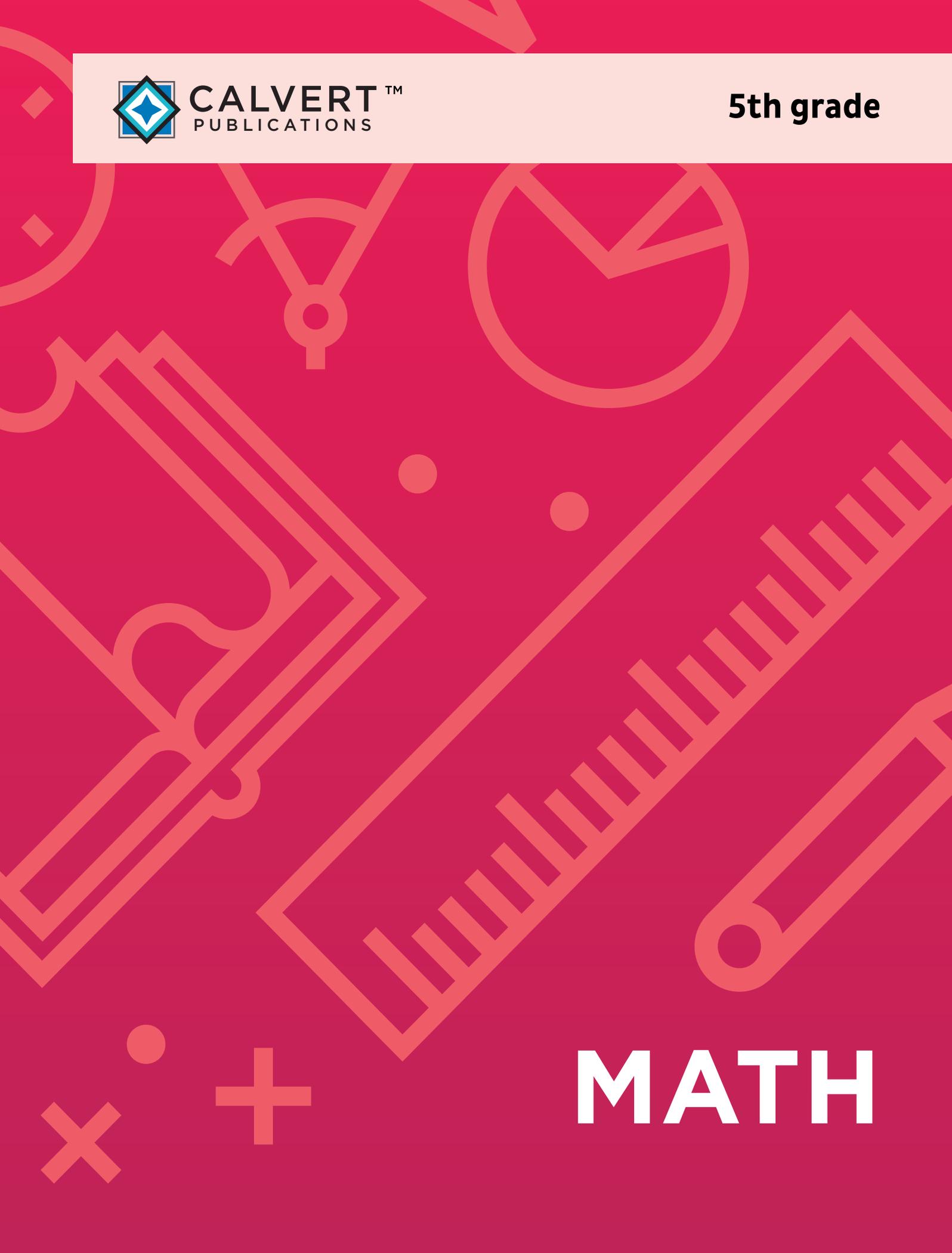




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**MATH**

# MATH 501

## PLACE VALUE, ADDITION, AND SUBTRACTION

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Test | **Pull-out at the back of the booklet**

## Ordering Numbers

When we have to compare more than two numbers, we can order the numbers from smallest to largest. Remember to first count the number of digits in each number. Then, compare the digits in each place value, starting at the left.

### Example:

On earth, the five oceans cover about 139,500,000 square miles. Look at the table to see the approximate number of square miles each ocean covers.

OCEAN	AREA (IN SQUARE MILES)
Arctic Ocean	5,427,000
Atlantic Ocean	31,800,000
Indian Ocean	28,350,000
Pacific Ocean	63,800,000
Southern Ocean	7,800,000



Put the oceans in order from smallest to largest.

### Solution:

The Arctic and Southern oceans have seven digits, and the Atlantic, Indian, and Pacific oceans have eight digits. So, the Arctic and Southern oceans are smaller than the other three. Compare the digits from each place value, starting from the left—in the millions' place.

Arctic: **5**,427,000

Southern: **7**,800,000

### S-T-R-E-T-C-H

About how many times larger is the Pacific Ocean than the Atlantic Ocean?

Since 5 is less than 7, the Arctic Ocean is smaller than the Southern Ocean.

Now, compare the digits of the Atlantic, Indian, and Pacific oceans, starting from the left—in the ten millions' place. Here they are listed from smallest to largest.

Indian: **28**,350,000

Atlantic: **31**,800,000

Pacific: **63**,800,000

Listed from smallest to largest, the oceans are the Arctic, Southern, Indian, Atlantic, and Pacific.

## 2. ROUNDING AND ESTIMATING

In this lesson, we'll learn how to round whole numbers and decimal numbers. We'll use the number line and rounding rules to help us.



### Objectives

Read these objectives. When you have completed this section, you should be able to:

- Round whole numbers and decimals.
- Estimate sums and differences.
- Apply the Commutative, Associative, and Identity Properties of Addition to problem solving.
- Add and subtract numbers mentally.

### Vocabulary

Study these new words. Learning the meanings of these words is a good study habit and will improve your understanding of this unit.

**addend.** A number to be added.

## Adding Decimal Numbers

Do you remember how to add whole numbers? Vertically line up digits in the same place value. Then, add digits in the same place value together, carrying if necessary.

In this lesson, we'll continue to add numbers together to solve problems. Now, we'll learn to add decimal numbers too!

Adding decimal numbers is just like adding whole numbers. Digits in the same place value get added together. So, tenths are added to tenths and hundredths are added to hundredths. To line up the place values correctly, the first step in adding is to line up the decimal points of the addends. Let's look at an example:

Let's add 0.14 and 0.59. Begin by writing the numbers vertically, lining up the decimal points.

$$\begin{array}{r} 0.14 \\ + 0.59 \\ \hline \end{array}$$

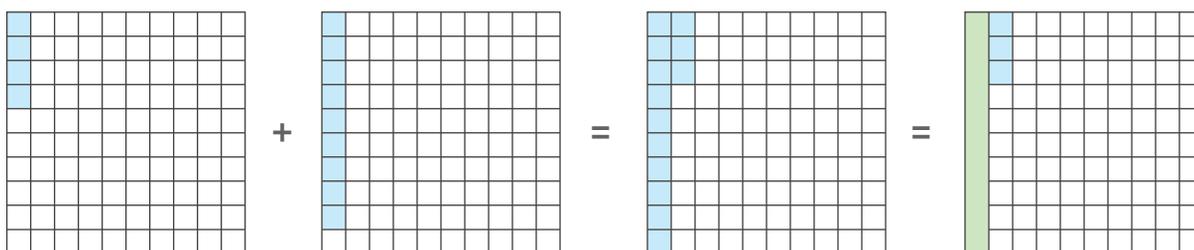
Next, we can add digits in the same place value. Just like with whole numbers, we'll start with the smallest place value, which is the hundredths. The sum of 4 and 9 is 13. So, write a 3 below the line and carry the 1 to the tenths. We do this because 13 hundredths is the same as 1 tenth and 3 hundredths. This is sometimes called **regrouping**.

$$\begin{array}{r} 1 \\ 0.14 \\ + 0.59 \\ \hline 0.73 \end{array}$$

### This might help!

13 hundredths is the same as 1 tenth and 3 hundredths. On the grids below, each little square represents 1 hundredth. And, each column (10 squares) represents 1 tenth.

A grid can help us visualize the problem:



Now, let's add the tenths.  
The sum of 1, 1, and 5 is 7.

$$\begin{array}{r} 1 \\ 0.14 \\ + 0.59 \\ \hline 0.73 \end{array}$$

### Keep in mind...

The first grid in the model below represents the one-tenth that was carried over from the hundredths' place.



**Circle the correct letter and answer.**

3.26

Skill in adding and subtracting decimals is useful for calculating \_\_\_\_\_ .

- a. the total children in a classroom
- b. the number of hours in two days
- c. the amount saved in buying a pair of \$32.95 blue jeans on sale for \$26.50



**Add. Circle the correct letter and answer.**

3.27

$22.3 + 4.1 + 78$

- a. 34.2
- b. 71.1
- c. 104.4
- d. 141.3



**Subtract.**

3.28

$88.9 - 26.7$  \_\_\_\_\_

3.29

$8.17 - 2.91$  \_\_\_\_\_

3.30

$12 - 4.6$  \_\_\_\_\_

3.31

$37.88 - 20$  \_\_\_\_\_

3.32

$27.29 - 6.4$  \_\_\_\_\_



**Circle the correct letter and answer.**

3.33

At Smith’s Grocery Store, almonds are on sale for \$3.14 per pound. Walnuts are on sale for \$2.65 per pound. How much more per pound are almonds than walnuts?

- a. \$1.51
- b. \$0.49
- c. \$0.51
- d. \$0.59

3.34

At Smith’s Grocery Store, almonds are on sale for \$3.14 per pound. Walnuts are on sale for \$2.65 per pound. If Mrs. Hook buys one pound of each, how much change will she get back from \$10? (Hint: You’ll need to add and subtract to solve this problem.)

- a. \$5.79
- b. \$5.21
- c. \$4.21
- d. \$4.79

3.35

In gym class, Jeremiah ran one mile in 7.2 minutes, Arianna ran one mile in 6.75 minutes, and Raven ran one mile in 7.08 minutes. How much longer did it take Jeremiah to run than Raven?

