

math

MAMMOTH

Grade 7-A Worktext

The language of algebra

Integers

Solving one—
step equations

Rational
numbers

Equations and inequalities



L
i
g
h
t
B
l
u
e
S
e
r
i
e
s

Contents

| | |
|---|----|
| Foreword | 5 |
| Chapter 1: The Language of Algebra | |
| Introduction | 6 |
| The Order of Operations | 11 |
| Expressions and Equations | 15 |
| Properties of the Four Operations | 18 |
| Simplifying Expressions | 22 |
| Growing Patterns 1 | 26 |
| The Distributive Property | 29 |
| Chapter 1 Review | 34 |
| Chapter 2: Integers | |
| Introduction | 36 |
| Integers | 41 |
| Addition and Subtraction on the Number Line | 45 |
| Addition of Integers | 49 |
| Subtraction of Integers | 52 |
| Adding or Subtracting Several Integers | 56 |
| Distance and More Practice | 58 |
| Multiplying Integers | 62 |
| Dividing Integers | 67 |
| Negative Fractions | 69 |
| The Order of Operations | 73 |
| Chapter 2 Mixed Review | 75 |
| Chapter 2 Review | 77 |
| Chapter 3: Solving One–Step Equations | |
| Introduction | 80 |
| Solving Equations | 82 |
| Addition and Subtraction Equations | 89 |
| Multiplication and Division Equations | 93 |

| | |
|------------------------------|-----|
| Word Problems | 97 |
| Constant Speed | 100 |
| Chapter 3 Mixed Review | 107 |
| Chapter 3 Review | 110 |

Chapter 4: Rational Numbers

| | |
|---|-----|
| Introduction | 112 |
| Rational Numbers | 116 |
| Adding and Subtracting Rational Numbers | 124 |
| Multiply and Divide Rational Numbers 1 | 130 |
| Multiply and Divide Rational Numbers 2 | 134 |
| Many Operations with Rational Numbers | 141 |
| Scientific Notation | 145 |
| Equations with Fractions | 148 |
| Equations with Decimals | 153 |
| Chapter 4 Mixed Review | 156 |
| Chapter 4 Review | 158 |

Chapter 5: Equations and Inequalities

| | |
|---------------------------------------|-----|
| Introduction | 163 |
| Two-Step Equations | 167 |
| Two-Step Equations: Practice | 172 |
| Growing Patterns 2 | 176 |
| A Variable on Both Sides | 180 |
| Some Problem Solving | 186 |
| Using the Distributive Property | 189 |
| Word Problems | 195 |
| Inequalities | 198 |
| Word Problems and Inequalities | 203 |
| Graphing | 205 |
| An Introduction to Slope | 209 |
| Speed, Time and Distance | 214 |
| Chapter 5 Mixed Review | 219 |
| Chapter 5 Review | 222 |

Foreword

Math Mammoth Grade 7-A and Grade 7-B worktexts comprise a complete math curriculum for seventh grade mathematics studies. This is a pre-algebra course, and students can continue to an algebra 1 curriculum after studying it.

The curriculum meets the Common Core Standards (CCS) for grade 7 but also exceeds them in several places. For example, we study linear equations in more depth than required in grade 7 CCS. The Pythagorean Theorem is not part of the CCS for grade 7 (it is in the CCS for grade 8). You can access a document detailing the alignment information either on the Math Mammoth website or in the download version of this curriculum.

We start out with an introduction to basic algebra, which is in many ways a review of the same topics from 6th grade. The first chapter reviews the order of operations, the concepts of an expression and equation, and the distributive property. Students learn about the commutative and associative properties of addition and multiplication, and they simplify expressions that do not involve negative numbers.

In chapter 2, we study integers and their operations in detail. Some of this is, again, review from 6th grade, and some of it is new. The four operations of integers are explained with the help of two visual models: the number line and counters, hopefully providing an intuitive understanding of the processes. Students need to be able to add, subtract, multiply, and divide integers when they learn to solve equations in chapter 3. In the end of the chapter students also learn about negative fractions.

The lesson about distance contains a formula that may look unfamiliar to the teacher. You can find the distance between two integers by taking the absolute value of their difference. In symbols, the distance between a and b is $|a - b|$. The idea behind the formula is simple, though, and most people use that idea instinctively without knowing about the formula. For example, how far apart from each other are 14 and 92? To solve that mentally, we find their difference, but we take that difference in a positive sense. In other words, we do not calculate $14 - 92 = -78$ and state that the distance is negative 78 units, but instead, we say the distance is 78 units because distance is always positive. The absolute value takes care of that: it turns any negative quantity into a positive one.

In the next chapter (chapter 3) students study simple one-step equations. They already know the basics of how to solve these types of equations from 6th grade, but this time we use negative numbers in them.

Chapter 4 is titled *Rational Numbers*, which are simply fractions and certain decimals, so the student is already very familiar with them. The goal of the chapter is to be able to add, subtract, multiply, and divide both positive and negative fractions and decimals. We also solve simple equations involving fractions and decimals and learn about scientific notation.

The last chapter in part 7-A focuses on linear equations. The student learns to solve various types of linear equations and practices using those in simple word problems. We study linear inequalities but not to the same depth as linear equations. Lastly, the student graphs linear equations and is introduced to the concept of slope, which is the steepness of a line. The student will continue studying these topics with more details in an algebra 1 course.

In part 7-B, students study ratios, proportions, percent, geometry, the Pythagorean Theorem, statistics, and probability.

I wish you success in teaching math!

Maria Miller, the author

Chapter 1: The Language of Algebra

Introduction

In the first chapter of *Math Mammoth Grade 7* we review all of the sixth grade algebra topics and also study some basic properties of the operations.

The main topics are the order of operations, expressions, and simplifying expressions in several different ways. The main principles are explained and practiced both with visual models and in abstract form, and the lessons contain varying practice problems that approach the concepts from various angles.

This chapter is like an introduction that lays a foundation for the rest of the year. For example, when we study integers in the next chapter, students will once again simplify expressions, just with negative numbers. Then when we study equations in chapters 3 and 5, students will again simplify expressions, use the distributive property, and solve equations.

The Lessons in Chapter 1

| | page | span |
|---|------|---------|
| The Order of Operations | 11 | 4 pages |
| Expressions and Equations | 15 | 3 pages |
| Properties of the Four Operations | 18 | 4 pages |
| Simplifying Expressions | 22 | 4 pages |
| Growing Patterns 1 | 26 | 3 pages |
| The Distributive Property | 29 | 5 pages |
| Chapter 1 Review | 34 | 2 pages |

Helpful Resources on the Internet

Order of operations

Otter Rush

Practice exponents in this otter-themed math game.

http://www.mathplayground.com/ASB_Otter_Rush.html

Choose A Math Operation

Choose the mathematical operation(s) so that the number sentence is true. Practice the role of zero and one in basic operations or operations with negative numbers. Helps develop number sense and logical thinking.

<http://www.homeschoolmath.net/operation-game.php>

Order of Operations Quiz

A 10-question online quiz that includes two different operations and possibly parentheses in each question. You can also modify the quiz parameters yourself.

<http://www.thatquiz.org/tq-1/?-j8f-lk-p0>

The Order of Operations Millionaire

Answer multiple-choice questions that have to do with the order of operations, and win a million. Can be played alone or in two teams.

<http://www.math-play.com/Order-of-Operations-Millionaire/order-of-operations-millionaire.html>

Exploring Order of Operations (Object Interactive)

The program shows an expression, and you click on the correct operation (either +, −, ×, ÷ or exponent) to be done first. The program then solves that operation, and you click on the *next* operation to be performed, *etc.*, until it is solved. Lastly, the resource includes a game where you click on the falling blocks in the sequence that the order of operations would dictate.

http://www.learnalberta.ca/content/mejhm/html/object_interactives/order_of_operations/use_it.html

Make 24 Game

Arrange the number cards, the operation symbols, and the parentheses, so that the expression will make 24.

http://www.mathplayground.com/make_24.html

Order of Operations Practice

A simple online quiz of 10 questions. Uses parentheses and the four operations.

<http://www.onlinemathlearning.com/order-of-operations-practice.html>

Writing expressions

Algebraic Symbolism Matching Game

Match each verbal statement with its algebraic expression.

<http://www.quia.com/mc/319817.html>

Algebraic Expressions Millionaire

For each question you have to identify the correct mathematical expression that models a given word expression.

<http://www.math-play.com/Algebraic-Expressions-Millionaire/algebraic-expressions-millionaire.html>

Expressions: Expressions and Variables Quiz

Choose an equation to match the word problem or situation.

http://www.softschools.com/quizzes/math/expressions_and_variables/quiz815.html

Words into Equations Battleship Game

Practice expressions such as quotient, difference, product, and sum.

<http://www.quia.com/ba/210997.html>

Rags to Riches - Verbal and Algebraic Expressions

Translate between verbal and algebraic expressions in this quest for fame and fortune.

<http://www.quia.com/rr/520475.html>

Algebra Noodle

Play a board game against the computer while modeling and solving simple equations and evaluating simple expressions. Choose level 2 (level 1 is too easy for 7th grade).

<http://www.free-training-tutorial.com/math-games/algebra-noodle.html>

Matching Algebraic Expressions with Word Phrases

Five sets of word phrases to match with expressions.

<http://www.mrmaisonet.com/index.php?/Algebra-Quizzes/Matching-Algebraic-Expressions-With-Word-Phrases.html>

Practice with Algebraic Representation

Practice problems with self-check answer keys about translating algebraic expressions into verbal expressions and vice versa.

<http://www.regentsprep.org/Regents/math/ALGEBRA/AV1/PAIgRep.htm>

Properties of the operations

Properties of Operations at Quizlet

Includes explanations, online flashcards, and a test for the properties of operations (commutative, associate, distributive, inverse, and identity properties). The inverse and identity properties are not covered in this chapter of Math Mammoth but can be learned at the website. The identity property refers to the special numbers that do not change addition or multiplication results (0 and 1).

<http://quizlet.com/2799611/properties-of-operation-flash-cards/>

Commutative/associative/distributive properties matching game

Match the terms and expressions in the two columns.

http://www.quia.com/cm/61114.html?AP_rand=1554068841

Properties of Multiplication

Simple online practice about the commutative, associative, distributive, and identity properties of multiplication.

<http://www.aaamath.com/pro74b-propertiesmult.html>

Properties of Multiplication

Simple online practice about the commutative, associative, distributive, and identity properties of multiplication.

