily recommend you take a look! Many of our customers love using these resources to supplement the bookwork. You can use these resources as you see fit for extra practice, to illustrate a concept better and even just for some fun. Enjoy!

https://l.mathmammoth.com/gr8ch4

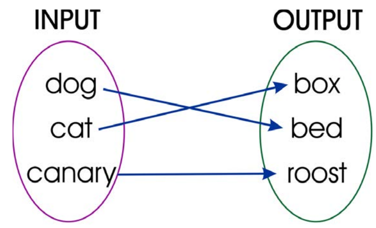
Scan me

Functions

A **function** is a rule or a relationship between two sets that assigns **exactly one output for each input.** We also use the word **mapping** for a function.

Example 1. The illustration below shows a simple function that maps each animal to its favorite sleeping place.

Each animal has a sleeping place, and only one, so this is a function.

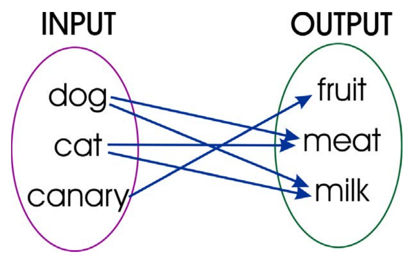


Example 2. The table lists the name of seven children, and the month when each child has their birthday. Notice that several of them have their birthday in December. Is this a function?

Input	Allie	Julie	Danny	Juan	Pete	Bob	Samantha
Output	September	December	December	June	August	December	February

Yes. The definition only requires that there has to be exactly one output for each input; **the outputs don't have to be unique.**

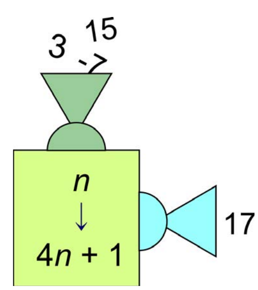
1. The relationship shown on the right is *not* a function. Why?



2. A function machine "ingests" a number (the input) and "spits out" another (the output) based on some rule. This function machine turns any number n into $4n + 1$.

   a. Number −7 is just going in. What will be the output?

   b. Number 17 just came out. What was the input?



3. Potatoes costs $3 per kilogram. Fill in the tables #1 and #2.

   Does each table represent a function? Explain.

#1

(Input) **Weight**	(Output) **Cost**
1 kg	$3
2 kg	
3 kg	
5 kg	
12 kg	

#2

(Input) **Cost**	(Output) **Weight**
$12	
$30	
$48	
$72	
$90	

4. The table lists seven children, and each child's favorite color.

Input	Allie	Julie	Danny	Juan	Pete	Bob	Samantha
Output	pink and blue	blue	gray	yellow	blue and red	?	purple

Is this a function? If not, change it in some manner(s) so it *is* a function.

5. T is a function that maps the name of a month to the number of days in it.

a. Create a depiction of T using a diagram like in example 1.

b. If you reverse the inputs and outputs, is the resulting relationship a function? Explain.

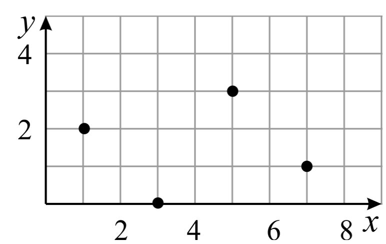
If the inputs and outputs are numbers, we can plot **a graph of the function** in the coordinate grid. Each input-output pair is viewed as an ordered pair (a single point).

We also use the terms "independent variable" for the input, and "dependent variable" for the output.

Example 3. Let F be the function (1, 2), (3, 0), (5, 3), (7, 1).

Note: A function *can* be given as a list of ordered pairs.

The image on the right is the plot of F; yet the plot is *not* F. The function F is the specific list of inputs and outputs, or the relationship itself.