- a. 0.12 minutes
- b. 0.45 minutes
- c. 0.33 minutes
- d. 0.6 minutes



# MATH 502

# MULTIPLYING WHOLE NUMBERS AND DECIMALS

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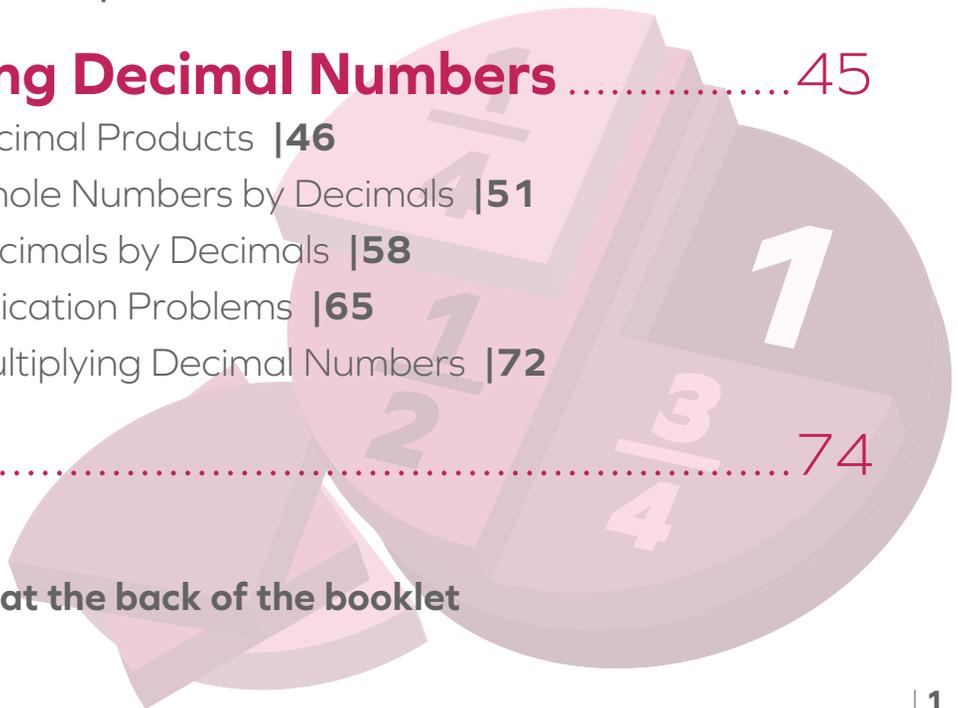
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Test | **Pull-out at the back of the booklet**



## Review: Basic Math Facts

As we saw in the cartoon, multiplication is just repeated addition. Nutmeg and Pepper had two different ways to find the same answer.

Pepper:  $5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 40$       Adding 5 eight times.

Nutmeg:  $5 \times 8 = 40$       Multiplying 5 and 8.

Both methods get the same result, but multiplying is usually much faster (and takes less bark!). Nutmeg's answer is called a **product**. A product is the result of multiplying two or more numbers, which are called **factors**. You're probably very good at finding the product of two factors that are each 12 or less. Take a look at the following multiplication table to help you review your math facts.

x	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

## Exponents

There are actually several different possible answers to the riddle! Let's look at a few of them. Read the poem carefully again.

The person telling the riddle (the narrator) is on his way to St. Ives. But, the narrator never says that all the others are going to St. Ives too. He only says that he met them along the way. So, one possible answer to this riddle is that only one person is going to St. Ives: the narrator himself!

Another way to read the riddle is to assume that everyone is headed for St. Ives—the narrator, man, wives, sacks, cats, and kittens! Let's see if we can figure out that answer. First, we'll have to find the number of things in each group. Then, we'll add all the groups together.

Let's make a list of those headed to St. Ives:

Narrator: **1**

Man: **1**

Wives: **7**

Now, it gets a little trickier.

Each of the 7 wives has 7 sacks.

So, there are  $7 \times 7$  sacks, or 49 total sacks.

Sacks:  $7 \times 7 = \mathbf{49}$

And each of the 49 sacks has 7 cats. So, there are  $49 \times 7$  cats, or 343 cats.

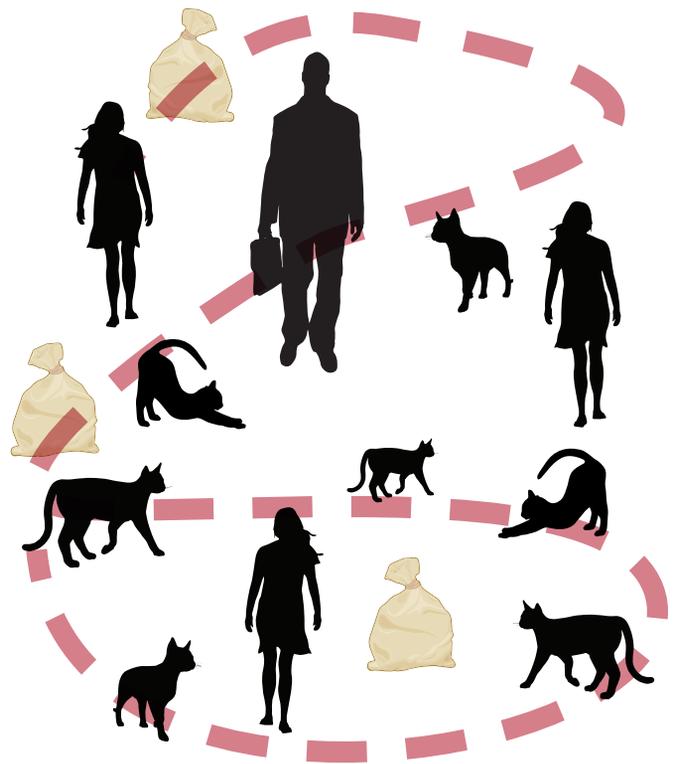
Cats:  $7 \times 7 \times 7 = 49 \times 7 = \mathbf{343}$

$$\begin{array}{r} 6 \\ 49 \\ \times 7 \\ \hline 343 \end{array}$$

And finally, each of the 343 cats has 7 kittens. So, there are  $343 \times 7$  kittens, or 2,401 kittens.

Kittens:  $7 \times 7 \times 7 \times 7 = 343 \times 7 = \mathbf{2,401}$

$$\begin{array}{r} 32 \\ 343 \\ \times 7 \\ \hline 2,401 \end{array}$$



### S-T-R-E-T-C-H

Another way to interpret the riddle is that it is just asking for the number of wives, sacks, cats and kittens that are going to St. Ives. That would be 2,800 things rather than 2,802, since the narrator and man wouldn't be included. Can you think of any other possible answers?

## Multiplying Whole Numbers by Decimals

Patrick and his two friends are participating in a relay race at camp. Each one will run through an obstacle course that is 0.14 miles long. What is the total distance that each team of three will run to finish the race?

There are two different ways we can solve this problem. We can add 0.14 together three times.

Or, since repeated addition is the same as multiplication, we can multiply 0.14 by 3.

How do we multiply decimal numbers and whole numbers?

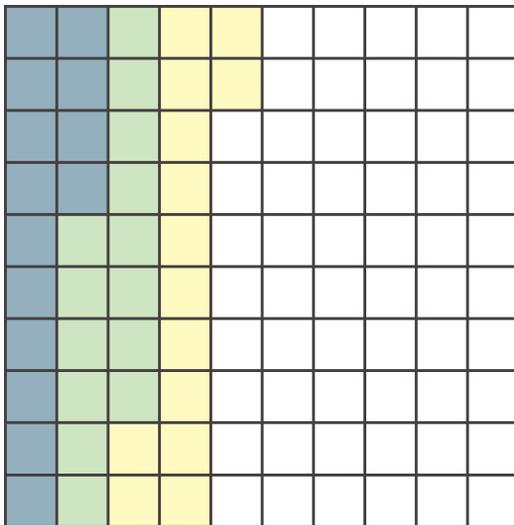
In this lesson, we'll learn how to multiply using a grid or paper and pencil.



### Multiplying a Whole Number by a Decimal Number Using a Grid

Let's look back to the problem from the introduction to this lesson. We need to multiply 0.14 miles by 3. Remember that we can use a grid to represent decimal numbers. We can also use a grid to represent multiplication of decimal numbers.

Take a look:



10 squares  
10 squares  
10 squares  
10 squares  
2 squares

0.10 0.10 0.10 0.10 0.02

$$0.14 \times 3 = 0.42$$

#### Keep in mind...

Decimal numbers that are less than 1 are written with a zero to the left of the decimal point. So, we write 0.14, not .14.

Patrick and his two friends ran a total of 0.42 miles.

Let's try some examples together. We'll follow each of the steps in order to solve.

**Example:**

Pete planted a fast-growing tree in his backyard. He has been told that it will probably grow about 0.75 feet each month of the first year. How much can he expect the tree to grow in the first year?

**Solution:**

- **Read the problem carefully.**
- **Identify what you're trying to find.**  
We need to find how much the tree grew in the first year.

- **Make a plan.** We are told that the tree grows about 0.75 feet each month. Since there are 12 months in a year, we should multiply 0.75 feet by 12 to find how much it will grow in one year.

- **Solve the problem.**

$$\begin{array}{r} 0.75 \\ \times 12 \\ \hline 150 \\ 750 \\ \hline 9.00 \end{array}$$

- **Check that your answer makes sense.**  
We found that the tree will grow about 9 feet in one year, which is what the problem was asking. This number makes sense because in ten months the tree would grow 7.5 feet (multiplying by 10 moves the decimal point one place to the right). Because one year is a little more than ten months, it makes sense that our answer is a little more than 7.5 feet.

**The tree will grow about 9 feet in the first year.**

**Keep in mind...**

When multiplying with decimal numbers, the product will have the same number of decimal places as there are in both factors.



# MATH 503

## DIVIDING WHOLE NUMBERS AND DECIMALS

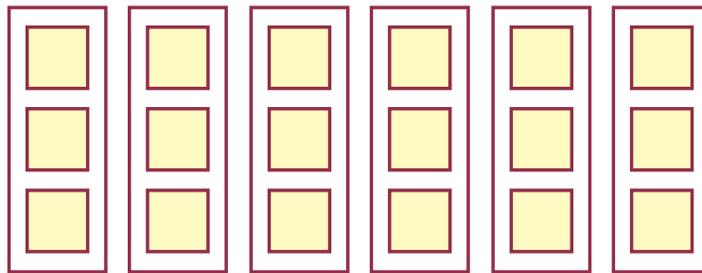
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So, each squirrel should get four pieces of licorice. The division problem we just did can be written as  $20 \div 5 = 4$ . The symbol for division is  $\div$ . The first number (20) is called the **dividend**. It is the amount being divided. The second number (5) is the **divisor**. The divisor is the number of parts that the dividend is being evenly divided into. The last number (4) is called the **quotient**. The quotient is the result of dividing two numbers. It represents the quantity that will be in each group.

$$\begin{array}{r} \text{quotient} \\ \text{divisor} \overline{) \text{dividend}} \end{array}$$

**Example:**

What division problem is modeled?

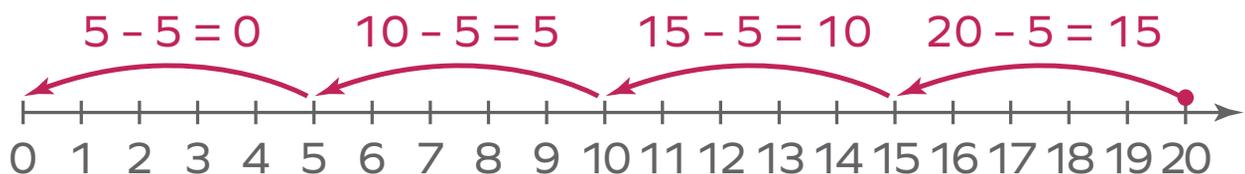


**Solution:**

The model shows 18 total yellow squares. These yellow squares are divided into six equal groups. In each group, there are three squares. So, the division problem is  $18 \div 6 = 3$ . The dividend is 18, the divisor is 6, and the quotient is 3.