<http://www.aaamath.com/pro74ax2.htm>

Properties of the Operations Scatter Game

Drag the corresponding items to each other to make them disappear.

<http://quizlet.com/763838/scatter>

Associative, Distributive and Commutative Properties

Examples of the various properties followed by a simple self-test.

<http://www.mathwarehouse.com/properties/associative-distributive-and-commutative-properties.php>

Simplifying expressions

Simplifying Algebraic Expressions Quiz

An online quiz of 15 questions.

<http://www.quia.com/quiz/1200540.html>

BBC Bitesize - Simplifying Algebraic Terms

A 10-question online quiz on simplifying expressions.

<http://www.bbc.co.uk/bitesize/quiz/q14530139>

The distributive property

Factor the Expressions Quiz

Factor expressions such as $3x + 15$ into $3(x + 5)$.

<http://www.thatquiz.org/tq-0/?-jh00-l3-p0>

Distributive Property Practice

Guided practice for applying the distributive property, such as writing $-8(-7a + 10)$ as $56a - 80$.

<http://www.hstutorials.net/dialup/distributiveProp.htm>

Distributive Property Battleship

Practice simplifying expressions using the distributive property while playing battleship game against the computer.

<http://www.quia.com/ba/15357.html>

Evaluate expressions

Late Delivery

Help Postie the postman deliver letters while evaluating simple expressions.

<http://www.bbc.co.uk/schools/mathsfile/shockwave/games/postie.html>

Escape Planet

Choose the equation that matches the words.

http://www.harcourtschool.com/activity/escape_planet_6/

Evaluating Expressions Quiz

Includes ten multiple-choice questions.

<http://www.mrmaisonet.com/index.php?/Algebra-Quizzes/Evaluating-Expressions.html>

Writing & Evaluating Expressions Quiz

This quiz has 12 multiple-choice questions and tests both evaluating and writing expressions.

<http://www.quibblo.com/quiz/aWAUlc6/Writing-Evaluating-Expressions>

Terms/constant/coefficient

Coefficients, Like Terms, and Constants

How to find and name the coefficients, like terms, and constants in expressions.

<http://mathcentral.uregina.ca/qq/database/qq.09.07/h/maddie1.html>

Identifying Variable Parts and Coefficients of Terms

After the explanations, you can generate exercises by pushing the button that says “new problem.” The script shows you a multiplication expression, such as $-(3e)(3z)m$, and you need to identify its coefficient and variable part, effectively by first simplifying it.

http://www.onemathematicalcat.org/algebra_book/online_problems/id_var_part_coeff.htm#exercises

Tasty Term Treats

A lesson followed by a simple game where you drag terms into Toby's bowl and non-terms into the trash can.

http://mathstar.lacoe.edu/lessonlinks/menu_math/var_terms.html

Algebra - basic definitions

Clear definitions with illustrations of basic algebra terminology, including term, coefficient, constant, and expression.

<http://www.mathsisfun.com/algebra/definitions.html>

General

Fill and Pour

Fill and pour liquid with two containers until you get the target amount. A logical thinking puzzle.

http://nlvm.usu.edu/en/nav/frames_asid_273_g_2_t_4.html

Balance Beam Activity

A virtual balance that poses puzzles where the student must think algebraically to find the weights of various figures. Includes three levels.

<http://mste.illinois.edu/users/pavel/java/balance/index.html>

Algebraic Reasoning

Find the value of an object based on two scales.

http://www.mathplayground.com/algebraic_reasoning.html

Algebra Puzzle

Find the value of each of the three objects presented in the puzzle. The numbers given represent the sum of the objects in each row or column.

http://www.mathplayground.com/algebra_puzzle.html

Algebra Balance Scales

Model the given equation on a balance. Then choose which operation is done to both sides, until the equation is solved. This version of the activity involves only positive whole numbers.

http://nlvm.usu.edu/en/nav/frames_asid_201_g_4_t_2.html

Algebraic Expressions - Online Assessment

During this online quiz you must simplify expressions, combine like terms, use the distributive property, express word problems as algebraic expressions and recognize when expressions are equivalent. Each incorrect response will allow you to view a video explanation for that problem.

<http://www.mrmaisonet.com/index.php/?Algebra-Quizzes/Online-Assessment-Algebraic-Expressions.html>

(This page intentionally left blank.)

Simplifying Expressions

Example. Simplify $2x \cdot 4 \cdot 5x$.

Notice, this expression contains only multiplications (because $2x$ and $5x$ are also multiplications).

Since we can multiply in any order, we can write this expression as $2 \cdot 4 \cdot 5 \cdot x \cdot x$.

Now we multiply 2, 4, and 5 to get 40. What is left to do? The part $x \cdot x$, which is written as x^2 .

So, $2x \cdot 4 \cdot 5x = 40x^2$.

Note: The equals sign used in $2x \cdot 4 \cdot 5x = 40x^2$ signifies that the two expressions are equal no matter what value x has. That equals sign does not signify an equation that needs to be solved.

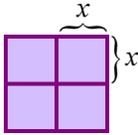
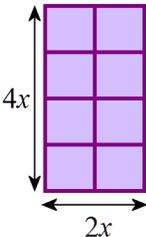
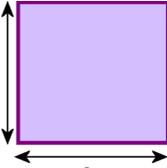
Similarly, we can simplify the expression $x + x$ and write $2x$ instead. That whole process is usually written as $x + x = 2x$.

Again, the equals sign there does not indicate an equation to solve, but just the fact that the two expressions are equal. In fact, if you think of it as an equation, *any* number x satisfies it! (Try it!)

1. Simplify the expressions.

| | | |
|--------------------|--|--|
| a. $p + 8 + p + p$ | b. $p \cdot 8 \cdot p \cdot p \cdot p$ | c. $2p + 4p$ |
| d. $2p \cdot 4p$ | e. $5x \cdot 2x \cdot x$ | f. $y \cdot 2y \cdot 3 \cdot 2y \cdot y$ |

2. Write an expression for the area and perimeter of each rectangle.

| | |
|--|---|
| <p>a. </p> <p>area =</p> <p>perimeter =</p> | <p>b. </p> <p>area =</p> <p>perimeter =</p> |
| <p>c. </p> <p>area =</p> <p>perimeter =</p> | <p>d. </p> <p>area =</p> <p>perimeter =</p> |

3. **a.** Sketch a rectangle with sides $2b$ and $7b$ long.

b. What is its area?

c. What is its perimeter?

4. **a.** The perimeter of a rectangle is $24s$.
Sketch one such rectangle.

What is its area?

Hint: there are many possible answers.

b. Find the area and perimeter of your rectangle
in (a) if s has the value 3 cm.

5. **a.** Which expression below is for an area of a rectangle? Which one is for a perimeter?

$$4a + 4b \quad 2a \cdot 2b$$

b. Sketch the rectangle.

6. **a.** Find the value of the expressions $3p$ and $p + 3$ for different values of p .

| Value of p | $3p$ | $p + 3$ |
|--------------|------|---------|
| 0 | | |
| 0.5 | | |
| 1 | | |
| 1.5 | | |
| 2 | | |
| 2.5 | | |
| 3 | | |
| 3.5 | | |
| 4 | | |

b. Now, look at the table. Can you tell which is larger, $3p$ or $p + 3$?

Some review! In algebra, a **term** is an expression that consists of numbers and/or variables that are multiplied together. A single number or variable is also a term.

- Examples.**
- $2xy$ is a term, because it only contains multiplications, a number, and variables.
 - $(5/7)z^3$ is a term. Remember, the exponent is a shorthand for repeated multiplication.
 - Addition and subtraction separate the individual terms in an expression from each other. For example, the expression $2x^2 - 6y^3 + 7xy + 15$ has four terms, separated by the plus and minus signs.
 - $s + t$ is *not* a term, because it contains addition. Instead, it is a sum of *two* terms, s and t .

The number by which a variable or variables are multiplied is called a **coefficient**.

- Examples.**
- The term $0.9ab$ has the coefficient 0.9.
 - The coefficient of the term m^2 is 1, because you can write m^2 as $1 \cdot m^2$.

If the term is a single number, such as $7/8$, we call it a **constant**.

Example. The expression $1.5a + b^2 + 6/7$ has three terms: $1.5a$, b^2 , and $6/7$. The last term, $6/7$, is a constant.

7. Fill in the table.

| Expression | the terms in it | coefficient(s) | Constants |
|---------------|-----------------|----------------|-----------|
| $(5/6)s$ | | | |
| w^3 | | | |
| $0.6x + 2.4y$ | | | |
| $x + 3y + 7$ | | | |
| $p \cdot 101$ | | | |
| $x^5y^2 + 8$ | | | |

The two terms in the expression $2x^2 + 5x^2$ are **like terms**: they have the same variable part (x^2). Because of that, we can add the two terms to simplify the expression. To do that, simply add the coefficients 2 and 5 and use the same variable part: $2x^2 + 5x^2 = 7x^2$. It is like adding 2 apples and 5 apples.

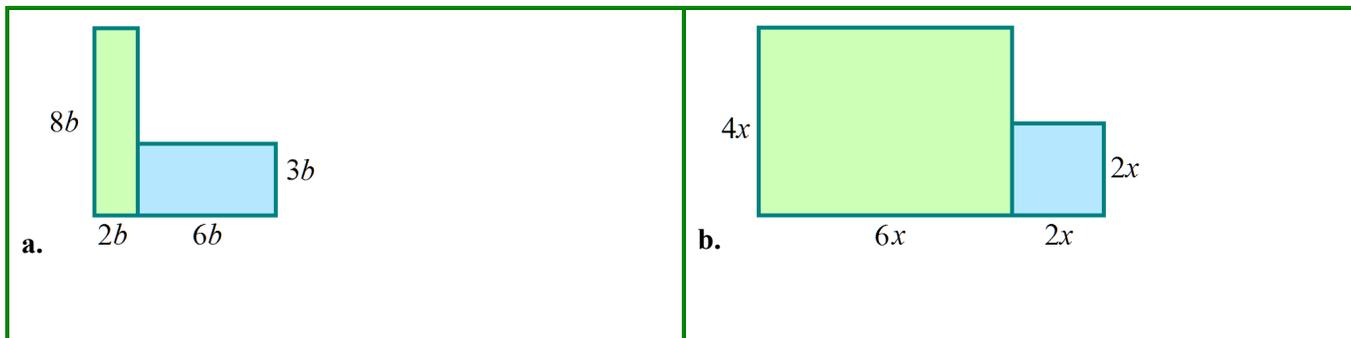
However, you cannot add (or simplify) $2x + 7y$. That would be like adding 2 apples and 7 oranges.

