Sample Lessons

# Developing Fractions Sense™

## Student Workbook B

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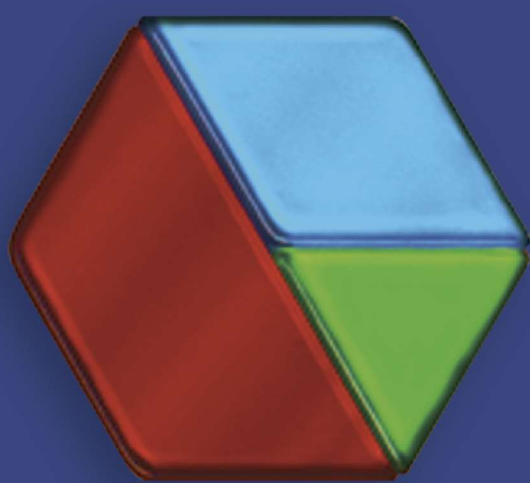


Student Name: ______________________________

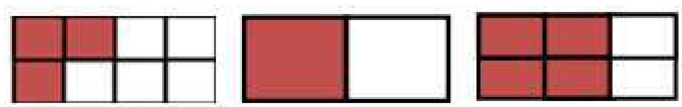
Teacher Name: ______________________________

Grade: ______________ Year: ____________________

# Lesson 11

## Comparing Fractions Smaller and Larger than $\frac{1}{2}$

Let's compare the fractions $\frac{3}{8}$ and $\frac{4}{6}$ to the benchmark of $\frac{1}{2}$. We know that $\frac{3}{8} < \frac{1}{2}$ since the numerator of $\frac{3}{8}$ is less than half its denominator. We know that $\frac{4}{6} > \frac{1}{2}$ since the numerator of $\frac{4}{6}$ is greater than half of its denominator. Therefore, since $\frac{3}{8}$ is less than $\frac{1}{2}$, while $\frac{4}{6}$ is more than $\frac{1}{2}$, it follows that $\frac{3}{8} < \frac{4}{6}$ , as shown on the diagram below.



$\frac{3}{8} < \frac{1}{2}$ $\frac{1}{2}$ $\frac{4}{6} > \frac{1}{2}$

In general, we can compare two fractions by seeing if one is less than $\frac{1}{2}$ and the other is more than $\frac{1}{2}$. We will then know that the fraction less than $\frac{1}{2}$ is the smaller fraction.

A. In each example below, insert the correct symbol: "<", "=" or ">".

a. $\frac{3}{8} \bigcirc \frac{6}{10}$ b. $\frac{4}{6} \bigcirc \frac{3}{8}$ c. $\frac{3}{4} \bigcirc \frac{2}{6}$ d. $\frac{2}{5} \bigcirc \frac{4}{7}$

B. Review Lesson #6: Please insert "=" or "≠" to make each statement true.

a. $\frac{6}{5} \bigcirc \frac{2}{5} + \frac{3}{5}$ b. $\frac{5}{2} + \frac{2}{2} \bigcirc \frac{3}{2} + \frac{2}{2} + \frac{2}{2}$ c. $\frac{2}{10} + \frac{3}{10} \bigcirc \frac{4}{10} + \frac{1}{10}$

C. Review Lesson #8: In each example below, insert the correct symbol: "<", "=" or ">". Example: $\frac{5}{6} < \frac{8}{7}$, since $\frac{5}{6} < 1$ and $\frac{8}{7} > 1$.

a. $\frac{7}{8} \bigcirc \frac{12}{10}$ b. $\frac{4}{6} \bigcirc \frac{7}{5}$ c. $\frac{4}{3} \bigcirc \frac{3}{4}$ d. $\frac{2}{5} \bigcirc \frac{5}{2}$

D. Review Lesson #9. Please enter the missing number:

a. $\frac{1}{2} = \frac{\quad}{12}$ b. $\frac{1}{2} = \frac{\quad}{14}$ c. $\frac{1}{2} = \frac{\quad}{24}$ d. $\frac{1}{2} = \frac{\quad}{120}$

**What we have learned:**

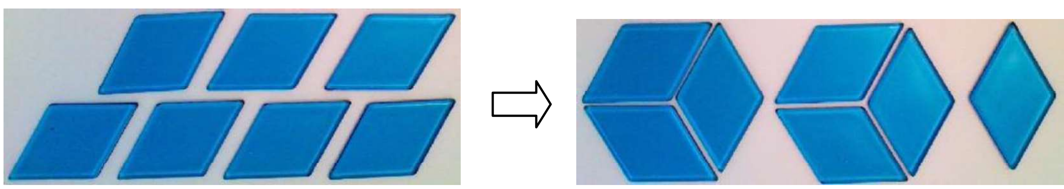
We can compare two fractions to the benchmark of $\frac{1}{2}$ by seeing if one fraction is larger than $\frac{1}{2}$ and the other is smaller than $\frac{1}{2}$. Example: $\frac{3}{5} > \frac{4}{10}$ since $\frac{3}{5} > \frac{1}{2}$ and $\frac{4}{10} < \frac{1}{2}$.