**Division as Repeated Subtraction**

Another way to think of division is as repeated subtraction. For example, we could solve the division problem  $20 \div 5$  by seeing how many times we would have to subtract 5 from 20 in order to end up with nothing left. This can be shown on a number line:



In order to get from 20 to 0 on the number line, we have to subtract 5, four times.

## Steps of Long Division

Now, let's find the exact quotient. To divide, we'll use **long division**, which is a method for dividing with larger numbers. This method breaks up the process into smaller steps.

Here are the steps of long division:

**Draw** the long division sign. Write the dividend on the inside of the sign and the divisor on the outside of the sign.

$$\text{divisor} \overline{) \text{dividend}}$$

**Divide.** See how many times the divisor goes into the first part of the dividend.

**Multiply.** Multiply the result of the division step by the divisor.

**Subtract.** Subtract the result of the multiplication step from the first part of the dividend. This difference should never be larger than the divisor. If it is, then we didn't use the largest number of times that the divisor went into the dividend.

**Bring down** the next digit from the dividend and repeat the division, multiplication, and subtraction steps.

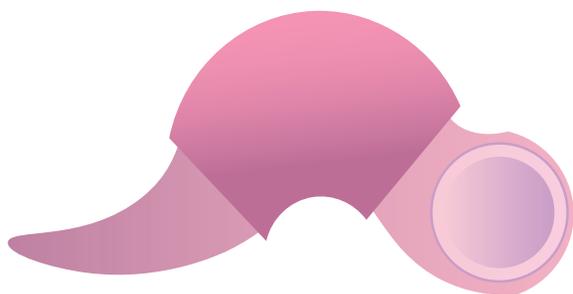
**Repeat** the steps until all digits from the dividend have been brought down.

At the end of the problem, the digits above the division sign represent the quotient.

$$\text{quotient} \\ \text{divisor} \overline{) \text{dividend}}$$

### This might help!

In the division step, we are looking for the largest number of times the divisor goes into the dividend, *without* going over the dividend. For example, 4 goes into 14 three times, because  $4 \times 3 = 12$ . When multiplying by 4, 12 is as close as we can get to 14 without going over 14.



| A worm divided



**Solution (continued):**

Now, let's use long division to find the exact quotient. 27 does not go into 3, but it does go into 32, one time. So, write a 1 in the quotient (above the 2). Then, multiply, subtract, and bring down the 4.

$$\begin{array}{r} 1 \\ 27 \overline{)324} \\ \underline{27} \\ 54 \end{array}$$

Next, 27 goes into 54 two times. So, write a 2 in the quotient (above the 4). Then, multiply and subtract.

$$\begin{array}{r} 12 \\ 27 \overline{)324} \\ \underline{27} \\ 54 \\ \underline{54} \\ 0 \end{array}$$

There are 12 pounds of oranges in each box. Remember that to check our answer we can use multiplication. Multiply the quotient by the divisor to get the dividend.

$$\begin{array}{r} 12 \\ \times 27 \\ \hline 84 \\ 270 \\ \hline 324 \end{array}$$

**Keep in mind...**

When subtracting, the result must be smaller than the divisor. If it's not, then we need to try a different number in the quotient.

**Example:**

A basketball team scored 3,675 points over the course of the season. If there were 35 games, how many points did they average per game?



**Solution:**

To find the average number of points the basketball team scored per game, we have to divide the number of points by the number of games. So we need to find the quotient of 3,675 and 35.

Let's begin dividing. 35 does not go into 3, but it does go into 36, one time. So, write a 1 in the quotient (above the 6). Then, multiply, subtract, and bring down the 7.

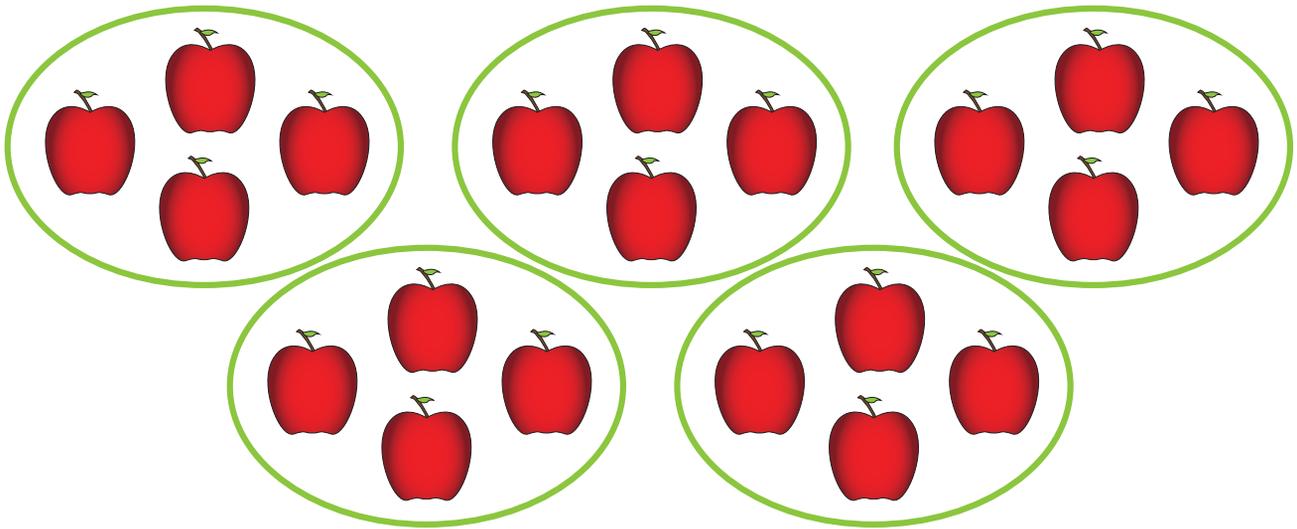
$$\begin{array}{r} 1 \\ 35 \overline{)3,675} \\ \underline{35} \\ 17 \end{array}$$



## Dividing Decimals by Whole Numbers

Let's use an example to review what it means to divide with whole numbers. If we have 20 apples that need to be divided between 5 people, we want to see how many apples each person will get. So, the dividend, or 20, is the amount that is being divided. The divisor, or 5, is the number of groups that we are evenly splitting the dividend into. And, the quotient is the amount that will be in each group after dividing. For this example, the quotient is 4, because each person will get 4 apples. We can use a picture, or model, to show this division:

$$20 \text{ apples} \div 5 \text{ people} = 4 \text{ apples for each person}$$

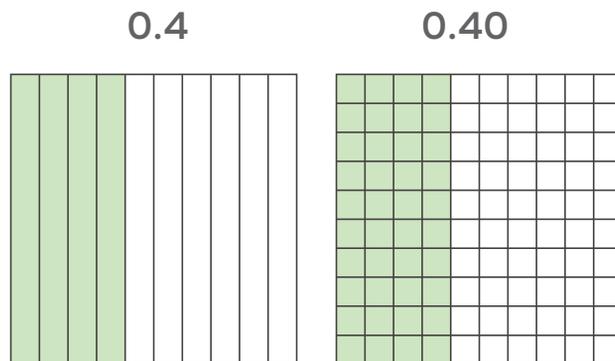


We can apply this same idea to dividing decimal numbers. In this lesson, we'll explore how to divide a decimal number into groups. We'll use models and long division to help us find quotients.

### Dividing Decimal Numbers Using a Grid

Remember that a decimal number can be represented using a grid.

For example, the decimal number 0.4 can be represented on a tenths grid by shading in four tenths. Or, it can be represented on a hundredths grid by shading in forty hundredths.



**Example:**

Each ticket for a soccer game is \$7.35. The cost for parking is \$4.50. There are five people in Mark's car. Each person is going to pay for their own ticket, and they plan to split the cost for parking. What will be the total cost for each person to go to the game?

**Solution:**

- **Read the problem carefully.**
- **Identify what you're trying to find.** We want to find the total cost (including the ticket and parking) each person will have to pay.
- **Make a plan.** We should divide the parking fee (\$4.50) by 5 to find out how much each person will have to pay for parking. Then, we can add the costs for parking and the ticket to find the total cost.
- **Solve the problem.**

Divide the parking fee by 5.

$$\begin{array}{r} 0.90 \\ 5 \overline{)4.50} \\ \underline{45} \phantom{0} \\ 00 \end{array}$$

Multiply  $9 \times 5$ .  
Subtract and bring down the 0.

Add each person's parking and ticket costs.

$$\begin{array}{r} 1 \\ 7.35 \\ + 0.90 \\ \hline 8.25 \end{array}$$

**This might help!**

5 goes into 0, zero times. So, a zero is written in the quotient. Since there are no other digits to bring down, the division is complete.

- **Check that your answer makes sense.** We found that each person will have to pay a total of \$8.25 to attend the game, which is what the problem was asking. To check our answer, we can estimate and compare. Round \$4.50 to a number that is compatible with 5. Then, divide.

Estimate:  $\$5 \div 5 = \$1$  and  $\$7.35 + \$1 = \$8.35$

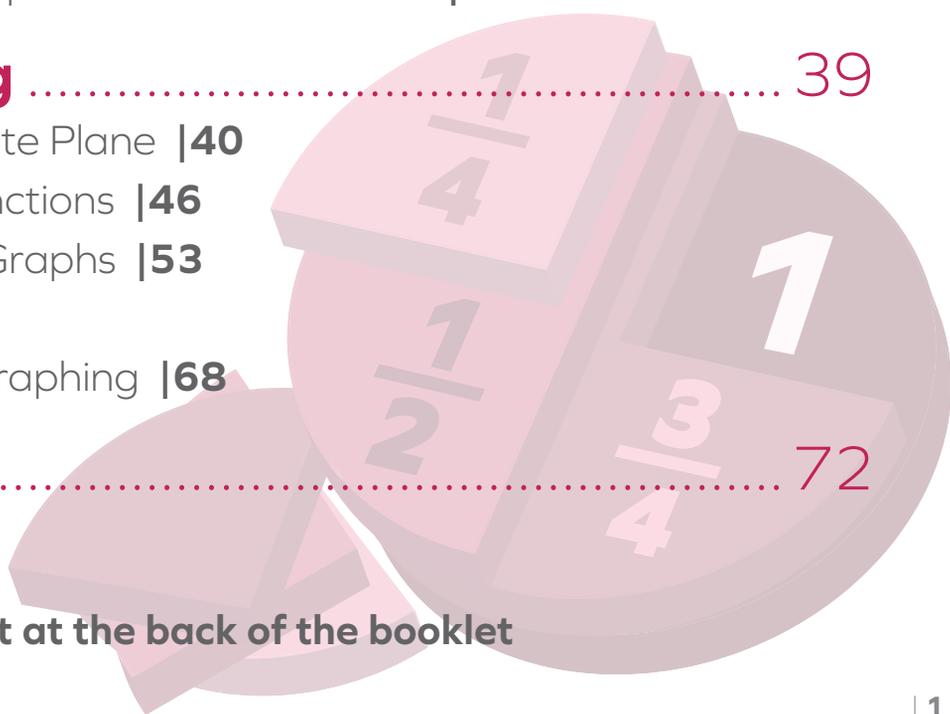
Since our answer and the estimate are close to each other, our answer makes sense.

Each person in Mark's car will have to pay a total of \$8.25.