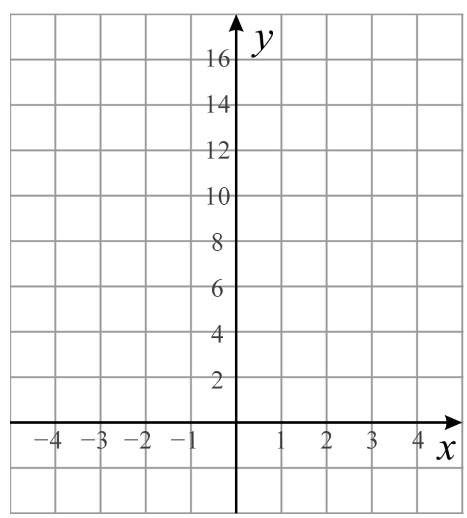
6. Let G be the function that maps each integer from −4 to 4 to its square minus one.

a. Fill in the table, listing the ordered pairs of G.

Input (x)	−4	−3	−2						
Output (y)	15								

b. Make a plot of G.

c. If you reversed the inputs and the outputs, would the relationship still be a function? Explain.



Example 4. Mary bicycled from her home to a friend's house. The table shows the distance (d) Mary had covered at specific amounts of time (t).

Input (t)	5 min	10 min	12 min	13 min	15 min	18 min	20 min	22 min
Output (d)	0.8 km	1.5 km	1.9 km	1.9 km	1.9 km	2.4 km	2.7 km	3 km

We say that **distance is a function of time.** The output variable, or the dependent variable, is always said to be a function of the input (or independent) variable. This means that for each moment of time (input) there is a specific distance she has traveled (output).

Is it true in reverse? Is *time* a function of *distance*?

This means we consider distance as the input, and time as the output. If yes, then for each distance (input), there is exactly one time (output). Is that so in this case?

7. Is Age a function of Name? Explain.

Is Name a function of Age? Explain.

Name	Age
FenFen	14
Larry	15
Pierre	13
Sam	12
Amy	14

Age	Name
14	FenFen
15	Larry
13	Pierre
12	Sam
14	Amy

8. Choose the relationships that are functions.

(1)

Rainfall (mm)	2	0	0	5	0	13	0
Day of month	6	7	8	9	10	11	12

(2) Let S be a rule that takes any number x as input, and gives $4x + 1$ as output.

(3) Input is a zip code, output is a person that lives there.

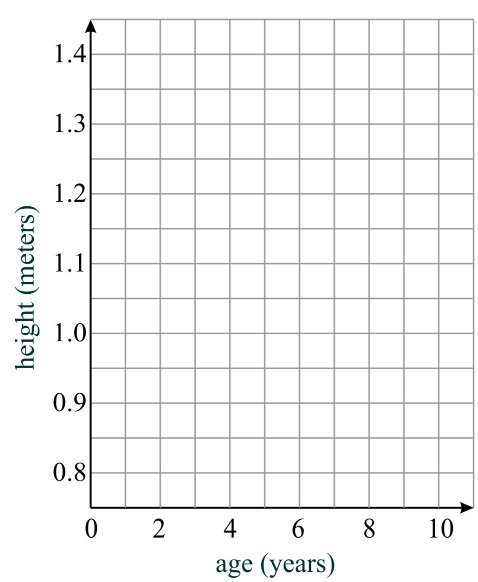
(4) Input is a person's first name, output is their bank account number.

9. Plot the following points that give the age (in years) and the height (in meters) of various children.

(2, 0.8) (5, 1.05) (10, 1.40) (9, 1.31) (6, 1.17) (5, 1.09)

a. Is this a function? Explain.

b. What is the independent variable? The dependent variable?



(Optional content; beyond the CSS)

The **domain** of a function is the set of inputs. The **range** of a function is the set of outputs.

Let's go back to example 3, where we had kindergartners and their birthday months.

Input	Allie	Julie	Danny	Juan	Pete	Bob	Samantha
Output	September	December	December	June	August	December	February

The domain of this function is the list of the children's names. To write it as a set, we enclose the items of the set in curly brackets: {Allie, Julie, Danny, Juan, Pete, Bob, Samantha}.