## Lesson 18

### Converting Improper Fractions to Mixed Fractions: Using Fraction Blocks

**Fractions such as $\frac{1}{2}$ and $\frac{5}{6}$ are called *proper fractions*. In these fractions, the numerator is smaller than the denominator and the value of the fraction is less than 1. On the other hand, fractions such as $\frac{3}{2}$ and $\frac{7}{6}$ are called *improper fractions*. In these fractions, the numerator is larger than the denominator and the value of the fraction is more than 1.**

**Our goal is to express $\frac{7}{3}$ as a *mixed fraction*, that is, as a whole number and a proper fraction. If we consider the yellow block as the whole, we can try to see how many wholes we can make from the 7 blue blocks at the left below. We recall that one whole is the same size as three blue blocks. Hence, seven blue blocks can be arranged as shown on the right below. We see therefore that $\frac{7}{3} = 2\frac{1}{3}$.**



**A. Use your fraction blocks to express each improper fraction as a mixed fraction:**

| | | | |
|---|---|---|---|
| **a.** $\frac{3}{2} =$ | **b.** $\frac{5}{2} =$ | **c.** $\frac{4}{3} =$ | **d.** $\frac{5}{3} =$ |
| **e.** $\frac{7}{3} =$ | **f.** $\frac{8}{3} =$ | **g.** $\frac{7}{6} =$ | **h.** $\frac{8}{6} =$ |
| **i.** $\frac{9}{6} =$ | **j.** $\frac{10}{6} =$ | **k.** $\frac{11}{6} =$ | **l.** $\frac{13}{6} =$ |

**B. Use your fraction blocks to express each fraction as a whole number.**

| | | | |
|---|---|---|---|
| **a.** $\frac{2}{2} =$ | **b.** $\frac{4}{2} =$ | **c.** $\frac{6}{2} =$ | **d.** $\frac{3}{3} =$ |
| **e.** $\frac{6}{3} =$ | **f.** $\frac{9}{3} =$ | **g.** $\frac{6}{6} =$ | **h.** $\frac{12}{6} =$ |

**What we have learned:**
**We can use the fraction blocks to express some improper fractions as mixed fractions by first forming as many wholes as possible.**

## Lesson 24

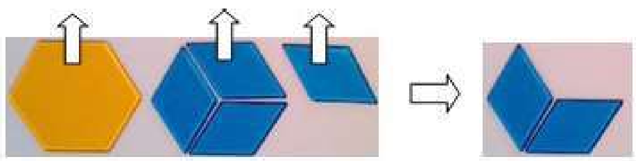
### Subtracting Mixed Fractions by Making an Exchange: Using Fraction Blocks

**Let's consider the following subtraction problem:**

$$2\frac{1}{3} - 1\frac{2}{3}$$

**Considering the yellow block as the whole, we set up $2\frac{1}{3}$ using the fraction blocks.**

**Since the problem calls for subtracting $1\frac{2}{3}$, we need to take away one yellow block and two blue blocks. Whereas we do have one yellow block available to take away, we do not have two blue blocks available! However, we can solve this difficulty by *making an exchange*: We can exchange one yellow block for three blue blocks and then carry out the required action, as shown below. We see that $2\frac{1}{3} - 1\frac{2}{3} = \frac{2}{3}$.**



**A. Please use your fraction blocks to solve, making an exchange as needed:**

**a.** $3 - 1\frac{1}{2} =$ **b.** $2\frac{1}{6} - 1\frac{5}{6} =$ **c.** $1\frac{3}{6} - \frac{4}{6} =$

**B. Review Lesson #21: Find the sum. Example: $2\frac{3}{5} + 1\frac{4}{5} = 3 + \frac{7}{5} = 3 + 1\frac{2}{5} = 4\frac{2}{5}$.**

**a.** $2\frac{3}{4} + 1\frac{3}{4} =$ **b.** $2\frac{1}{5} + 1\frac{1}{5} + 1\frac{4}{5} =$

**C. Review Lesson #22: Please use your fraction blocks to find the answer:**

**a.** $2\frac{5}{6} + \frac{2}{6} - 1\frac{3}{6} =$ **b.** $3\frac{1}{2} + \frac{3}{2} - 1\frac{1}{2} =$ **c.** $2\frac{2}{3} + 1\frac{2}{3} - 1\frac{1}{3} =$