Example. Simplify $6x - x - 2x + 9x$. The terms are like terms, so we simply add or subtract the coefficients: $6 - 1 - 2 + 9 = 12$ and tag the variable part x to it. The expression simplifies to $12x$.

8. Simplify the expressions.

| | | |
|------------------|------------------------|--------------------------|
| a. $5p + 8p - p$ | b. $p^2 + 8p^2 + 3p^2$ | c. $12a^2 - 8a^2 - 3a^2$ |
|------------------|------------------------|--------------------------|

9. Write an expression for the total area.



In the following problems, write an expression for part (a), and then for part (b) write an equation and solve it. Don't skip writing the equation, even if you can solve the problem without it, because we are practicing writing equations! You don't have to use algebra to solve the equations—you can solve them in your head or by guessing and checking.

10. a. The length of a rectangle is 4 meters and its width is w .
What is its perimeter? Write an expression.

b. Let's say the perimeter has to be 22 meters. How wide is the rectangle then?
Write an equation for this situation, using your expression from (a).

Remember, you do not have to use algebra to solve the equation—you can solve it in your head or by "guess and check." But do write the equation.

11. a. Linda borrows six books from the library each week, and her mom borrows two.
How many books, in total, do both of them borrow in w weeks? Write an expression.

b. How many weeks will it take them to have borrowed 216 books? Write an equation.

12. a. Alice buys y containers of mints for \$6 apiece. A fixed shipping cost of \$5 is added to her order. What is her total cost? Write an expression.

b. Alice's total bill was \$155. How many containers of mints did she buy?
Write an equation.

Puzzle Corner

a. What is the total value, in cents, if Ashley has n dimes and m quarters?
Write an expression.

b. The total value of Ashley's coins is 495 cents. How many dimes and quarters can she have?
Hint: make a table to organize the possibilities.

(This page intentionally left blank.)

Chapter 2: Integers

Introduction

This chapter deals with integers, which are signed (positive or negative) whole numbers. We begin with a review of addition and subtraction of integers from 6th grade. Then we study in detail multiplication and division of integers and conclude with negative fractions and the order of operations.

The first lesson reviews the concepts of integers, absolute value, the opposite of an integer, and simple inequalities on a number line. The next lessons present the addition and subtraction of integers through two visual models: first as movements on a number line, and then using positive and negative counters. These lessons also endeavor to connect the addition and subtraction of integers with various situations from real life.

The lesson *Subtraction of Integers* includes this important principle: Any subtraction can be converted into an addition (of the number of opposite sign) and vice versa. This principle allows us to calculate not only subtractions such as $5 - (-7)$ but also problems that contain both addition and subtraction. These mixed problems become simple sums after the subtractions have been converted into additions. Converting subtractions into additions or vice versa is also important when simplifying expressions. For example, $5 + (-x)$ can be simplified to $5 - x$.

Next, we study the distance between two integers. This can be found by taking the absolute value of their difference: the distance between x and y is $|x - y|$. Students learn to use this formula to find distances between integers, and they also compare the result the formula gives to the answer they get by logical thinking.

The lesson *Multiplying Integers* not only teaches the mechanics of how to multiply integers but also gives both intuitive understanding and formal justification for the shortcut, “a negative times a negative makes a positive.” This formal justification using the distributive property introduces and illustrates the type of careful and precise reasoning that mathematicians use in proofs.

The next lesson, on the division of integers, leads naturally into the topic of negative fractions in the following lesson. The final topic is a review of the order of operations where we perform several operations at a time with integers.

The Lessons in Chapter 2

| | page | span |
|---|------|---------|
| Integers | 41 | 4 pages |
| Addition and Subtraction on the Number Line | 45 | 4 pages |
| Addition of Integers | 49 | 3 pages |
| Subtraction of Integers | 52 | 4 pages |
| Adding and Subtracting Many Integers | 56 | 2 pages |
| Distance and More Practice | 58 | 4 pages |
| Multiplying Integers | 62 | 5 pages |
| Dividing Integers | 67 | 2 pages |
| Negative Fractions | 69 | 4 pages |
| The Order of Operations | 73 | 2 pages |
| Chapter 2 Mixed Review | 75 | 2 pages |
| Chapter 2 Review | 77 | 3 pages |

Helpful Resources on the Internet

Ordering integers

Number Balls

Click the balls in the ascending order of numbers.

<http://www.sheppardsoftware.com/mathgames/numberballs/numberballsAS2.htm>

Negative Numbers Hat Game

Put the hats with numbers on the people's heads in the right order.

<http://www.primaryresources.co.uk/online/negnumorder.swf>

Order Negative Numbers

Drag and drop the numbers in the right order onto the ladder (scroll down the page a bit to see the activity).

http://www.bbc.co.uk/bitesize/ks3/maths/number/negative_numbers/revision/2/

Compare Integers Quiz

A 10-question online quiz where you compare two integers. You can also modify the quiz parameters to include sums, differences, products, and quotients, which makes it more challenging.

<http://www.thatquiz.org/tq-8/?-j11-11i-p0>

Integers in Space

This is an asteroids-style game with a twist: where you get points for shooting the rocks in space in the correct order — either starting from the smallest or from the greatest integer.

<http://www.mathwarehouse.com/games/our-games/arithmetric-games/integers-in-space/>

Diamond Drop

Drag integers to the empty spaces in comparison sentences (such as $___ < ____$, $___ = ____$, and $___ > ____$) as they fall from the top of the screen. (The link does not work when clicked from the PDF file; please copy and paste it to your browser window.)

http://oame.on.ca/CLIPS/swfPlayer.html?swfURL=lib/CL005_IntegersRepresentCompareOrder/CL005_C02_A05_C_DiamondDrop/CL005_C02_A05_C_DiamondDrop.swf

Absolute value and opposites

Number Balls - Absolute Value

Click the balls in the ascending order of numbers.

<http://www.onlinemathlearning.com/absolute-value-game.html>

Absolute Value Boxes

You are shown expressions with absolute value (such as $|-11|$, $-|8|$ and $|3|$). Choose the one that has either the greatest or the least value.

<http://www.xpmath.com/forums/arcade.php?do=play&gameid=96>

Absolute Value Quiz

Find the absolute value of each integer or sum.

http://www.softschools.com/quizzes/math/absolute_value/quiz1035.html

Absolute Value Quiz - harder

This quiz includes mixed operations and absolute value.

<http://www.softschools.com/testing/math/theme2.html>

Introducing Integers - Activities

Choose from six different interactive tasks: place integers on a number line, identify integers on a number line, identify opposites of integers, compare two integers, order four integers, and solve absolute values.

http://mathstar.lacoe.edu/newmedia/integers/intro/activities/intro_numberline.html

Addition and subtraction

The section for “all operations” has more games for adding and subtracting.

Spider Match

Choose pairs of numbers that add to the given integer. Can be played as a multi-player game or against the computer.

<http://www.arcademics.com/games/spider-match/spider-match.html>

Orbit Integers

Practice integer addition in this racing game. Can be played as a multi-player game or against the computer.

<http://www.arcademics.com/games/orbit-integers/orbit-integers.html>

Integer warp

Practice integer multiplication in this racing game. Can be played as a multi-player game or against the computer.

<http://www.arcademics.com/games/integer-warp/integer-warp.html>

Math Lines Integers

Combine positive and negative balls to make the target number.

http://www.mathplayground.com/math_lines_integers.html

Subtracting Integers Activity

Model subtraction problems by dragging individual + or – signs off the board. You may need to add neutral pairs (a positive-negative pair) to the working space to solve the problem.

<http://mathstar.lacoe.edu/newmedia/integers/subtract/activities/activities.html>

Find the missing integers

Fill in the missing integer in addition equations such as $-23 + \underline{\quad} = -8$.

http://www.aaamath.com/g8_65_x3.htm

Color Chips Addition

The user drags positive/negative chips to the working area, then combines them in pairs to see the sum.

http://nlvm.usu.edu/en/nav/frames_asid_161_g_2_t_1.html

Color Chips Subtraction

Drag positive/negative chips and zero pairs into working area as instructed, then subtract.

http://nlvm.usu.edu/en/nav/frames_asid_162_g_3_t_1.html

Integer Football

The football player advances either towards the positive or the negative yard lines.

http://www.mathgoodies.com/games/integer_game/football.html

Line Jumper

You see a number line and an addition or subtraction problem. Click the right answer on the number line.

<http://www.funbrain.com/linejump>

Space Coupe to the Rescue

By choosing a positive or negative number, the player controls the vertical position of a spaceship. If the spaceship reaches the same vertical position as a virus pod, the pod is destroyed.

<http://pbskids.org/cyberchase/games/negativenumbers>

Red and Black Triplematch Game for Adding Integers

This is a fun card game with 2-5 people to practice adding integers.

<http://mathmamawrites.blogspot.com/2010/07/black-and-red-triplematch-card-game-for.html>

Adding and Subtracting Integers Gizmos from Explorelearning.com

Interactive simulations that illustrate adding and subtracting integers on a number line or with chips. Includes an exploration guide and assessment questions. You can get a 5-minute access for free, or a free 30-day trial account.

<http://www.explorelearning.com/index.cfm?method=cResource.dspResourcesForCourse&CourseID=211>

Multiplication and division

The section for "all operations" has more games that practice multiplication and division.

Integer Warp (Race)

A one-minute race with up to four spaceships (players). Answer integer multiplication problems correctly to speed up your spaceship.

<http://www.arcademicskillbuilders.com/games/integer-warp/integer-warp.html>

Integers Multiplication Blocks

Click on numbers whose product is equal to the given target number. Score as high as you can in the given time limit.

<http://www.xpmath.com/forums/arcade.php?do=play&gameid=58>

Integers Multiplication -144 to 144

Solve as many integer multiplication problems as you can within one minute with these online flashcards.

<http://www.xpmath.com/forums/arcade.php?do=play&gameid=17>

Integers Division -81 to 81

Solve as many integer division problems as you can within one minute with these online flashcards.

<http://www.xpmath.com/forums/arcade.php?do=play&gameid=16>

All operations / General

Rags to Riches with Integers

In this game you find absolute value of integers, compare integers, and do simple calculations with integers.

<http://www.quia.com/rr/41496.html>

Integers Jeopardy

A jeopardy game where the questions involve adding, subtracting, multiplying, and dividing integers.