The range of this function is {September, December, June, August, February}.

10. **a.** Change some thing(s) in this table so it is a function.

    b. Give the domain of the function.

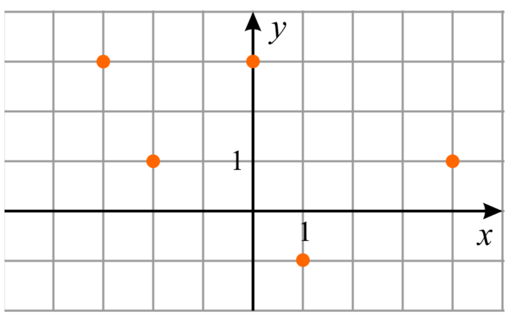
    c. Give the range of the function.

Input	Output
Name	**Grade level**
Jenny	8
Pedro	7
Ann	8
Marsha	
Rob	9
Ann	6

11. Let F be the function that maps a number x to $2x + 1$.
    Let the set {0, 1, 2, 3, 4, 5} be its domain.
    What is its range?

12. Give the domain and range of each function.

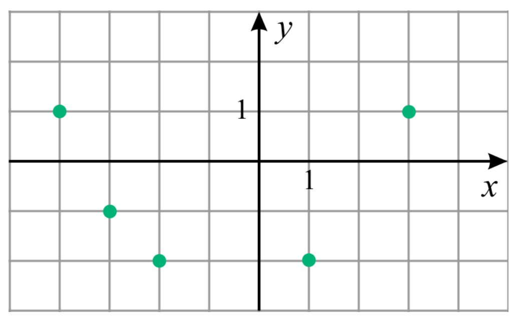
a.



Domain:

Range:

b.



Domain:

Range:

13. Let S be the function that allows any word from this sentence as the input, and the output is the number of letters in it. What is the range of this function?

14. G is a function that maps a number x to $x - 5$.
    If the set {0, 5, 10, 15, 20} is its range, what is its domain?

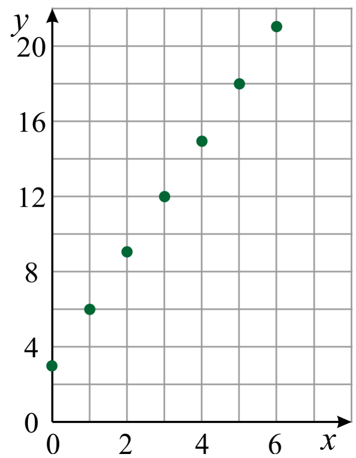
Linear Functions and the Rate of Change 1

If the graph of a function consists of points that fall on a single line, it is a **linear function**.

We will define a linear function in a different manner later, but for now, this is sufficient, so let's look at some examples.

Example 1. The input and output values in the table below define a function. Notice the patterns: the x-values increase by ones, and the y-values increase by 3s.

Input (x)	0	1	2	3	4	5	6
Output (y)	3	6	9	12	15	18	21



The graph shows that the points fall on a line. This is a linear function.

The **rate of change** of a function is the rate at which the output values change as compared to the change in the input values.

We calculate it as the ratio of $\dfrac{\text{change in output values}}{\text{change in input values}}$.

In the context of this graph, **rate of change =** $\dfrac{\text{difference in } y\text{-values}}{\text{difference in } x\text{-values}}$.

In this case, each time the x-values increase by 1, the y-values increase by 3. **The rate of change is 3/1 = __3__.**

Example 2. The price of bananas is a function of their weight. What is the rate of change?

Weight in kg (input)	0	2	5	10	12	15
Price in \$ (output)	0	5	12.50	25	30	37.50

Check how much the output (price) changes for a certain change in the input (the weight). For example, when the weight increases from 0 to 2 kg, the price increases from \$0 to \$5, or by \$5. This happens also when the weight increases from 10 to 12 kg: the price increases \$5 (from \$25 to \$30).

$$\text{Rate of change} = \frac{\$5}{2\text{ kg}} = \$2.50/\text{kg}$$

Note that if the independent and dependent variables have units, **we include the units in the rate of change**.