**What we have learned:**

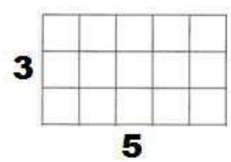
**In order to model a subtraction problem using the fraction blocks, we sometimes first need to exchange the yellow block for blocks of another color.**

# Lesson 32

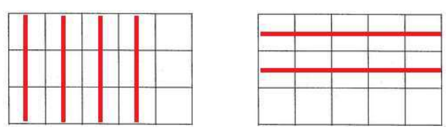
## Comparing Fractions with Different Denominators: Part I

**It is easy to compare two fractions when they have the same denominator. For example, we know that $\frac{5}{6} > \frac{3}{6}$ since 5 copies of $\frac{1}{6}$ are more than 3 copies of $\frac{1}{6}$.**

**Let's now consider how we might visually compare the size of two fractions such as $\frac{4}{5}$ and $\frac{2}{3}$ which do not have a common denominator. Since the denominators of the given fractions are 5 and 3, let's use a 5 by 3 rectangular grid to represent the whole.**




**We notice above that the rectangular grid is composed of 15 small boxes of the same size. Therefore the area of each small box is $\frac{1}{15}$ of the whole area. Now, let's mark off each of the fractions $\frac{4}{5}$ and $\frac{2}{3}$ on a different copy of this rectangle, as shown below.**



$$\frac{4}{5} = \frac{12}{15} \qquad\qquad \frac{2}{3} = \frac{10}{15}$$

**We indicate the fraction $\frac{4}{5}$ by marking four of the five columns on the first rectangle, and we indicate $\frac{2}{3}$ by marking off two of the three rows on the second rectangle.**

**We can now count the number of small boxes marked off by each fraction, namely 12 in the first rectangle and 10 in the second rectangle. From the first rectangle we see that $\frac{4}{5} = \frac{12}{15}$ and from the second rectangle we see that $\frac{2}{3} = \frac{10}{15}$. Since $\frac{12}{15} > \frac{10}{15}$ we can conclude that $\frac{4}{5} > \frac{2}{3}$. *Hence, the rectangular grids enabled us to compare the two fractions by visually transforming them into equivalent fractions having the same denominator.***

**What we have learned:**

**We can compare two fractions by using two equal rectangular grids whose sides have the lengths of the given denominators, and then marking off a fraction on each grid.**

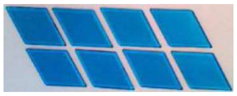
# Lesson 39

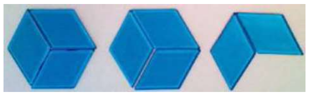
## Understanding Word Problems Involving Multiplication of a Fraction by a Whole Number

**Let's see how we might solve the following word problem using our fraction blocks:** ***A cake recipe requires two-thirds of a cup of flour. You are interested in baking four cakes. How much flour do you need?***

**Solution:** **Let's use the letter $n$ to represent the amount of flour needed to bake 4 cakes. Since we need $\frac{2}{3}$ of a cup for each cake, we need to multiply this by 4 in order to get the amount of flour needed to bake 4 cakes. Hence, the equation for this problem is $n = 4 \times \frac{2}{3}$.**

**Let's now represent this problem visually. We will designate the yellow block to represent a full cup of flour. Hence, two blue blocks would represent two-thirds of a cup of flour, the amount needed to bake one cake. Since we need to bake four cakes, we need to quadruple these two blocks, as indicated at the left below.**





**We notice that if we combine the eight blue blocks into wholes, we get $2\frac{2}{3}$.**

**Answer:** **We need $2\frac{2}{3}$ cups of flour to bake four cakes.**

**A. If $\frac{2}{3}$ of a cup of flour is needed to bake a cake, use your fraction blocks to determine how much flour would be needed to bake each of the following:**

**a. 2 cakes:______________________ b. 3 cakes: ______________________**

**B. Review Lesson #37. Express each product below as a fraction. Example: $8 \times \frac{1}{5} = \frac{8}{5}$.**

**a.** $3 \times \frac{1}{4} =$ **b.** $2 \times \frac{1}{6} =$ **c.** $7 \times \frac{1}{3} =$ **d.** $5 \times \frac{1}{10} =$

**C. Review Lesson #38. Express each product as a fraction. Example: $4 \times \frac{2}{5} = \frac{8}{5}$.**

**a.** $2 \times \frac{3}{6} =$ **b.** $3 \times \frac{5}{8} =$ **c.** $5 \times \frac{3}{4} =$ **d.** $4 \times \frac{3}{10} =$

**What we have learned:**

**We can model word problems involving multiplication of a fraction by a whole number by duplicating as many times as needed the visual model for the fraction.**