<http://www.math-play.com/Integers-Jeopardy/Integers-Jeopardy.html>

Integers Mystery Picture Game

Solve expressions with many operations.

<http://www.dositey.com/2008/addsub/Mystery11.htm>

Solve For Unknown Variable - Integer Review

Find the value of an unknown variable in a given addition or subtraction equation with integers.

<http://www.mrmaisonet.com/index.php?/Algebra-Quizzes/Solve-For-Unknown-Variable-Integer-Review.html>

Arithmetic Four (Connect the Four game)

Practice any or all of the four operations with integers. First you answer a math problem, then you move in a connect-the-four game.

<http://www.shodor.org/interactivate/activities/ArithmeticFour>

Fruit Shoot Game: Mixed Integer Operations

Practice all four operations with integers while shooting fruits. You can choose the difficulty level and the speed.

http://www.sheppardsoftware.com/mathgames/fruitshoot/FS_Mixed_Integers.htm

Student CLIPS in Mathematics

Activities, video clips, and games for middle school math topics, including integers.

<http://oame.on.ca/CLIPS/>

Create Integers Worksheets

Use the basic operations worksheet generator for integer worksheets by choosing the range of numbers to be from negative to positive.

<http://www.homeschoolmath.net/worksheets/basic-operations-worksheets.php>

Flashcards with Negative Numbers

Interactive flashcards at AplusMath.com for integer addition, subtraction, multiplication, and division.

<http://www.aplusmath.com/Flashcards/sub-nflash.html>

How to Teach Integers

An article for the teacher about how to teach integer operations.

<http://www.homeschoolmath.net/teaching/integers.php>

Free Downloadable Integer Fact Sheets

http://www.homeschoolmath.net/download/Add_Subtract_Integers_Fact_Sheet.pdf

http://www.homeschoolmath.net/download/Multiply_Divide_Integers_Fact_Sheet.pdf

The History of Negative Numbers

While they seem natural to us now, in the past negative numbers have spurred controversy and been called “fictitious” and other names.

http://nrich.maths.org/public/viewer.php?obj_id=5961

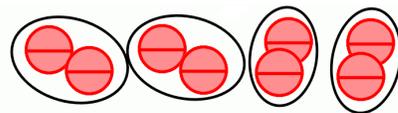
http://www.classzone.com/books/algebra_1/page_build.cfm?content=links_app3_ch2&ch=2

(This page intentionally left blank.)

Dividing Integers

Divide a negative number by a positive

The image illustrates $(-8) \div 4$, or eight negatives divided into four groups. We can see the answer is -2 .



Any time a negative integer is divided by a positive integer, we can illustrate it as so many negative counters divided evenly into groups. The answer will be negative.

Divide a positive integer by a negative. For example, $24 \div (-8) = ?$

Remember, multiplication is the opposite operation to division. Let's write the answer to $24 \div (-8)$ as s . Then from that we can write a multiplication:

$$24 \div (-8) = s \Rightarrow (-8)s = 24$$

(You could use an empty line instead of s , if the variable s confuses you.)

The only number that fulfills the equation $(-8)s = 24$ is $s = -3$. Therefore, $24 \div (-8) = -3$.

Similarly, each time you divide a positive integer by a negative integer, the answer is negative.

Divide a negative integer by a negative. For example, $(-24) \div (-8) = ?$

Again, let's denote the answer to $-24 \div (-8)$ with y , and then write a multiplication sentence.

$$-24 \div (-8) = y \Rightarrow (-8)y = -24$$

The only number that fulfills the equation $(-8)y = -24$ is $y = 3$. Therefore, $-24 \div (-8) = 3$.

Similarly, each time you divide a negative integer by a negative integer, the answer is positive.

Summary. The symbols below show whether you get a positive or negative answer when you multiply or divide integers. Notice that the rules for multiplication and division are the same!

Multiplication

$$\oplus \cdot \ominus = \ominus$$

$$\ominus \cdot \oplus = \ominus$$

$$\ominus \cdot \ominus = \oplus$$

$$\oplus \cdot \oplus = \oplus$$

Examples

$$4 \cdot (-5) = -20$$

$$-4 \cdot 5 = -20$$

$$-4 \cdot (-5) = 20$$

$$4 \cdot 5 = 20$$

Division

$$\oplus \div \ominus = \ominus$$

$$\ominus \div \oplus = \ominus$$

$$\ominus \div \ominus = \oplus$$

$$\oplus \div \oplus = \oplus$$

Examples

$$20 \div (-5) = -4$$

$$-20 \div 5 = -4$$

$$-20 \div (-5) = 4$$

$$20 \div 5 = 4$$

Here is a shortcut for *multiplication* and *division* (NOT for addition or subtraction):

- If both numbers have the same sign (both are positive *or* negative), the answer is positive.
- If the numbers have different signs, the answer is negative.

1. Divide.

a. $-50 \div (-5) = \underline{\hspace{2cm}}$

$-12 \div 2 = \underline{\hspace{2cm}}$

b. $(-8) \div (-1) = \underline{\hspace{2cm}}$

$14 \div (-2) = \underline{\hspace{2cm}}$

c. $81 \div (-9) = \underline{\hspace{2cm}}$

$-100 \div (-10) = \underline{\hspace{2cm}}$

2. Multiply. Then use the same numbers to write an equivalent division equation.

| | | |
|---|--|---|
| a. $-5 \cdot (-5) = \underline{\hspace{2cm}}$ $\underline{\hspace{2cm}} \div \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$ | b. $9 \cdot (-6) = \underline{\hspace{2cm}}$ $\underline{\hspace{2cm}} \div \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$ | c. $-80 \cdot 8 = \underline{\hspace{2cm}}$ $\underline{\hspace{2cm}} \div \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$ |
|---|--|---|

3. Four people shared a debt of \$280 equally. How much did each owe? Write an integer division.

4. In a math game, you get a negative point for every wrong answer and a positive point for every correct answer. Additionally, if you answer in 1 second, your negative points from the past get slashed in half!

Angie had accumulated 14 negative and 25 positive points in the game. Then she answered a question correctly in 1 second. Write an equation for her current "point balance."

5. Complete the patterns.

| a. | b. | c. |
|---|---|--|
| $12 \div 4 = \underline{\hspace{2cm}}$ | $\underline{\hspace{2cm}} \div (-7) = -3$ | $60 \div \underline{\hspace{2cm}} = 2$ |
| $8 \div 4 = \underline{\hspace{2cm}}$ | $\underline{\hspace{2cm}} \div (-7) = -2$ | $40 \div \underline{\hspace{2cm}} = 2$ |
| $4 \div 4 = \underline{\hspace{2cm}}$ | $\underline{\hspace{2cm}} \div (-7) = -1$ | $20 \div \underline{\hspace{2cm}} = 2$ |
| $0 \div 4 = \underline{\hspace{2cm}}$ | $\underline{\hspace{2cm}} \div (-7) = 0$ | $-20 \div \underline{\hspace{2cm}} = 2$ |
| $(-4) \div 4 = \underline{\hspace{2cm}}$ | $\underline{\hspace{2cm}} \div (-7) = 1$ | $-40 \div \underline{\hspace{2cm}} = 2$ |
| $(-8) \div 4 = \underline{\hspace{2cm}}$ | $\underline{\hspace{2cm}} \div (-7) = 2$ | $-60 \div \underline{\hspace{2cm}} = 2$ |
| $(-12) \div 4 = \underline{\hspace{2cm}}$ | $\underline{\hspace{2cm}} \div (-7) = 3$ | $-80 \div \underline{\hspace{2cm}} = 2$ |
| $(-16) \div 4 = \underline{\hspace{2cm}}$ | $\underline{\hspace{2cm}} \div (-7) = 4$ | $-100 \div \underline{\hspace{2cm}} = 2$ |

6. Here's a funny riddle. Solve the math problems to uncover the answer.

E $\underline{\hspace{2cm}} \div (-8) = 2$

N $-12 \cdot (-5) = \underline{\hspace{2cm}}$

E $(-144) \div 12 = \underline{\hspace{2cm}}$

E $3 \cdot (-12) = \underline{\hspace{2cm}}$

H $\underline{\hspace{2cm}} \div 12 = -5$

T $-4 \cdot (-9) = \underline{\hspace{2cm}}$

N $-15 \div \underline{\hspace{2cm}} = -5$

E $\underline{\hspace{2cm}} \cdot (-6) = 0$

V $-45 \div \underline{\hspace{2cm}} = 5$

G $-1 \cdot (-9) = \underline{\hspace{2cm}}$

I $-27 \div 9 = \underline{\hspace{2cm}}$

I $-7 \cdot \underline{\hspace{2cm}} = -84$

S $-48 \div 6 = \underline{\hspace{2cm}}$

N $3 \cdot \underline{\hspace{2cm}} = -24$

Why is six afraid of seven? Because....

-8 -12 -9 -36 60

| | | | | |
|--|--|--|--|--|
| | | | | |
|--|--|--|--|--|

0 12 9 -60 36

| | | | | |
|--|--|--|--|--|
| | | | | |
|--|--|--|--|--|

3 -3 -8 -16

| | | | |
|--|--|--|--|
| | | | |
|--|--|--|--|

(This page intentionally left blank.)

Chapter 3: One-Step Equations

Introduction

The goal of this chapter is that students learn to solve one-step equations that involve integers.

The first lesson reviews the concept of an equation and how to model equations using a pan balance (scale). The basic principle for solving equations is that, when you perform the same operation on both sides of an equation, the two sides remain equal.

The chapter presents two alternatives for keeping track of the operations to be performed on an equation. The one method, writing the operation under each side of the equation, is common in the United States. The other method, writing the operation in the right margin, is common in Finland. Either is adequate, and the choice is just a matter of the personal preference of the teacher.

The introduction to solving equations is followed by a chapter on addition and subtraction equations and another on multiplication and division equations. All the equations are easily solved in only one step of calculations. The twofold goal is to make the student proficient in manipulating negative integers and also to lay a foundation for handling more involved equations in Chapter 5.

In the next lesson, students write equations to solve simple word problems. Even though they could solve most of these problems without using the equations, the purpose of the lesson is to make the student proficient in writing simple equations before moving on to more complex equations from more difficult word problems.

The last topic, in the lesson *Constant Speed*, is solving problems with distance (d), rate or velocity (v), and time (t). Students use the equivalent formulas $d = vt$ and $v = d/t$ to solve problems involving constant or average speed. They learn an easy way to remember the formula $v = d/t$ from the unit for speed that they already know, “miles per hour.”