This rate of change tells us that for each one-kilogram increase in weight, the price increases by \$2.50.

1. **a.** Calculate the rate of change in example 2, using the increase in weight from 5 to 10 kg, and the corresponding increase in price. Do you get the same rate of change as calculated in the example?

   b. Do the same using the input values 10 kg and 15 kg.

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Mixed Review Chapter 4

1. Solve

a. $10 + 3(a + 5) \;=\; 2(a - 6) - 4a$	**b.** $20x - 2(x + 1) \;=\; 10 - (x - 5)$
c. $\frac{1}{6}x - 1 \;=\; 1 + \frac{4}{5}x$	**d.** $2z + \frac{2}{5} \;=\; \frac{1}{4}z - 1$

2. Tell, without fully solving the equations, whether each equation has one unique solution, no solution, or an infinite number of solutions.

a. $y = 6 - 7y$

b. $6 - 7y = 2 - 7y$

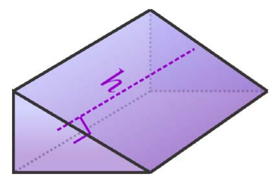
c. $-7y + 14 = 7(2 - y)$

d. $-7y - 2 = -2$

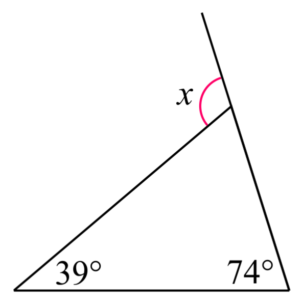
3. **a.** The volume of a cone is $V = \dfrac{A_b h}{3}$, where A_b is the area of the base and h is the height of the cone. Solve this for h.

b. What is the height of a circular cone with a volume of 20,900 cm^3 and a bottom radius of 25.0 cm?

4. Find the volume of this triangular prism, if its height is 36 cm and its base is a right triangle with 10 cm, 15 cm, and 18 cm sides.