# MATH 504

## ALGEBRA AND GRAPHING

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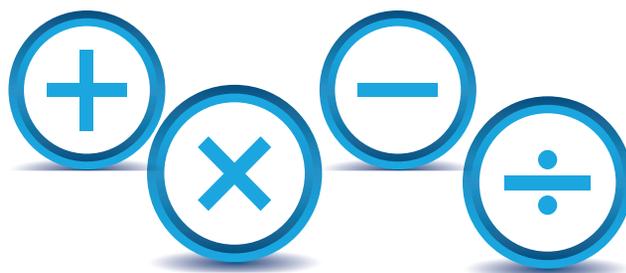


## The Order of Operations

What is the value of this expression?

$$12 - 4 \times 2 + 6$$

If you work from left to right, which seems to be a logical order, surprisingly you will get the wrong answer!



We normally follow steps in the order they are given. However, in mathematics, there is a special order in which operations are done. In this lesson, we will explore the **order of operations**, so that we can agree on how to evaluate expressions.

### Rules for the Order of Operations

$$12 - 4 \times 2 + 6$$

Is this expression equal to 22, 64, or 10? Depending on the order in which the operations are done, each of these answers is possible. You can see how it is important that everyone agrees on the order that operations should be done; otherwise different people would get different answers.

Here are the first few rules for the order of operations:

- **Multiplication and division are done first.** If both operations are in the same expression, they are done as they appear from left to right.
- **Addition and subtraction are done next.** If both operations are in the same expression, they are done as they appear from left to right.

Following these rules for the expression  $12 - 4 \times 2 + 6$ , we can see that there is multiplication.

$$4 \times 2 = 8, \text{ so } 8 \text{ can replace } 4 \times 2 \text{ in the expression: } 12 - 8 + 6$$

Next, there is addition and subtraction, but subtraction is first, looking from left to right.

$$12 - 8 = 4, \text{ so } 4 \text{ can replace } 12 - 8 \text{ in the expression: } 4 + 6$$

Finally, we can do the addition:

$$4 + 6 = 10$$

We could write this in a shorter form:

$12 - 4 \times 2 + 6 =$	• Multiply: $4 \times 2$
$12 - 8 + 6 =$	• Subtract: $12 - 8$
$4 + 6 = 10$	• Add: $4 + 6$

#### Make note!

When simplifying expressions, show your work one step at a time so that you can keep track of what you did.

## Using Multiplication Equations to Solve Real-Life Problems

As we've seen, we can represent real-life situations where there is an unknown quantity we want to discover.

### Example:

Tom plants several rows of 5 flowers in his garden. If there are a total of 40 flowers, how many rows did Tom plant?

### Solution:

The number of rows is unknown, so it will be the variable ( $r$ ). Because each row has 5 flowers, the number of rows is multiplied by 5 ( $5r$ ). We know that there are a total of 40 flowers, so  $5r = 40$ .

To solve for  $r$ , we can think of the family of facts that include 5 and 40. If you are not sure of your times facts, you could substitute values for  $r$  until you find the value that makes the equation true.

$$5r = 40$$

We know  $5 \times 10 = 50$ , which is too high, so we'll start with a lower number.

$$5 \times 9 = 45$$

Still too high, so let's try 8.

$$5 \times 8 = 40.$$

So, there are 8 rows of flowers in Tom's garden.

### Test tip

You can solve algebraic equations by substituting values for the variable until both sides of the equation are equal. This can be helpful on a test where you may have only 3 or 4 numbers to try.



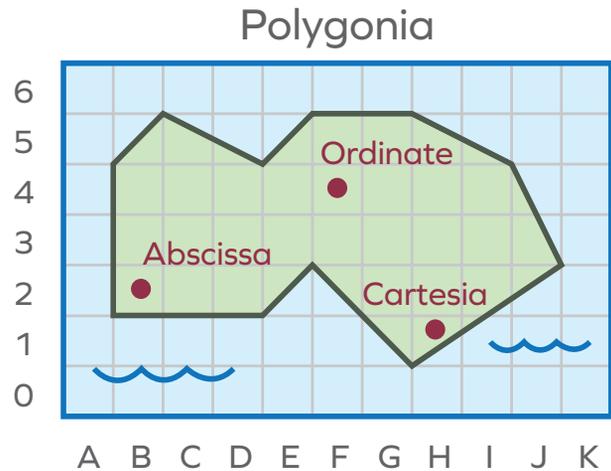
## The Coordinate Plane

This is a map of the island of Polygonia.

Places on the island can be located by using a grid. Each square of the grid is named by the intersection of a column (named by letters) and a row (named by numbers).

There are three cities on Polygonia: Abscissa (located in B2), Ordinate (located in F4), and Cartesia (located in H1).

In mathematics, we use a similar system to name the location of points in two-dimensional space, called the **coordinate plane**. In this lesson, you will learn how to use the coordinate plane.

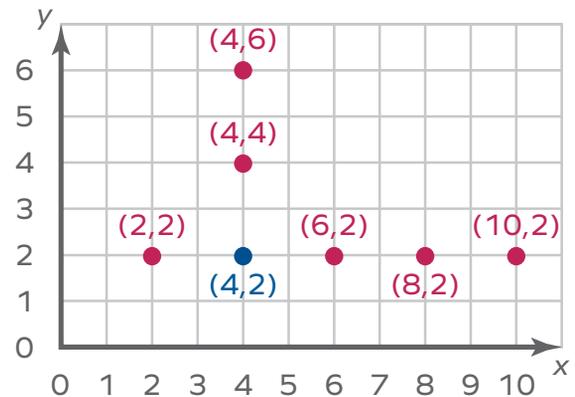


## Graphing Ordered Pairs in Quadrant I of the Coordinate Plane

The grid system used on the Polygonia map is commonly used on city maps to help people locate places. The coordinate plane can be used in a similar way, but it is used to name the location of a point. Let's see how the coordinate plane works.

You can think of the **x-axis** and **y-axis** as number lines that go on forever. The x-axis tells us the distance a point is horizontally from the **origin** (shown as the **x-coordinate**), and the y-axis tells us the distance a point is vertically from the origin (shown as the **y-coordinate**). Each pair of these coordinates (an ordered pair) describes a unique location in the coordinate plane.

The point located at (4, 2) is the only possible point at that location. There are many points that are 4 away from the origin horizontally, or 2 away from the origin vertically. But there is only one point that is 4 away vertically *and* 2 away horizontally.



### Key point!

The x-coordinate is always listed first, followed by the y-coordinate. This is done so that everybody names points the same way and there is no confusion.

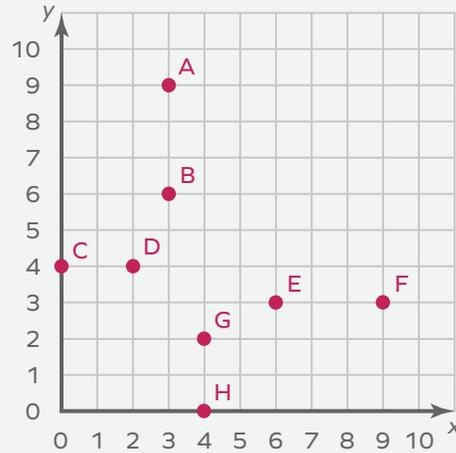
## SELF TEST 3: GRAPHING

Each numbered question = 6 points

Circle the correct letter and answer.

**3.01** What point is located at  $(0, 4)$  on this graph?

- a. point C
- b. point D
- c. point G
- d. point H

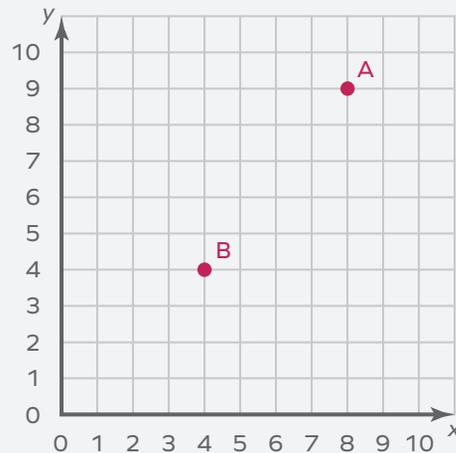


**3.02** Using the graph from the previous question, what point is located at  $(9, 3)$ ?

- a. point A
- b. point B
- c. point E
- d. point F

**3.03** What is the location of point A on this graph?

- a.  $(8, 8)$
- b.  $(9, 8)$
- c.  $(9, 9)$
- d.  $(8, 9)$



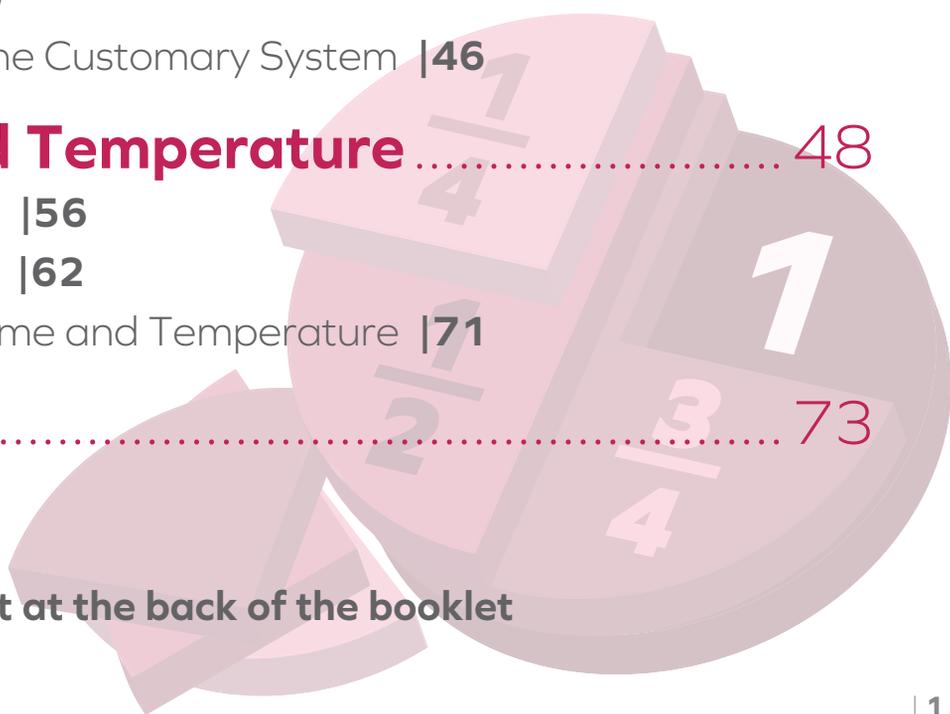
**3.04** Using the graph from the previous question, what is the location of point B?

- a.  $(3, 4)$
- b.  $(4, 4)$
- c.  $(4, 3)$
- d.  $(3, 3)$

# MATH 505

## MEASUREMENT

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## Capacity

A glass of water contains 500 milliliters.

A bottle of soda holds 2 liters.

A bathtub holds about 200 liters of water.

You can begin to get an idea of how much a liter and milliliter are by thinking of these containers. In this lesson we will explore metric units of capacity and learn more about using these units.



### Comparing Units of Capacity within the Metric System

You know that the liter is the basic unit of capacity in the metric system (the International System of Units). Liters and milliliters are the most commonly used units, so we will focus on those two units in this lesson.