The Lessons in Chapter 3

| | page | span |
|---|------|---------|
| Solving Equations | 82 | 7 pages |
| Addition and Subtraction Equations | 89 | 4 pages |
| Multiplication and Division Equations | 93 | 4 pages |
| Word Problems | 97 | 3 pages |
| Constant Speed | 100 | 7 pages |
| Chapter 3 Mixed Review | 107 | 3 pages |
| Chapter 3 Review | 110 | 2 pages |

Helpful Resources on the Internet

Model Algebra Equations

Model an equation on a balance using algebra tiles (tiles with numbers or the unknown x). Then, solve the equation according to instructions by placing -1 tiles on top of $+1$ tiles or vice versa. Includes one-step and two-step equations.

<http://www.mathplayground.com/AlgebraEquations.html>

One-Step Equation Game

Choose the correct root for the given equation (multiple-choice), and then you get to attempt to shoot a basket. The game can be played alone or with another student. The equations in the first game involve addition and subtraction, and in the second game (down the page) multiplication and division.

<http://www.math-play.com/One-Step-Equation-Game.html>

Exploring Equations E-Lab

Choose which operation to do to both sides of an equation in order to solve one-step multiplication and division equations.

<http://www.harcourtschool.com/activity/elab2004/gr6/12.html>

Battleship

An interesting game where the student must choose the right solution to a 1-step equation every time she hits an enemy ship.

<http://www.quia.com/ba/36544.html>

Algebra Meltdown

Solve simple equations using function machines to guide atoms through the reactor. But don't keep the scientists waiting too long or they blow their tops.

<http://www.mangahigh.com/en/games/algebrameltdown>

Practice Translating and Solving Equations

Translate verbal sentences into an equation and solve algebraically. Includes a self-check answer key.

<http://www.regentsprep.org/Regents/math/ALGEBRA/AE2/PTransSolvEq.htm>

Distance, Speed, and Time from BBC Bitesize

Instruction, worked out exercises, and an interactive quiz relating to constant speed, time, and distance. A triangle with letters D, S, and T helps students remember the formulas for distance, speed, and time.

http://www.bbc.co.uk/bitesize/standard/maths_i/numbers/dst/revision/1/

Speed problems from Slider Math

Click on the correct speed from three choices when a distance and time are given. Often, you need to convert units in your head in order to find the correct answer.

<http://www.slidermath.com/probs/Problem2.shtml>

Absorb Advanced Physics - Speed

An online tutorial that teaches the concept of average speed with the help of interactive simulations and exercises.

<http://www.absorblearning.com/advancedphysics/demo/units/010101.html#Describingmotion>

(This page intentionally left blank.)

Multiplication and Division Equations

| | |
|---|--|
| <p>Do you remember how to show simplification? Just cross out the numbers and write the new numerator above the fraction and the new denominator below it.</p> <p>Notice that the number you divide by (the 5 in the fraction at the right) isn't indicated in any way!</p> | $\frac{\cancel{35}^7}{\cancel{55}_{11}} = \frac{7}{11}$ |
| <p>We can simplify expressions involving variables in exactly the same way.</p> <p>In the examples on the right, we cross out the <i>same number</i> from the numerator and the denominator. That is based on the fact that a number divided by itself is 1. We could write a little "1" beside each number that is crossed out, but that is usually omitted.</p> | $\frac{\cancel{2}x}{\cancel{2}} = x \quad \frac{\cancel{5}s}{\cancel{5}} = s$ $\frac{\cancel{4}x}{\cancel{4}} = x$ |
| <p>In this example, we simplify the fraction $3/6$ into $1/2$ the usual way.</p> | $\frac{\cancel{3}x}{\cancel{6}_2} = \frac{1}{2}x \quad \text{or} \quad \frac{x}{2}$ |
| <p>Notice: We divide both the numerator and the denominator by 8, but <u>this leaves -1 in the denominator</u>. Therefore, the whole expression simplifies to $-z$ instead of z.</p> | $\frac{\cancel{8}z}{-\cancel{8}} = \frac{z}{-1} = -z$ |

1. Simplify.

| | | |
|---------------------|--------------------|--------------------|
| a. $\frac{8x}{8}$ | b. $\frac{8x}{2}$ | c. $\frac{2x}{8}$ |
| d. $\frac{-6x}{-6}$ | e. $\frac{-6x}{6}$ | f. $\frac{6x}{-6}$ |
| g. $\frac{6w}{2}$ | h. $\frac{6w}{w}$ | i. $\frac{6w}{-2}$ |

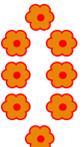
2. Draw the fourth and fifth steps of the pattern and answer the questions.



Step 1



Step 2



Step 3

a. How would you describe the growth of this pattern?

b. How many flowers will there be in step 39?

c. In step n ?

Now you should be ready to use multiplication and division to solve simple equations.

Example 1. Solve $-2x = 68$.

The unknown is being multiplied by -2 . To isolate it, we need to divide both sides by -2 . (See the solution on the right.)

We get $x = -34$. Lastly we check the solution by substituting -34 in the place of x in the original equation:

$$\begin{aligned} -2(-34) &\stackrel{?}{=} 68 \\ 68 &= 68 \quad \text{It checks.} \end{aligned}$$

$$-2x = 68 \quad \text{This is the original equation.}$$

$$\frac{-2x}{-2} = \frac{68}{-2} \quad \text{We divide both sides by } -2.$$

$$\frac{\cancel{-2}x}{\cancel{-2}} = \frac{68}{-2} \quad \text{Now it is time to simplify. We cross out the } -2 \text{ factors on the left side. On the right side, we do the division.}$$

$$x = -34 \quad \text{This is the final answer.}$$

Note: Most people combine the first 3 steps into one when writing the solution. Here they are written out for clarity.

3. Solve. Check your solutions.

| | |
|---------------|----------------|
| a. $5x = -45$ | b. $-3y = -21$ |
| c. $-4 = 4s$ | d. $72 = -6y$ |

4. Solve. Simplify the one side first.

| | |
|-------------------------|--------------------------|
| a. $-5q = -40 - 5$ | b. $2 \cdot 36 = -6y$ |
| c. $3x = -4 + 3 + (-2)$ | d. $5 \cdot (-4) = -10z$ |

| | |
|--|---|
| <p>Example 2. Solve $\frac{x}{-6} = -5$.</p> <p>Here the unknown is divided by -6. To undo that division, we need to <i>multiply</i> both sides by -6. (See the solution on the right.)</p> <p>We get $x = 30$. Lastly we check the solution:</p> $\frac{30}{-6} \stackrel{?}{=} -5$ $-5 = -5 \quad \checkmark$ | $\frac{x}{-6} = -5$ <p>This is the original equation.</p> $\frac{x}{-6} \cdot (-6) = -5 \cdot (-6)$ <p>We multiply both sides by -6.</p> $\frac{x}{\cancel{-6}} \cdot (\cancel{-6}) = 30$ $x = 30$ <p>Now it is time to simplify. We cross out the -6 factors on the left side, and multiply on the right.</p> <p>This is the final answer.</p> |
| <p>When writing the solution, most people would combine steps 2 and 3. Here both are written out for clarity.</p> | |

5. Solve. Check your solutions.

| | |
|---|--|
| <p>a. $\frac{x}{2} = -45$</p> | <p>b. $\frac{s}{-7} = -11$</p> |
| <p>c. $\frac{c}{-7} = 4$</p> | <p>d. $\frac{a}{-13} = -9 + (-11)$</p> |

6. Write an equation for each situation. Then solve it. Do not write the answer only, as the main purpose of this exercise is to practice writing equations.

- a.** A submarine was located at a depth of 500 ft. There was a shark swimming at $\frac{1}{6}$ of that depth. At what depth is the shark?

- b.** Three towns divided highway repair costs equally. Each town ended up paying \$21,200. How much did the repairs cost in total?

Example 3. Solve $-\frac{1}{5}x = 2$. Here the unknown is multiplied by a negative fraction, but do not panic!

You see, you can *also* write this equation as $\frac{x}{-5} = 2$, where the unknown is simply divided by negative 5.

So what should we do in order to isolate x ?

That is correct! Multiplying by -5 will isolate x . In the boxes below, this equation is solved in two slightly different ways, though both are doing essentially the same thing: multiplying both sides by -5 .

Multiplying a fraction by its reciprocal:

$$-\frac{1}{5}x = 2 \quad | \cdot (-5)$$

$$(-5) \cdot \left(-\frac{1}{5}\right)x = (-5) \cdot 2 \quad \text{Note that } -5 \text{ times } -1/5 \text{ is } 1.$$

$$1x = -10$$

$$x = -10$$

Canceling a common factor:

$$-\frac{1}{5}x = 2 \quad \text{rewrite the equation}$$

$$\frac{x}{-5} = 2 \quad | \cdot (-5)$$

$$\frac{\cancel{x}}{\cancel{-5}} \cdot (\cancel{-5}) = 2 \cdot (-5)$$

$$x = -10$$

Lastly we check the solution by substituting -10 in place of x in the original equation:

$$-\frac{1}{5}(-10) \stackrel{?}{=} 2$$

$$2 = 2 \quad \text{It checks.}$$

7. Solve. Check your solutions.

a. $\frac{1}{3}x = -15$

b. $-\frac{1}{6}x = -20$

c. $-\frac{1}{4}x = 18$

d. $-2 = -\frac{1}{9}x$

e. $-21 = \frac{1}{8}x$

f. $\frac{1}{12}x = -7 + 5$

(This page intentionally left blank.)

Chapter 4: Rational Numbers

Introduction

In this chapter we study *rational* numbers, which are numbers that can be written as a *ratio* of two integers. All fractions and whole numbers are rational numbers, and so are percents and decimals (except non-ending non-repeating decimals). Obviously, students already know a lot about rational numbers and how to calculate with them. Our focus in this chapter is to extend that knowledge to negative fractions and negative decimals.

The first lesson presents the definition of a rational number, how to convert rational numbers back and forth between their fractional and decimal forms, and a bit about repeating decimals (most fractions become repeating decimals when written as decimals). The next lesson deals with adding and subtracting rational numbers, with an emphasis on adding and subtracting negative fractions and decimals.

The next two lessons are about multiplying and dividing rational numbers. The first of the two focuses on basic multiplication and division with negative fractions and decimals. The second of the two compares multiplying and dividing in decimal notation to multiplying and dividing in fraction notation. Students come to realize that, though the calculations — and even the answers — may look very different, the answers are equal. The lesson also presents problems that mix decimals, fractions, and percents, and deals with real-life contexts for the problems and the importance of pre-estimating what a reasonable answer would be.