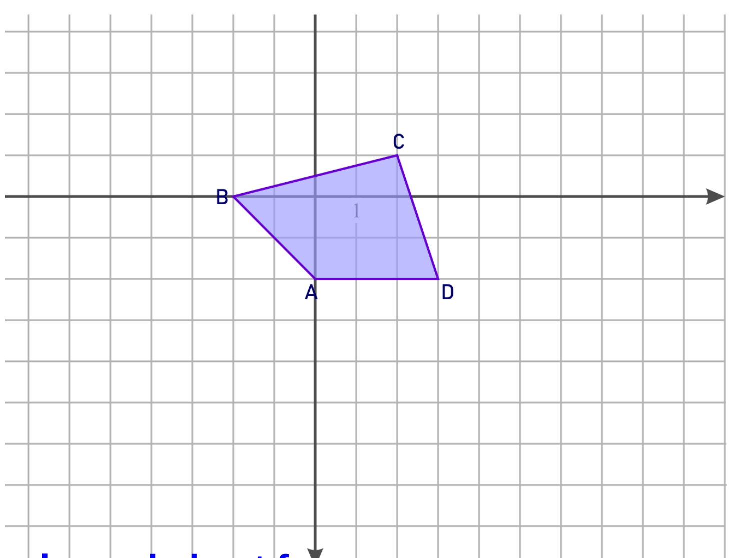


5. Find the measure of angle x. Explain your reasoning.

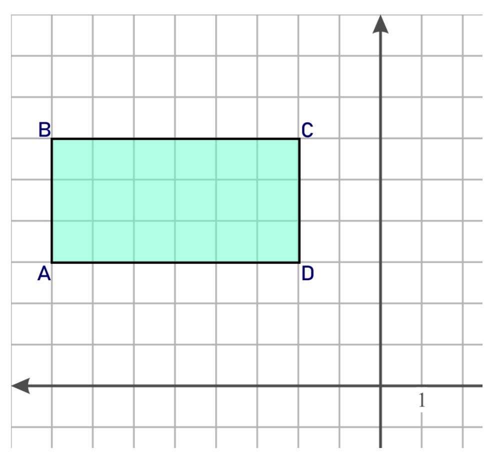


6. Dilate each figure with origin as center and with the given scale factor.

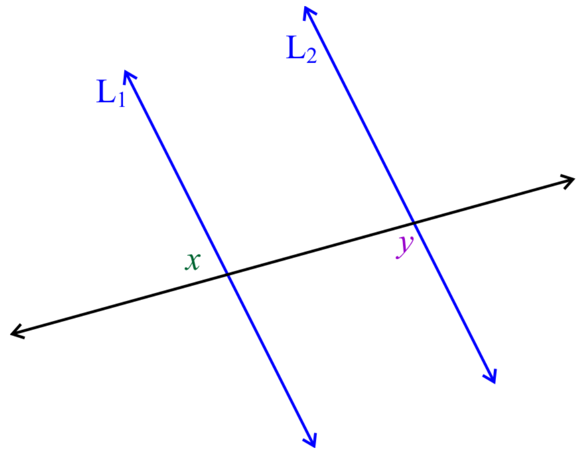
a. scale factor 3



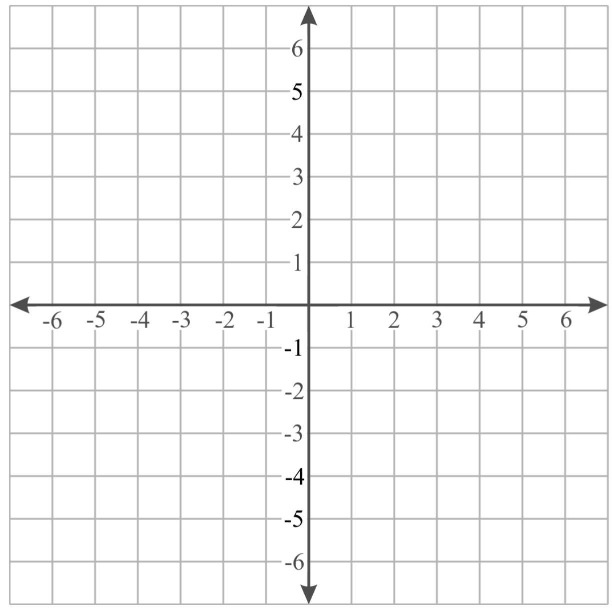
b. scale factor 1/2



7. Two parallel lines, L_1 and L_2, are cut by a transversal.
If angle x is 76°, find the angle measure of y, and explain how you know that.

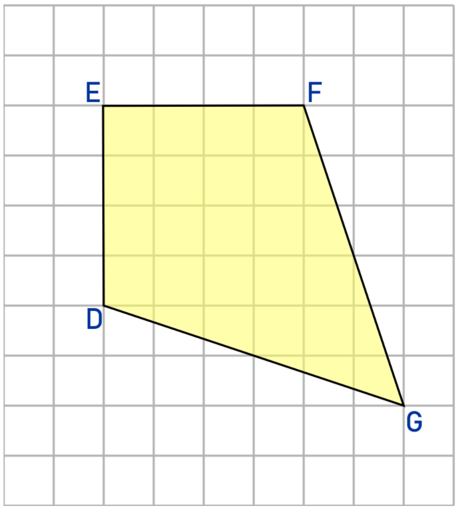


8. A triangle with vertices A(−1, 2), B(−3, 2), and C(−4, 4) is first reflected in the y-axis and then rotated 90° clockwise around the origin. What are the coordinates of the vertices of the resulting triangle?



9. Draw dilations.

a. Draw a dilation of kite DEFG from point E, with scale factor 1/2.



b. Draw a dilation of triangle ABC from point C, with scale factor 3.