- milliliter =  $\frac{1}{1,000}$  of a liter (1,000 milliliters = 1 liter)

Let's see if we can get an idea of the size of these units.

Remember, a cube one centimeter on each side would contain a **milliliter** of water. That's about the amount of water in a large raindrop. Although the United States does not use the International System of Units, most drink containers are labeled in liters or milliliters, and fluid ounces or quarts. A 500 mL glass of water holds close to 16 ounces, or 1 pint (500 mL  $\approx$  16.9 oz.).



Can you estimate the equivalent metric measure of 1 cup (8 oz)? Hopefully you said something from 225 mL to 240 mL!

A liter is 1,000 milliliters, and is similar to a quart (1 liter  $\approx$  1.06 quarts).

A gallon is about  $3\frac{3}{4}$  liters.

Bottles of soda often come in a 2-liter size.

Liters are used to measure capacity, from drink containers to bodies of water.

A bathtub holds about 200 liters of water.

#### Connections

Remember, a cube of water 1 centimeter on each side weighs a gram. So 1,000 milliliters (1 L) weigh 1,000 grams (1 kilogram). This means a 2-liter bottle of soda weighs about 2 kilograms.

# Length

## Comparing Units of Length within the Customary System

The **U.S. Customary System of Units** of length begins with the inch. An inch is about the length of the thumb from the tip to the first knuckle.

In fact, there is evidence that the inch was based on this length. Each of the other units of length are derived from the inch:

### CUSTOMARY UNITS OF LENGTH

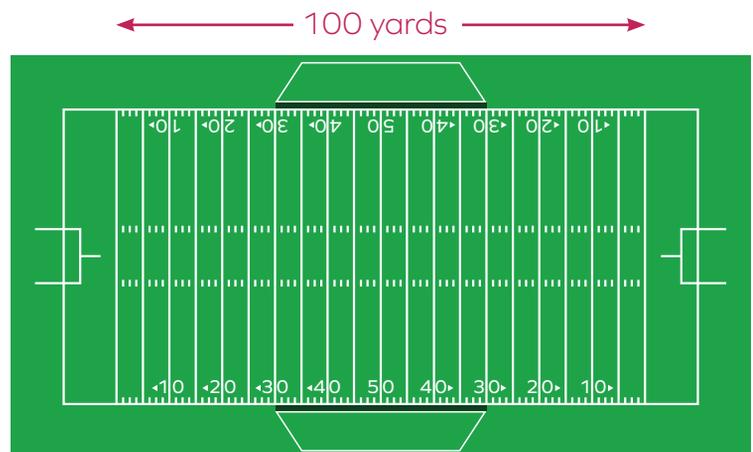
1 foot = 12 inches

1 yard = 3 feet

1 mile = 5,280 feet

It is likely that the foot measurement was based on the length of an important person's (such as a king's) foot, and the inch was defined as  $\frac{1}{12}$  of the foot. An adult male's foot would be about one foot long.

The next largest customary unit is the yard. It is equal to 3 feet, or 36 inches. Because the meter is equal to 39 inches, meters and yards are used to measure some of the same things. Yards are used for longer measurements that are less than a mile. For example, a football field is measured in yards and is 100 yards long.



Miles are used for still longer measurements such as the distance between cities. Los Angeles and New York are about 3,930 miles apart.



Let's try to estimate the length of a few things.

**Example:**

Estimate the length of the following, using customary units:

- the height of a flagpole
- the length of an envelope
- the distance from your home to the airport

**Solution:**

For each length, first we'll decide the appropriate unit and then make the estimate.

**The height of a flagpole:**

A flagpole would be taller than you, and you're probably between 5 and 6 feet tall. So feet, or even yards, would be appropriate. How many of "you" stacked up would be the height of the flagpole? Flagpoles come in different sizes, but most are about 20 to 25 feet tall, or around 7 yards.

**The length of an envelope:**

A postage stamp is about one inch on each side, so inches would be the appropriate unit. How many stamp widths would the envelope be? Envelopes come in different sizes, but most are between 8 and 9 inches long.

**The distance from your home to the airport:**

This length will be different for everybody, but most likely would be measured in miles since airports are usually a reasonable distance from residential areas. Could you walk to your airport? An adult could probably walk a mile in fifteen to twenty minutes, so it might take awhile!





### Complete this activity.

3.1

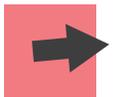
Match the terms with their definitions.

a. \_\_\_\_\_ analog clock

1. a clock that uses hands to display the time

b. \_\_\_\_\_ digital clock

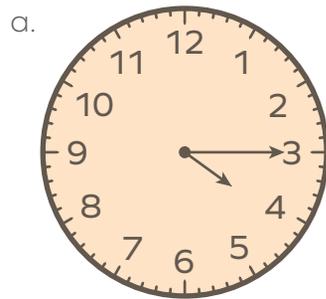
2. a clock that shows the time using digits



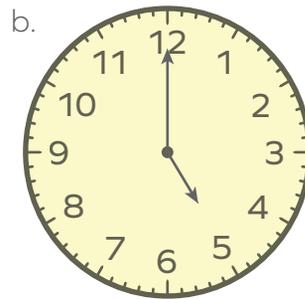
### Complete these activities.

3.2

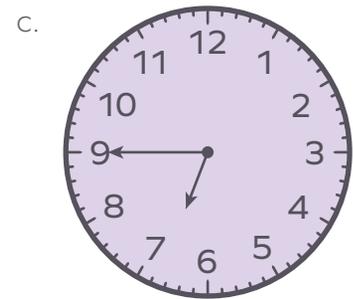
Write the correct time for each clock.



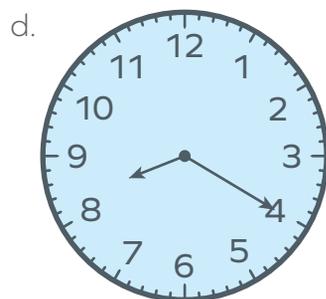
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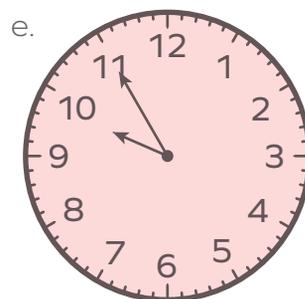
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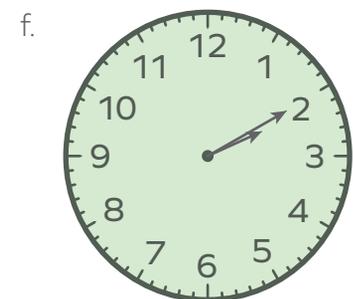
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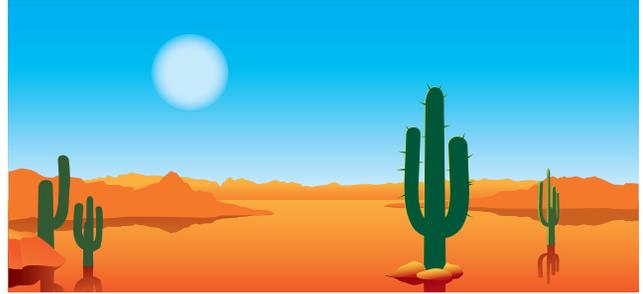
\_\_\_\_\_

# Temperature

If you lived in France and you knew the temperature outside was  $30^{\circ}$ , would that seem cold? It would be if it were  $30^{\circ}$  **Fahrenheit**, the temperature scale used in the United States. However, in France (and most of the world) the temperature scale **Celsius** is used, and  $30^{\circ}$  Celsius is actually quite warm ( $86^{\circ}$  Fahrenheit).



$30^{\circ}$  Fahrenheit



$30^{\circ}$  Celsius

In this lesson, we will explore these two temperature scales and learn to convert between them.

## Comparing Measurements of Temperature

Temperature is measured using a **thermometer**. A thermometer uses a scale that measures temperature in degrees. We measure temperature to see how hot or cold it is outside, for cooking, or to find our body temperature.

For each of these situations, we use different kinds of thermometers, but each use either the Celsius scale or the Fahrenheit scale, and sometimes both are shown.

You are probably familiar with the Fahrenheit temperature scale because it is used in the United States. However, the rest of the world (except for Belize, an English-speaking country in Central America) uses the Celsius scale.

The Celsius scale was set up so that  $0^{\circ}$  is the freezing point for water and  $100^{\circ}$  is the boiling point. Then, the scale was divided into 100 equal parts. For a long time, the scale was also called the *centigrade* scale (*centi-* means 100, and *gradus* is Latin for “steps”).

It is related to the metric system of measurement because it is based on a power of 10 ( $100 = 10 \times 10 = 10^2$ ).

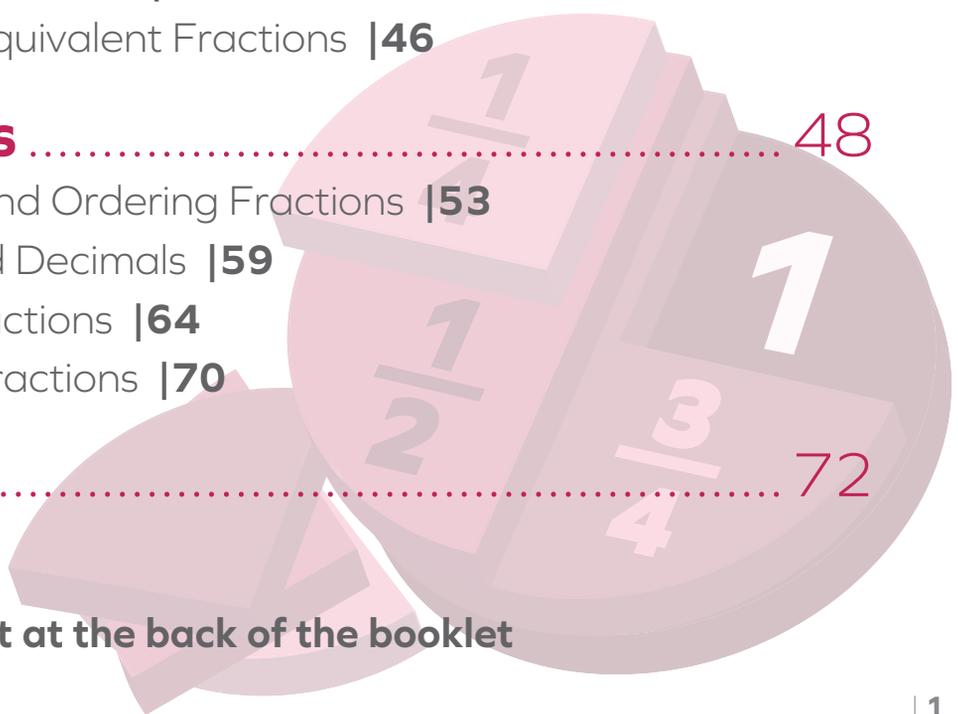
### Connections

A thermometer uses liquid mercury in a narrow glass tube to indicate the temperature. As it gets warmer, the mercury expands and takes up more space in the tube, moving the mercury up the scale. When it’s colder, the mercury contracts and takes up less space in the tube, moving the mercury down the scale.