The lesson *Multiple Operations with Rational Numbers* reviews the order of operations and applies it to fraction and decimal problems with more than one operation. It also presents a simple method to solve complex fractions, which are fractions that contain another fraction, either in the numerator, in the denominator, or in both.

After a lesson on scientific notation, the instructional portion of the chapter concludes with two lessons on solving simple equations that involve fractions and decimals.

The Lessons in Chapter 4

| | page | span |
|---|------|---------|
| Rational Numbers | 116 | 8 pages |
| Adding and Subtracting Rational Numbers | 124 | 6 pages |
| Multiply and Divide Rational Numbers 1 | 130 | 4 pages |
| Multiply and Divide Rational Numbers 2 | 134 | 7 pages |
| Many Operations with Rational Numbers | 141 | 4 pages |
| Scientific Notation | 145 | 3 pages |
| Equations with Fractions | 148 | 5 pages |
| Equations with Decimals | 153 | 3 pages |
| Chapter 4 Mixed Review | 156 | 2 pages |
| Chapter 4 Review | 158 | 5 pages |

Helpful Resources on the Internet

Rational numbers

Practice with Irrational and Rational Numbers

Two pages of exercises where you need to tell whether numbers are rational or irrational. Includes a self-check answer key.

<http://www.regentsprep.org/Regents/math/ALGEBRA/AOP1/PRatNos.htm>

<http://www.regentsprep.org/Regents/math/ALGEBRA/AOP1/Prat.htm>

Rational and Irrational Numbers Game

Drag each number into the correct bin to classify them as rational or irrational. Fast-paced.

<http://www.math-play.com/rational-and-irrational-numbers-game/rational-and-irrational-numbers-game.html>

Classifying Numbers

Drag the given numbers to the correct sets. This chapter of Math Mammoth does not teach about square roots and irrational numbers but you can probably do these activities, if you note that most square roots are irrational, and that the set of whole numbers is $\{0, 1, 2, 3, 4, \dots\}$.

http://www.softschools.com/math/classifying_numbers/

http://www.softschools.com/math/classifying_numbers/real_rational_integer_whole_natural_irrational_number_table/

Number System Muncher

“Munch” or select all the numbers from the grid that are in the specified set. Again, this chapter of Math Mammoth does not teach about square roots and irrational numbers but you can probably play the game, if you note the following: Taking a square root is the opposite operation of squaring. For example, $\sqrt{25} = 5$ because $5^2 = 25$. Therefore, $\sqrt{25}$ is actually a natural number (5). However, most square roots, such as $\sqrt{5}$ and $\sqrt{13}$ are irrational.

<http://staff.argyll.epsb.ca/jreed/math9/strand1/munchers.htm>

Recurring Decimals and Fractions

Two games (Grade or No Grade and Fling the Teacher) where you answer multiple-choice questions about repeating decimals.

<https://sites.google.com/a/revisemaths.org.uk/revise/number-files/recdefrac-gong.swf?attredirects=0>

<https://sites.google.com/a/revisemaths.org.uk/revise/number-files/recdefrac-fling.swf?attredirects=0>

Terminating and Repeating Decimal Numbers Practice

Tell whether whether the quotient of the following problems are terminating decimal numbers or repeating decimal numbers.

<http://www.studyzone.org/mtestprep/math8/e/reptermdecimals6p.cfm>

Terminating vs. Repeating Decimals Game

A card game that practices repeating and terminating decimals. Students create fractions from their cards and then turn them into decimals to see if they are terminating or repeating. Several fun twists to score extra points! This game costs \$1 (per download).

<http://www.teacherspayteachers.com/Product/Terminating-VS-Repeating-Decimals-Game-425199>

Terminating and Repeating Decimals Worksheet

A 10-question online quiz about repeating decimals.

<http://worksheets.tutorvista.com/terminating-and-repeating-decimals-worksheet.html>

Converting Repeating Decimals to Fractions

A lesson that explains the method for writing repeating decimals as fractions.

<http://www.basic-mathematics.com/converting-repeating-decimals-to-fractions.html>

The four operations with rational numbers

Power Football

Practice the four operations with decimals with a football game. Choose “all of the above” (all operations), level “medium” or “hard,” and “algebra style” to practice the concepts studied in this chapter.

<http://www.funbrain.com/football/>

Adding and Subtracting Rational Numbers Test

A 15-question test with mostly multiple-choice questions about adding, subtracting, and comparing rational numbers.

<http://teachers.henrico.k12.va.us/math/hcpsalgebra1/Documents/examviewweb/ev2-2.htm>

Adding and Subtracting Rational Numbers Worksheets

Generate a worksheet for adding and subtracting negative fractions and decimals.

http://www.math-aids.com/Algebra/Algebra_1/Basics/Add_Sub_Rational.html

Add and Subtract Fractions Quiz

A multiple-choice quiz of five questions about adding and subtracting negative fractions and solving simple equations. Refresh the page to get a different set of questions.

<http://www.phschool.com/webcodes10/index.cfm?fuseaction=home.gotoWebCode&wcprefix=asa&wcsuffix=0204>

Multiply and Divide Fractions Quizzes

A multiple-choice quiz of five questions. Refresh the page to get a different set of questions.

<http://www.phschool.com/webcodes10/index.cfm?fuseaction=home.gotoWebCode&wcprefix=asa&wcsuffix=0205>

<http://www.glencoe.com/sec/math/studytools/cgi-bin/msgQuiz.php4?isbn=0-07-829635-8&chapter=2&lesson=4>

Multiply and Divide Rational Numbers Quiz

A multiple-choice quiz of five questions.

http://www.softschools.com/quizzes/math/multiply_rational_numbers/quiz3285.html

Scientific notation

Scientific Notation

Interactive practice where you write the given number in scientific notation.

<http://www.xpmath.com/forums/arcade.php?do=play&gameid=21>

Scientific Notation Quiz

Write numbers in scientific notation, and vice versa. You can modify the quiz parameters to your liking, such as changing the difficulty level or the duration of the quiz.

<http://www.thatquiz.org/tq-c/?-j820-l6-p0>

Scientific Notation Quizzes

Short, multiple-choice quizzes on scientific notation.

<http://www.glencoe.com/sec/math/studytools/cgi-bin/msgQuiz.php4?isbn=0-02-833051-X&chapter=2&lesson=9>

<http://www.studyzone.org/mttestprep/math8/g/scientificnotationquiz.cfm>

General

Equations Quiz

A five-question quiz on solving simple one-step equations that involve decimals. Refresh the page to get a different set of questions.

<http://www.phschool.com/webcodes10/index.cfm?fuseaction=home.gotoWebCode&wcprefix=ara&wcsuffix=0404>

Fraction Four

Choose “algebra” as the question type to solve equations that involve fractions in this connect-the-four game.

<http://www.shodor.org/interactivate/activities/FractionFour/>

Equations with Fractions Quiz

<http://www.phschool.com/webcodes10/index.cfm?fuseaction=home.gotoWebCode&wcprefix=bj&wcsuffix=0508>

7th Grade Numbers and Operations Jeopardy

A jeopardy game with questions about absolute value, ordering rational numbers, adding and subtracting rational numbers, and multiplying & dividing rational numbers.

<http://www.math-play.com/7th-Grade-Numbers-and-Operations-Jeopardy/7th-Grade-Numbers-and-Operations-Jeopardy.html>

(This page intentionally left blank.)

Multiply and Divide Rational Numbers 1

In real life we often combine **fractions, decimals, ratios, and percents** — rational numbers in different forms — in the same situation. You need to be able to easily calculate with them in their different forms.

In this lesson, we will concentrate on multiplying and dividing *decimals* and *fractions* because percentages are usually rewritten as decimals and ratios as fractions before calculating with them.

To multiply decimals

Shortcut: First multiply as if there were no decimal points. Then put the decimal point in the answer so that the number of decimal digits in the answer is the SUM of the number of the decimal digits in all the factors.

Example 1. Solve $-0.2 \cdot 0.09$.

Multiply $2 \cdot 9 = 18$. The answer will have three decimals *and* be negative (Why?), so the answer is -0.018 .

Multiply fractions and mixed numbers

1. Change any mixed numbers to fractions.
2. Multiply using the shortcut (multiply the numerators; multiply the denominators).

Example 2. $-\frac{4}{5} \cdot \left(-5\frac{1}{8}\right)$

$$= -\frac{4}{5} \cdot \left(-\frac{41}{8}\right)$$

A negative times a negative makes a positive, so we can drop the minus signs in the next step.

$$= \frac{4 \cdot 41}{5 \cdot 8} = \frac{1 \cdot 41}{5 \cdot 2} = \frac{41}{10} = 4\frac{1}{10}$$

1. Write the rational numbers in their four forms.

| ratio | fraction | decimal | percent | ratio | fraction | decimal | percent |
|---------|-----------------|---------|---------|-------|------------------|---------|---------|
| a. 2:5 | = $\frac{2}{5}$ | = 0.4 | = 40% | d. | = $\frac{7}{20}$ | = | = |
| b. 3:4 | = | = | = | e. | = | = | = 55% |
| c. 4:25 | = | = | = | f. | = | = 0.85 | = |

2. Multiply these in your head.

| | | |
|------------------------|-----------------------------------|-----------------------|
| a. $0.1 \cdot 6.5$ | b. $-0.08 \cdot 0.006$ | c. $-0.09 \cdot 0.02$ |
| d. $-0.2 \cdot (-1.6)$ | e. $-0.8 \cdot 1.1 \cdot (-0.02)$ | f. 0.8^2 |
| g. $(-0.5)^2$ | h. $(-0.2)^3$ | i. $(-0.1)^5$ |

3. Multiply

| | | |
|--|--|---|
| a. $-\frac{1}{7} \cdot \left(-\frac{3}{8}\right)$ | b. $\frac{1}{5} \cdot \left(-2\frac{1}{2}\right)$ | c. $-\frac{2}{9} \cdot \frac{5}{6} \cdot \frac{3}{10}$ |
| d. $-3\frac{1}{4} \cdot \frac{5}{2}$ | e. $\frac{7}{18} \cdot \left(-\frac{12}{27}\right)$ | f. $\frac{8}{7} \cdot \left(-\frac{3}{10}\right) \cdot 1\frac{1}{2}$ |

4. Multiply using the regular multiplication algorithm (write one number under the other).

| | |
|----------------------------------|--------------------------------|
| a. $12.5 \cdot 2.5$ | b. $-0.088 \cdot 0.16$ |
| c. $-9.08 \cdot (-0.006)$ | d. $24 \cdot (-0.0087)$ |

To divide decimals

1. If the divisor has no decimal digits, you can divide using long division “as is.”
2. If the divisor does have decimal digits, multiply *both* the dividend and the divisor by the same number (usually a power of ten) to make the divisor into a whole number. Now with that whole number divisor, performing the long division has become straightforward.

Example 3. Solve $6 \div 0.5$ without a calculator.

Since 0.5 fits into 6 exactly twelve times, the answer is 12. So mental math was sufficient in this case.

Example 4. Solve $-92.91 \div 0.004$ without a calculator.

It may be easier to write the problem using a fraction line:

$$\frac{-92.91}{0.004} = \frac{-929.1}{0.04} = \frac{-9291}{0.4} = \frac{-92910}{4}$$

Notice how we multiply both the dividend and the divisor repeatedly by 10 until the divisor becomes a whole number (4). (You could, of course, simply multiply them both by 1,000 to start with.) Then we use long division

The long division gives us the absolute value of the final answer, but we still need to apply the correct sign. So $-92.91 \div 0.004 = -23,227.5$.

Does this make sense? Yes. The answer has a very large absolute value because 0.004 is a very tiny number, and thus it “fits” into 92.91 multitudes of times.

$$\begin{array}{r} 23227.5 \\ 4 \overline{) 92910.0} \\ \underline{-8} \\ 12 \\ \underline{-12} \\ 09 \\ \underline{-8} \\ 11 \\ \underline{-8} \\ 30 \\ \underline{-28} \\ 20 \\ \underline{-20} \\ 0 \end{array}$$

5. Divide using mental math.

| | | |
|-----------------------|-----------------|-----------------------|
| a. $-0.88 \div 4$ | b. $8.1 \div 9$ | c. $72 \div 10000$ |
| d. $-1.6 \div (-0.2)$ | e. $8 \div 0.1$ | f. $0.8 \div (-0.04)$ |

6. Multiply both the dividend and the divisor by the same number so that you get a divisor that is a *whole number*. Then divide using long division. If necessary, round your answer to three decimal digits.

| | |
|--------------------|----------------------|
| a. $27.6 \div 0.3$ | b. $2.088 \div 0.06$ |
|--------------------|----------------------|

To divide fractions and mixed numbers

1. Change any mixed numbers to fractions.
2. Divide using the shortcut. (Change the division into a multiplication by the reciprocal of the divisor.)

Example 5. $\frac{4}{5} \div \left(-2\frac{1}{2}\right)$

$$= \frac{4}{5} \div \left(-\frac{5}{2}\right)$$

$$= \frac{4}{5} \cdot \left(-\frac{2}{5}\right) = -\frac{8}{25}$$

The answer makes sense, because $2\frac{1}{2}$ does not fit into $\frac{4}{5}$, not even half-way.

7. Divide.

a. $-\frac{2}{9} \div \frac{6}{7}$

b. $\frac{9}{8} \div \left(-1\frac{1}{2}\right)$

c. $-10 \div \frac{5}{6}$

d. $-\frac{1}{9} \div \left(-\frac{1}{3}\right)$

e. $10\frac{1}{5} \div \left(-2\frac{1}{3}\right)$

f. $10 \div \frac{1}{6}$

(This page intentionally left blank.)

Chapter 5: Equations and Inequalities

Introduction

In this chapter we delve deeper into our study of equations. Now the equations require two or more steps to solve and may contain parentheses. The variable may appear on both sides of the equation. Students will also write equations to solve simple word problems.

There is also another lesson on patterns of growth, which may seem to be simply a fascinating topic, but in reality presents the fundamentals of a very important concept in algebra — that of linear functions (although they are not mentioned by that name) — and complements the study of lines in the subsequent lessons.

After the section about equations, the text briefly presents the basics of inequalities and how to graph them on a number line. Students apply the principles for solving equations to solve simple inequalities and word problems that involve inequalities.

The last major topic is graphing. Students begin the section by learning to graph linear equations and continue on to the concept of slope, which in informal terms is a measure of the inclination of a line. More formally, slope can be defined as the ratio of the change in y -values to the change in x -values. The final lesson applies graphing to the previously-studied concepts of speed, time, and distance through graphs of the equation $d = vt$ in the coordinate plane.

The Lessons in Chapter 5

| | page | span |
|---------------------------------------|------|---------|
| Two-Step Equations | 167 | 5 pages |
| Two-Step Equations: Practice | 172 | 4 pages |
| Growing Patterns 2 | 176 | 4 pages |
| A Variable on Both Sides | 180 | 6 pages |
| Some Problem Solving | 186 | 3 pages |
| Using the Distributive Property | 189 | 6 pages |
| Word Problems | 195 | 3 pages |
| Inequalities | 198 | 5 pages |
| Word Problems and Inequalities | 203 | 2 pages |
| Graphing | 205 | 4 pages |
| An Introduction to Slope | 209 | 5 pages |
| Speed, Time and Distance | 214 | 5 pages |
| Chapter 5 Mixed Review | 219 | 3 pages |
| Chapter 5 Review | 222 | 6 pages |

Helpful Resources on the Internet

Simplifying Expressions

Factor the Expressions Quiz

Factor expressions. For example, $-4x + 16$ factors into $-4(x - 4)$.

<http://www.thatquiz.org/tq-0/?-jh00-l4-p0>

Simplifying Algebraic Expressions Practice Problems

Practice simplifying expressions such as $4(2p - 1) - (p + 5)$ with these 10 questions. Answer key included.

<http://www.algebra-class.com/algebraic-expressions.html>

Simplifying Algebraic Expressions (1)

Eight practice problems that you can check yourself about combining like terms and using the distributive property.

http://www.algebra-lab.org/lessons/lesson.aspx?file=Algebra_BasicOpsSimplifying.xml

Simplifying Algebraic Expressions (2)

An activity in which you match the expressions to their simplified forms.

http://www.media.pearson.com.au/schools/cw/au_sch_bull_md7_1/dnd/11_sim_a.html

Simplifying Algebraic Expressions (3)

You can check this five-question quiz from Glencoe yourself.

<http://www.glencoe.com/sec/math/studytools/cgi-bin/msgQuiz.php4?isbn=0-07-825200-8&chapter=3&lesson=2&&headerFile=4>

Equations

Balance when Adding and Subtracting Game

The interactive balance illustrates simple equations. Your task is to add or subtract x 's and 1's until you leave x alone on one side.

<http://www.mathsisfun.com/algebra/add-subtract-balance.html>

Algebra Balance Scales

Model the given equation on a balance. Then choose an operation to perform on both sides of the equation, and the computer will show the result. Continue until the equation is solved.

http://nlvm.usu.edu/en/nav/frames_asid_324_g_4_t_2.html

Equation Buster

Choose an operation to perform on both sides of the given equation, and the computer will show the result.

Continue until the equation is solved. The equations involve a variable on both sides, occasionally with a fractional coefficient (such as $y/2$).

http://mathsnet.net/l4_equation.html

Equation Match

A matching game with a hidden picture. Click on the two equations that have the same root. Choose level 2 to practice 7th grade concepts.

<http://www.bbc.co.uk/schools/mathsfle/shockwave/games/equationmatch.html>

Solve Equations Quiz

A 10-question online quiz where you need to solve equations with an unknown on both sides.

<http://www.thatquiz.org/tq-0/?-j102-l4-p0>

Algebra Four

This is a connect-the-four game from Shodor. To practice the types of equations we study in this chapter, choose "Level 1," and tick the boxes "Variable on both sides," "Distributive Property," and "Two-Step Problems" (don't check "Quadratic Equations").

<http://www.shodor.org/interactivate/activities/AlgebraFour/>

Two-Step Equations Game

Choose the correct root for the given equation (multiple-choice), and then you get to attempt to shoot a basket. The game can be played alone or with another student.

<http://www.math-play.com/Two-Step-Equations-Game.html>

Two-Step Equations

Here's another five-question quiz from Glencoe that you can check yourself.

<http://www.glencoe.com/sec/math/studytools/cgi-bin/msgQuiz.php4?isbn=0-07-825200-8&chapter=3&lesson=5&&headerFile=4>

Solving Two-Step Equations

Type the answer to two-step-equations such as $-4y + 9 = 29$, and the computer checks it. If you choose "Practice Mode," it is not timed.

<http://www.xpmath.com/forums/arcade.php?do=play&gameid=64>

Rags to Riches Equations

Choose the correct root to a linear equation.

<http://www.quia.com/rr/4096.html>

Inequalities

Inequality Quiz

A 10-question multiple choice quiz on linear inequalities (like the ones studied in this chapter).

<http://www.mrmaisonet.com/index.php?/Inequality-Quiz/Inequality-Quiz.html>

Inequalities

Here's another five-question quiz from Glencoe that you can check yourself.

<http://www.glencoe.com/sec/math/studytools/cgi-bin/msgQuiz.php4?isbn=0-07-825200-8&chapter=7&lesson=3&&headerFile=4>

Speed, Time, and Distance

Solving Distance Problems Practice

Five word problems about constant speed with solutions.

<http://www.studyzone.org/mtestprep/math8/g/distancep1.cfm>

Understanding Distance, Speed, and Time

An interactive simulation of two runners. You set their starting points and their speeds and observe their positions as the tool runs the simulation. It graphs the position of both runners in relation to time.

<http://www.nctm.org/standards/content.aspx?id=33191>

"Representing Motion" from GCSE BiteSize

An interactive quiz with various questions about speed, time, and distance.

http://www.bbc.co.uk/schools/gcsebitesize/science/add_aqa_pre_2011/forces/represmotionrev1.shtml

Speed - a lesson from Absorb

A comprehensive tutorial with interactive simulations and questions to check your understanding.

<http://www.absorblearning.com/advancedphysics/demo/units/010101.html#Describingmotion>

Graphing and Slope

Graph Linear Equations

A ten-question online quiz where you click on three points on the coordinate grid to graph the given equation.

<http://www.thatquiz.org/tq-0/?-j10g-l4-p0>

Find the Slope

A ten-question online quiz that asks for the slope of the given line.

<http://www.thatquiz.org/tq-0/?-j300-l4-p0>

Slope Slider

Use the sliders to change the slope and the y -intercept of a linear equation to see what effect they have on the graph of the line.