# MATH 506

# FACTORS AND FRACTIONS

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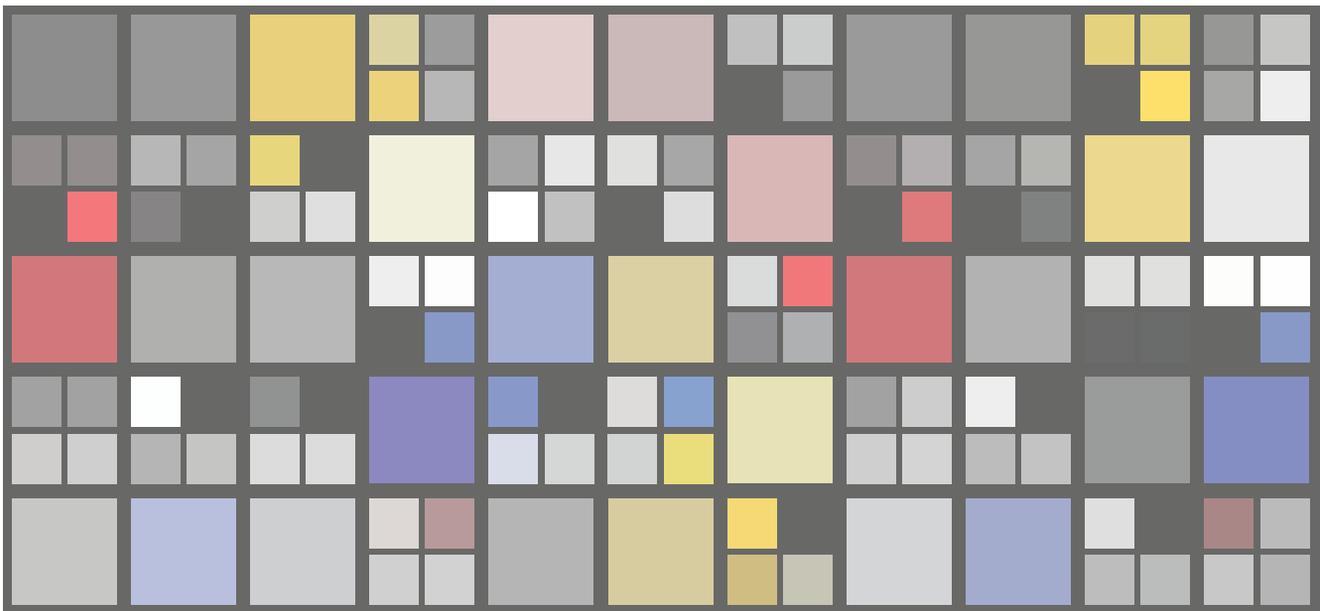


- 1.11** Which of the following lists all the factors of 36?
- a. 1, 2, 3, 4, 6, 12, 36                      b. 1, 2, 3, 4, 5, 6, 7, 9, 12, 36
- c. 2, 3, 4, 6, 9                                  d. 1, 2, 3, 4, 6, 9, 12, 18, 36
- 1.12** Which of the following numbers is 20 divisible by? (There may be more than one correct answer.)
- a. 2                      b. 3                      c. 4                      d. 5
- e. 6                      f. 9                      g. 10



**Complete these activities.**

- 1.13** List all the factors of 56. Tell whether 56 is prime or composite.
- Factors: \_\_\_\_\_
- Prime or composite? \_\_\_\_\_
- 1.14** List all the factors of 15. Tell whether 15 is prime or composite.
- Factors: \_\_\_\_\_
- Prime or composite? \_\_\_\_\_
- 1.15** List all the factors of 19. Tell whether 19 is prime or composite.
- Factors: \_\_\_\_\_
- Prime or composite? \_\_\_\_\_

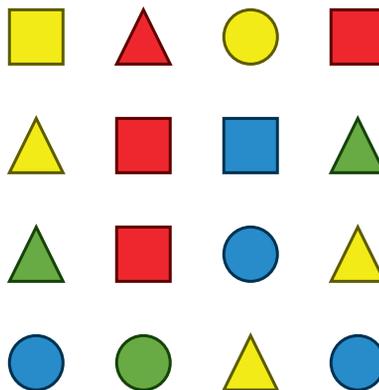


**Example:**

Use a fraction to describe the amount of triangles.

**Solution:**

There are sixteen total shapes. So, the denominator of the fraction is 16. Six of the shapes are triangles. So, the numerator of the fraction is 6.  $\frac{6}{16}$  of the shapes are triangles.

**Keep in mind...**

$\frac{6}{16}$  is read as six-sixteenths. The numerator of a fraction is read as a cardinal number (one, two, three, four, etc.). The denominator of a fraction is read as an ordinal number (third, fourth, fifth, sixth, etc.).

One exception is  $\frac{1}{2}$ , which is usually read as one-half.

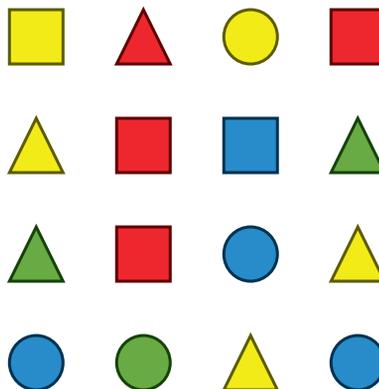
**Example:**

Use a fraction to describe the amount of green circles.

**Solution:**

There are sixteen total shapes. So, the denominator of the fraction is 16. Only one of the shapes is a green circle. So, the numerator of the fraction is 1.

$\frac{1}{16}$  of the shapes are green circles.

**Example:**

In Ella's class, there are ten boys and eleven girls. Use a fraction to represent the number of girls there are in her class.

**Solution:**

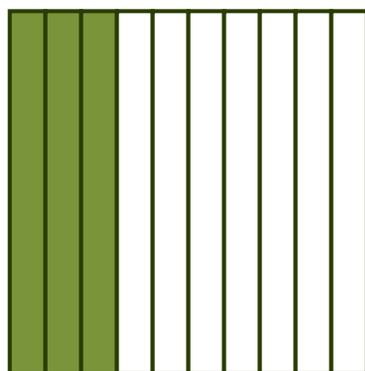
There are ten boys and eleven girls in Ella's class. That means the total class size is  $10 + 11$ , or 21. So, the denominator of our fraction is 21. Eleven of the students are girls.

So, the numerator of our fraction is 11.

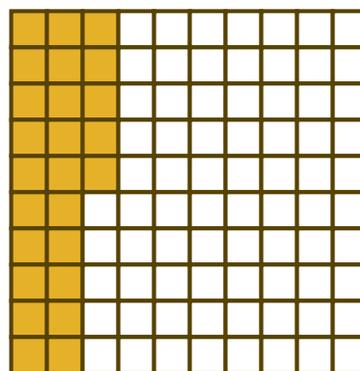
$\frac{11}{21}$  of the students in Ella's class are girls.

## Fractions and Decimals

You probably remember that fractions can be represented using a model. For example, the fraction  $\frac{3}{10}$ , or three-tenths, can be modeled on the grid on the left. The denominator of the fraction (10) tells how many total parts there are in the grid. The numerator (3) tells how many parts are represented, or shaded in.



$$\frac{3}{10}$$



$$\frac{25}{100}$$

And the fraction  $\frac{25}{100}$ , or twenty-five hundredths, can be modeled on the grid on the right. As you can see, 25 of the 100 squares are shaded in.

Fractions are used to represent part of a whole, so we can model fractions by shading in *part* of a grid. What other type of numbers represent part of a whole? Decimal numbers! In this lesson, we'll learn how to represent the same amount using either a fraction or a decimal.

### Converting Between Fractions and Decimals

Fractions and decimal numbers are both used to represent part of a whole. So, the same amount can be described using both a fraction and a decimal number. For example, in the models shown, we represented the fractions  $\frac{3}{10}$  and  $\frac{25}{100}$ . These same models can be described using decimal numbers.

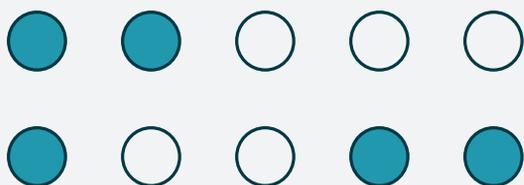
To represent the models as decimal numbers, use place value. Remember that the first place to the right of the decimal point represents the tenths. To represent three-tenths, write a three in the tenths place. So,  $\frac{3}{10}$  is the same as 0.3. The second place to the right of the decimal point represents the hundredths.

So, twenty-five hundredths, or  $\frac{25}{100}$ , is the same as 0.25.

#### Key point...

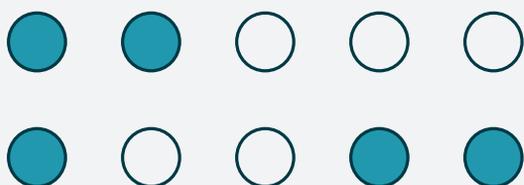
The first decimal place represents tenths, and the second decimal place represents hundredths. Use place value to convert fractions that have a denominator of 10 or 100 to decimal numbers.

**3.011** Use a fraction (in simplest form) to describe the shaded amount in this diagram.



- a.  $\frac{5}{10}$                       b.  $\frac{1}{2}$                       c.  $\frac{1}{5}$                       d.  $\frac{5}{8}$

**3.012** Use a decimal number to describe the shaded amount in this diagram.



- a. 0.05                      b. 0.15                      c. 0.2                      d. 0.5

**Complete these activities.**

**3.013** What is the LCM of 2 and 7? \_\_\_\_\_

**3.014** Round  $7\frac{3}{7}$  to the nearest whole number. \_\_\_\_\_

**3.015** Round  $\frac{7}{8}$  to the nearest whole number. \_\_\_\_\_

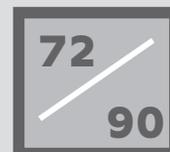


**Teacher check:**

Score \_\_\_\_\_

Initials \_\_\_\_\_

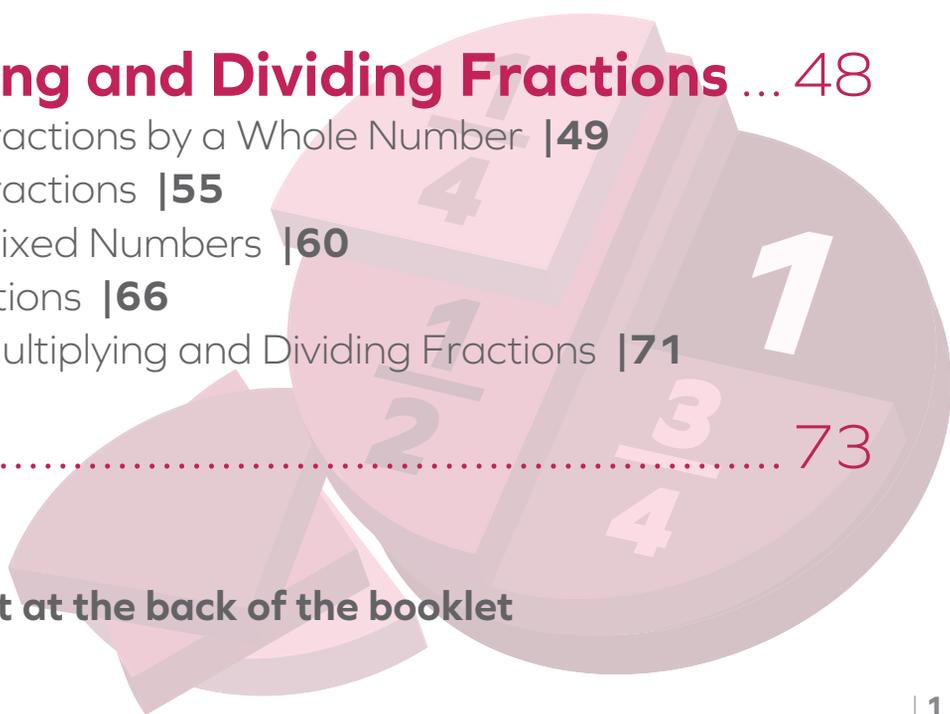
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# MATH 507

## FRACTION OPERATIONS

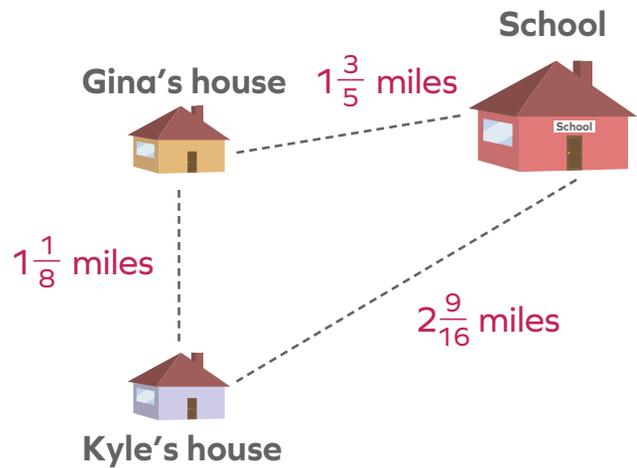
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## Estimating Sums and Differences

Kyle and Gina walk to school each morning. About how far does Kyle walk if he goes to Gina's house first?

Notice that the question we were asked began with the word *about*. That means that we can **estimate** to find the answer. Remember that an estimate is an amount that is close to the actual value. Estimating is helpful when you need a quick answer, but not necessarily an exact answer. In this lesson, we'll practice estimating sums and differences of fractions and mixed numbers.



### Estimating Sums of Fractions and Mixed Numbers

From the map, we can see that Kyle walks  $1\frac{1}{8}$  miles to get to Gina's house and another  $1\frac{3}{5}$  miles to get to school. To estimate the total distance that Kyle walks, we'll use rounding and then add to find the total. We'll round the mixed numbers to the nearest whole or half. To round each mixed number, look at the fraction part of the mixed number and compare the numerator to the denominator. If the numerator is very small compared to the denominator, then the mixed number rounds down to the previous whole number.  $1\frac{1}{8}$  is an example of a mixed number that would round down to the previous whole number—because 1 is very little compared to 8. We can see this on the number line.

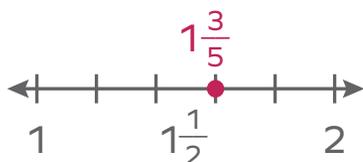


#### Think about it...

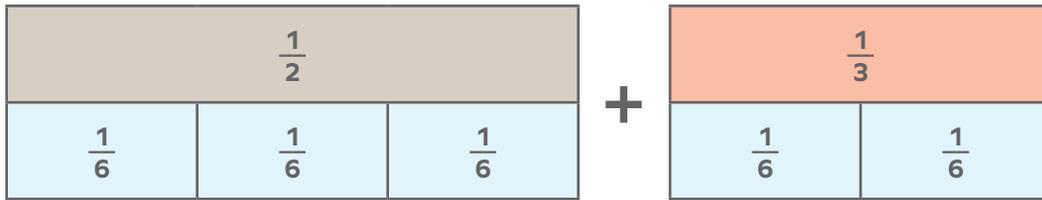
About how much further does Kyle walk than Gina each morning?

If the numerator is close to half of the denominator, then the mixed number rounds to halfway between the whole numbers.  $1\frac{3}{5}$  is an example of a mixed number that would round to the nearest half—because 3 is about half of 5.

Take a look:



Because 6 is a smaller number than 12, let's represent both  $\frac{1}{2}$  and  $\frac{1}{3}$  in sixths.



There are a total of five  $\frac{1}{6}$  bars. So,  $\frac{1}{2} + \frac{1}{3}$  is equal to  $\frac{5}{6}$ .

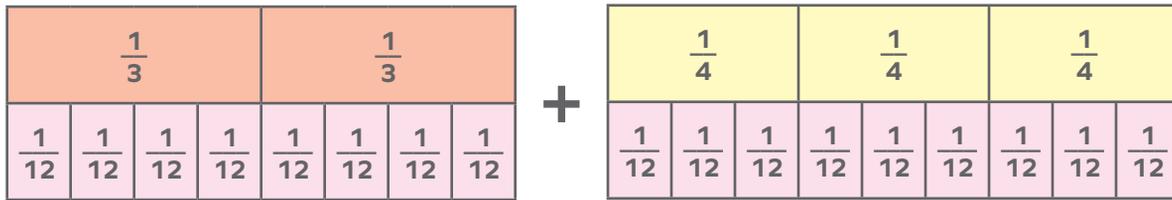
**Example:**

Find the sum using fraction bars.

$$\frac{2}{3} + \frac{3}{4}$$

**Solution:**

Both fractions can be represented in twelfths. Notice that  $\frac{2}{3}$  is the same as  $\frac{8}{12}$ , and  $\frac{3}{4}$  is the same as  $\frac{9}{12}$ .

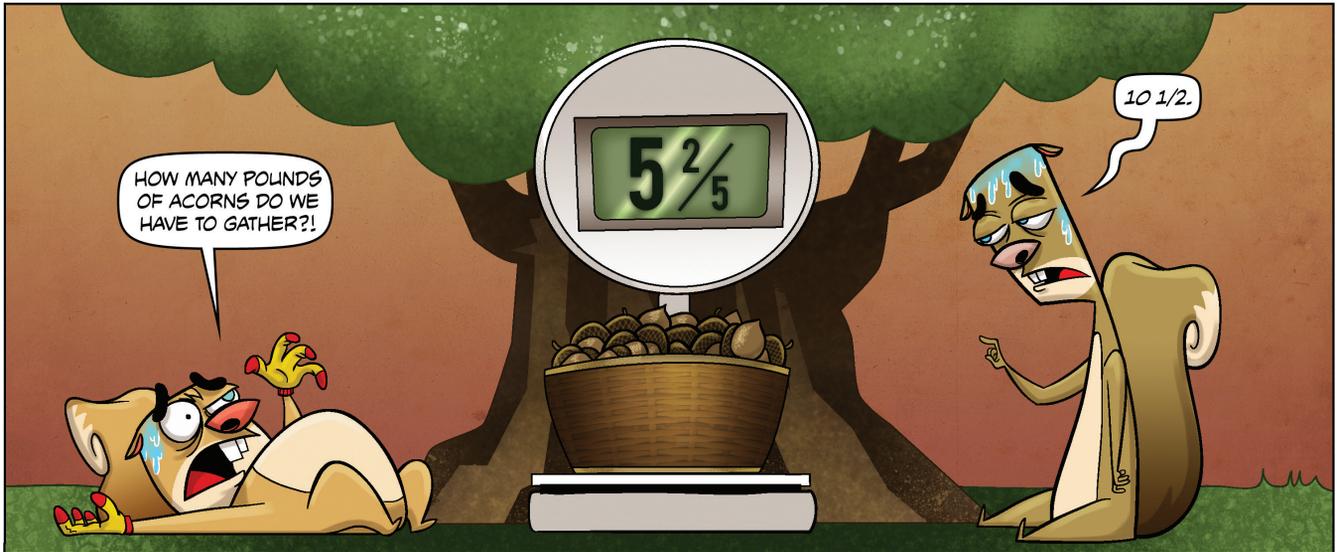


There are a total of seventeen  $\frac{1}{12}$  bars. So,  $\frac{2}{3} + \frac{3}{4}$  is equal to  $\frac{17}{12}$ , or  $1\frac{5}{12}$ .

**Keep in mind...**

To rewrite an improper fraction as a mixed number, divide the numerator by the denominator. The quotient is the whole number, and the remainder is the numerator. The denominator stays the same. Always write the fraction in simplest form.

## Subtracting Mixed Numbers



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In this lesson, we'll learn how to subtract with mixed numbers that have unlike denominators. We'll also help Nutmeg and Pepper figure out how many pounds of acorns they have yet to gather!

### Subtracting Mixed Numbers with Unlike Denominators

Subtracting mixed numbers with unlike denominators uses the same four steps as subtracting fractions with unlike denominators. Let's review those steps.

- Find the least common denominator (LCD) of the fractions.
- Rewrite each fraction using the LCD.
- Subtract the mixed numbers, using the rewritten fractions.
- Write the difference in simplest form.

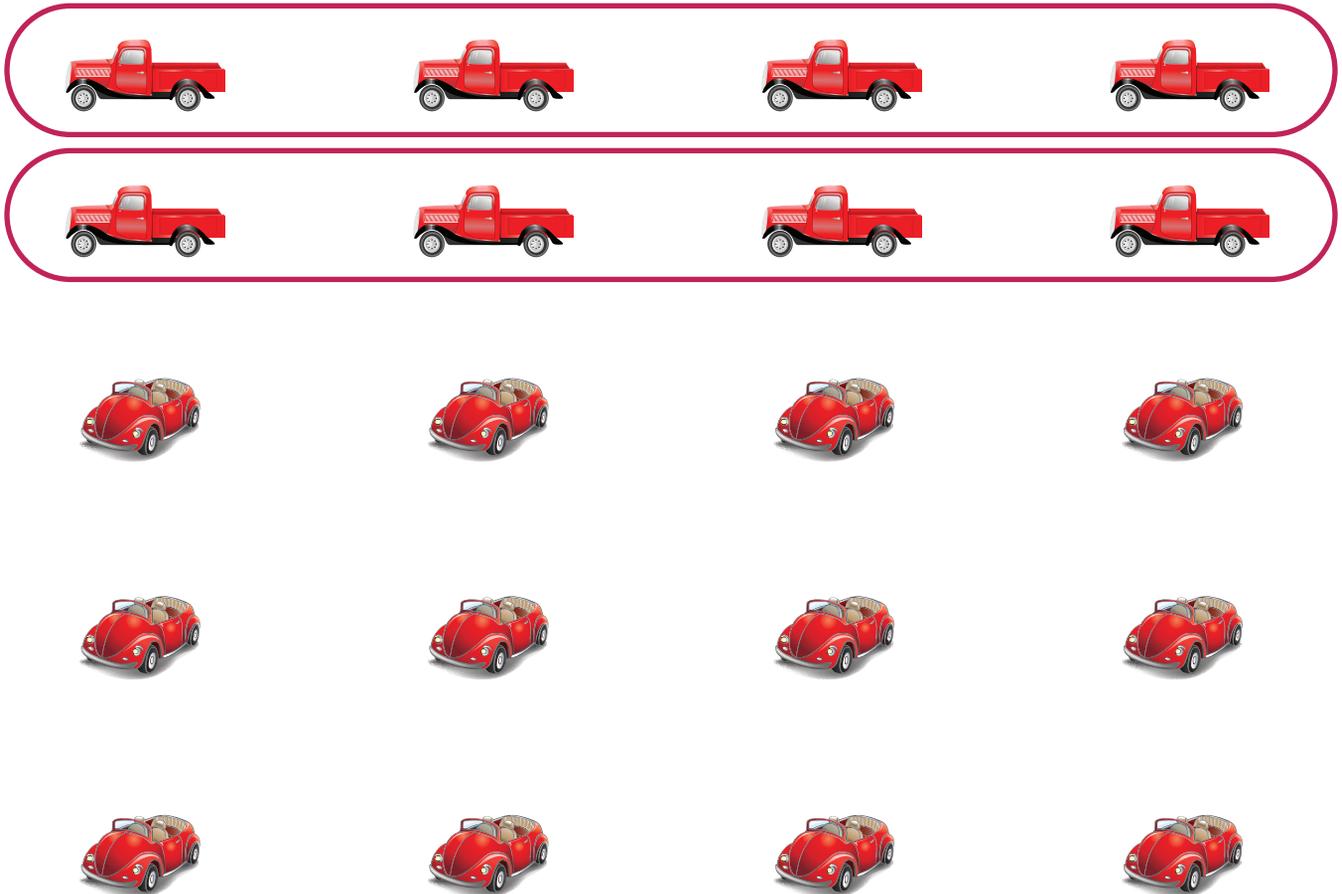
Using these steps, we can help Nutmeg and Pepper figure out how many more pounds of acorns they have yet to gather. Let's subtract the amount they've already gathered ( $5\frac{2}{5}$  pounds) from the total amount they need ( $10\frac{1}{2}$  pounds).