<http://www.shodor.org/interactivate/activities/SlopeSlider/>

Line Plotter

Practice drawing lines through a given point with a specified slope.

http://nlvm.usu.edu/en/nav/frames_asid_332_g_3_t_2.html

Graphing Equations Match

Match the given equations to their corresponding graphs.

<http://www.math.com/school/subject2/practice/S2U4L3/S2U4L3Pract.html>

General**Algebra Quizzes**

A variety of online algebra quizzes from MrMaisonet.com.

<http://www.mrmaisonet.com/index.php?/Algebra-Quizzes/>

Pre-algebra Quizzes

Pearson provides a variety of online algebra quizzes to support their *Algebra Readiness* textbook.

<http://www.phschool.com/webcodes10/index.cfm?fuseaction=home.gotoWebCode&wcprefix=bjk&wcsuffix=0099>

(This page intentionally left blank.)

A Variable on Both Sides

Example 1. Solve $2x + 8 = -5x$.

Notice that the unknown appears on both sides of the equation. To isolate it, we need to

- either subtract $2x$ from both sides—because that makes $2x$ disappear from the left side
- or add $5x$ to both sides—because that makes $-5x$ disappear from the right side.

$$\begin{array}{r|l}
 2x + 8 = -5x & + 5x \\
 2x + 8 + 5x = 0 & \text{(now add } 2x \text{ and } 5x \text{ on the left side)} \\
 7x + 8 = 0 & - 8 \\
 7x = -8 & \div 7 \\
 x = -8/7 &
 \end{array}$$

Check:

$$\begin{array}{l}
 2 \cdot (-8/7) + 8 \stackrel{?}{=} -5 \cdot (-8/7) \\
 -16/7 + 8 \stackrel{?}{=} 40/7 \\
 -2 \frac{2}{7} + 8 = 5 \frac{5}{7} \quad \checkmark
 \end{array}$$

Example 2. Solve $10 - 2s = 4s + 9$.

To isolate s , we need to

- either add $2s$ to both sides
- or subtract $4s$ from both sides.

The choice is yours. Personally, I like to keep the unknown on the left side and eliminate it from the right.

$$\begin{array}{r|l}
 10 - 2s = 4s + 9 & - 4s \\
 10 - 2s - 4s = 9 & \text{(now simplify } -2s - 4s \text{ on the left side)} \\
 10 - 6s = 9 & - 10 \\
 -6s = -1 & \div (-6) \\
 s = 1/6 &
 \end{array}$$

Check:

$$\begin{array}{l}
 10 - 2 \cdot (1/6) \stackrel{?}{=} 4 \cdot (1/6) + 9 \\
 10 - 2/6 \stackrel{?}{=} 4/6 + 9 \\
 9 \frac{4}{6} = 9 \frac{4}{6} \quad \checkmark
 \end{array}$$

1. Solve. Check your solutions (as always!).

a. $3x + 2 = 2x - 7$

b. $9y - 2 = 7y + 5$

2. Solve. Check your solutions (as always!).

a. $11 - 2q = 7 - 5q$

b. $6z - 5 = 9 - 2z$

c. $8x - 12 = -1 - 3x$

d. $-2y - 6 = 20 + 6y$

e. $6w - 6.5 = 2w - 1$

f. $5g - 5 = -20 - 2g$

Combining like terms

Remember, in algebra, a *term* is an expression that consists of numbers, fractions, and/or variables that are multiplied. This means that the expression $-2y + 7 + 8y$ has three terms, separated by the plus signs.

In the expression $-2y + 7 + 8y$, the terms $-2y$ and $8y$ are called **like terms** because they have the same variable part (in this case a single y). We can **combine** (add or subtract) like terms.

To do that, it helps to organize the terms in the expression in alphabetical order according to the variable part and write the constant terms last. We get $-2y + 8y + 7$ ($8y - 2y + 7$ is correct, too).

Next, we add $-2y + 8y$ and get $6y$. So the expression $-2y + 7 + 8y$ simplifies to $6y + 7$.

Example 3. Simplify $6y - 8 - 9y + 2 - 7y$.

First, we organize the expression so that the terms with y are written first, followed by the constant terms.

For that purpose, we **view each operation symbol** (+ or -) **in front of the term as the sign of each term**.

In a sense, you can imagine each plus or minus symbol as being “glued” to the term that follows it. Of course the first term, $6y$, gets a “+” sign.

$$\textcircled{+6y} \textcircled{-8} \textcircled{-9y} \textcircled{+2} \textcircled{-7y}$$

After reordering the terms, the expression becomes $6y - 9y - 7y - 8 + 2$.

Now we need to combine the like terms $6y$, $-9y$, and $-7y$. We do that by finding the sum of their coefficients 6, -9 , and -7 . Since $6 - 9 - 7 = -10$, we know that $6y - 9y - 7y = -10y$.

Similarly, we combine the two constant terms: $-8 + 2 = -6$.

Our expression therefore simplifies to $-10y - 6$.

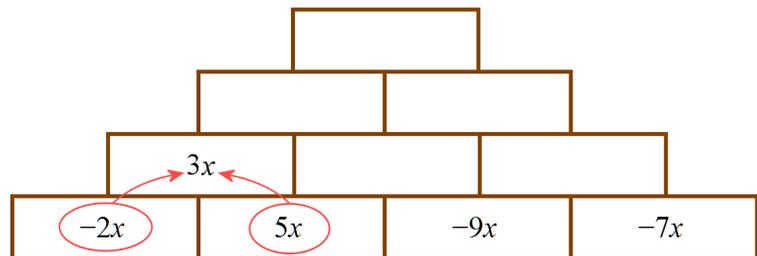
Why can we do it this way?

Because subtracting a term is the same as adding its opposite. In symbols,

$$\begin{aligned} & 6y \quad -8 \quad -9y + 2 \quad -7y \\ = & 6y + (-8) + (-9y) + 2 + (-7y). \end{aligned}$$

In other words, the expression $6y - 8 - 9y + 2 - 7y$ is the SUM of the terms $6y$, -8 , $-9y$, 2 , and $-7y$.

3. Fill in the pyramid! Add each pair of terms in neighboring blocks and write its sum in the block above it.



4. Organize the expressions so that the variable terms are written first, followed by constant terms.

a. $6 + 2x - 3x - 7 + 11$

b. $-s - 12 + 15s + 9 - 7s$

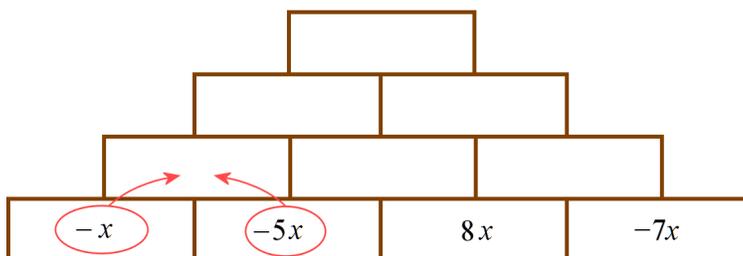
c. $-8 + 5t - 2 - 6t$

5. Simplify the expressions in the previous exercise.

6. Simplify.

| | |
|--|---|
| <p>a. $5x - 8 - 7x + 1$</p> | <p>b. $-6a - 15a + 9a + 7a$</p> |
| <p>c. $-8 + 7c - 11c + 8 - c$</p> | <p>d. $10 - 5x - 8x - 9 + x$</p> |

7. Fill in the pyramid! Add each pair of terms in neighboring blocks and write its sum in the block above it.



8. Find what is missing from the sums.

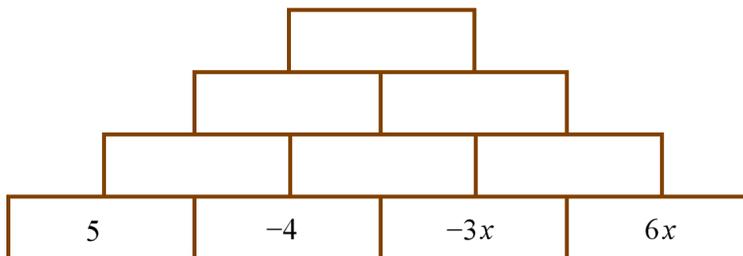
a. $8x + 2 + \underline{\hspace{2cm}} = 5x + 8$

b. $5b - 2 + \underline{\hspace{2cm}} = 2b + 7$

c. $-2z + \underline{\hspace{2cm}} = 1 - 5z$

d. $-4f + 3 + \underline{\hspace{2cm}} = -f - 1$

9. Fill in the pyramid! Add each pair of terms in neighboring blocks and write its sum in the block above it.



10. Simplify.

| | | |
|---|--------------------------------------|---|
| <p>a. $0.5y + 1.2y - 0.6y$</p> | <p>b. $-1.6v - 1 - v$</p> | <p>c. $-0.8k + 3 + 0.9k$</p> |
|---|--------------------------------------|---|

11. A challenge! Solve the equation $(-1/2)x - 6 + 8x + 7 - x = 0$.

Example 4. One or both sides of an equation may have several terms with the unknown. In that case, we need to combine the like terms (simplify) before continuing with the actual solution.

$$3x + 7 - 5x = 6x + 1 - 5x$$

$$-2x + 7 = x + 1$$

$$-3x + 7 = 1$$

$$-3x = -6$$

$$3x = 6$$

$$x = 2$$

On the left side, combine $3x$ and $-5x$.
On the right side, combine $6x$ and $-5x$.

$$-x$$

$$-7$$

$$\cdot (-1)$$

$$\div 3$$

12. Solve. Check your solutions.

a. $6x + 3x + 1 = 9x - 2x - 7$

b. $16y - 4y - 3 = -4y - y$

c. $-26x + 12x = -18x + 8x - 6$

d. $-9h + 4h + 7 = -2 + 5h + 9h + 8h$

13. Solve. Check your solutions.

| | |
|---|--|
| <p>a. $2x - 4 - 7x = -8x + 5 + 2x$</p> | <p>b. $-6 - 4z - 3z = 5z + 8 - z$</p> |
| <p>c. $8 - 2m + 5m - 8m = 20 - m + 5m - 2m$</p> | <p>d. $-x - x + 2x = 5 - 5x + 9x$</p> |
| <p>e. $-q + 2q - 5q - 6q = 20 - 7 - 9 + q$</p> | <p>f. $9 - s + 7 - 9s = 2 - 2s - 11$</p> |