$$\text{Subtract: } 10\frac{1}{2} - 5\frac{2}{5}$$

- **Find the LCD of the fractions.** List the multiples of the denominators.
  - 2:** 2, 4, 6, 8, **10**
  - 5:** 5, **10**, 15, 20
  - LCD = 10**

## Multiplying Fractions by a Whole Number

Let's figure out what  $\frac{2}{5}$  of 20 is. One way is to divide 20 into fifths, or five equal groups. Take a look:



When 20 was divided into five equal groups, or fifths, there were four cars in each group. And each group represented  $\frac{1}{5}$ . Because  $\frac{2}{5}$  of the collection are trucks, two of those groups represented the trucks. So, Dana's dad owns eight model trucks.

# MATH 508

## DATA ANALYSIS AND PROBABILITY

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The goal in statistics is to collect a large enough **random sample** to draw valid conclusions. A random sample is a sample in which every member of the population has an equal chance of being selected. Instead of measuring only basketball players to find the height of students, which would be a biased sample, the first ten names from each fifth grade class could be measured.

## Organizing Data Using a Frequency Table

Once we've collected the data, we need a way to organize it. This can be done using a frequency table. A frequency table lists each piece of data from the survey. A frequency table for the topic of favorite subject might look like this:

If we construct the table before doing the survey, the table can also be used to help collect the data. If we are asking what the favorite subject is, we don't know what the responses will be, but we can add a row for each subject as it is selected. Each time a subject is selected, a tally is added to that row. When the survey is complete, we can total the tallies.

SUBJECT	TALLIES	FREQUENCY
Math		12
Science		4
History		5
Language		2
Art		7

Make sure that the total number of tallies is the same as the number of people in your survey. For instance, in the class above there are thirty students. Was everyone surveyed? Yes, because the tallies add to thirty:

$$12 + 4 + 5 + 2 + 7 = 30$$

What can we conclude from the data in the frequency table? We can see that almost half of the class likes math (12 out of 30; 15 out of 30 would be  $\frac{1}{2}$ ), so it looks like it is the favorite subject.

Let's see how Nutmeg and Pepper constructed their frequency table for the nuts they have gathered for the winter.

Nutmeg and Pepper have collected acorns, walnuts, pecans, and almonds. So Nutmeg made a table with those categories. Pepper began going through the pile as Nutmeg tallied each nut. Then they totaled each category. What can you tell about the nuts that Nutmeg and Pepper collected? Of the 100 nuts they collected, half of the nuts (50) are acorns. There are half as many walnuts (25) as acorns.

NUT	TALLIES	TOTAL
Acorns		50
Walnuts		25
Pecans		5
Almonds		20

**1.26** 10 students were randomly sampled and asked their shoe size. Which line plot displays the data for this sample? 9, 7, 8, 10, 9, 10, 11, 8, 8, 9



**1.27** Which of the following can be found by using a line plot?  
(There may be more than one correct answer.)

- a. range                      b. mode                      c. median                      d. mean

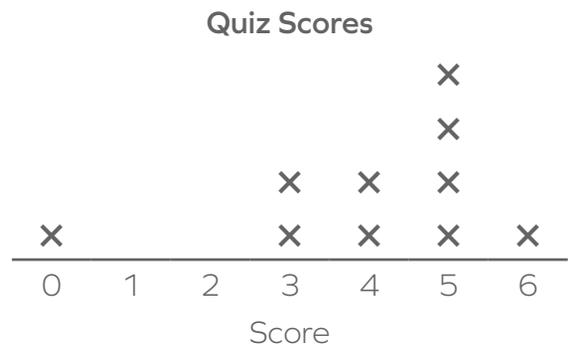
**1.28** A line plot has a range of 4, from 1 to 5, with 5 modes.  
How would you describe the graph?

- a. The graph has an outlier.  
b. The data is clustered around 3.  
c. Each column will be the same height.  
d. There is not enough information.



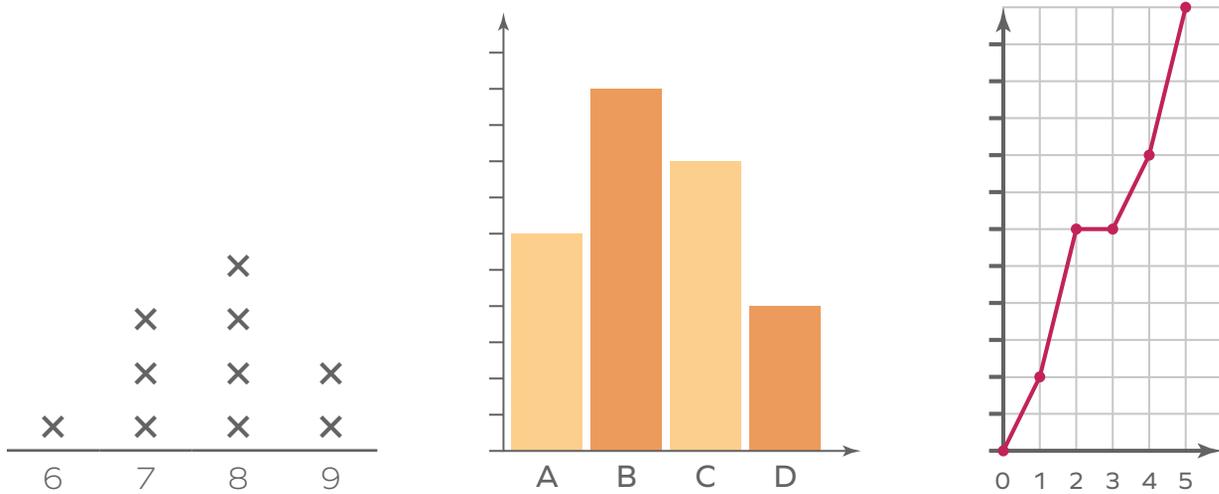
**Using the line plot below, match each measure with its correct value.**

- |             |                  |        |
|-------------|------------------|--------|
| <b>1.29</b> | a. _____ median  | 1. 5   |
|             | b. _____ outlier | 2. 4.5 |
|             | c. _____ mean    | 3. 0   |
|             | d. _____ range   | 4. 6   |
|             | e. _____ mode    | 5. 4   |



## Choosing the Right Graph

You have several tools for displaying data. Now, when you have a situation where you need to analyze a set of data, you must choose which type of graph to use.



In this lesson, we will explore how to decide which type of graph to use, and we'll also review one more type of graph that you've seen before.

## Using a Pictograph to Represent Data

Along with the graphs you've explored so far, there is one more graph called a **pictograph** that we will consider in choosing the right graph for a set of data.

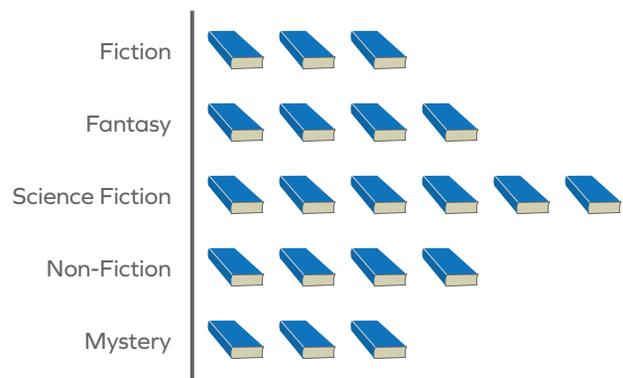
A pictograph uses a picture, or icon, as a unit to show the frequency of categorical data. Let's say we wanted to display the results of a survey on students' favorite type of book. We could use a picture of a book for each data point:

The graph is similar to a line plot because each book (like each x) represents a data point. However, it is also like a bar graph because it is used for categorical data and the books form bars like a bar graph.

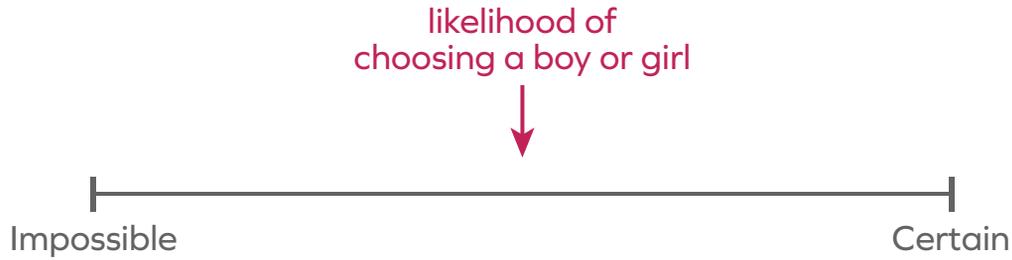
### This might help!

Notice that categories are shown on the left side, and the icons go horizontally. Pictographs and bar graphs can be set up horizontally or vertically, but usually pictographs are arranged horizontally and bar graphs are vertical.

### Favorite Type of Book



A third classroom has an equal number of boys and girls. It is just as likely that the teacher will choose a boy as a girl because there are the same numbers of boys as girls.



If an event is equally as likely to happen as not happen, it is halfway between impossible and certain. We would say that the event is **fair** because a boy is just as likely to be chosen as a girl. If two people played a game that depended on probability, it should be a fair game. It should give each person an equal chance to win.

All events will fall somewhere along the line between impossible and certain.



Let's look at an event and see if we can determine its likelihood.

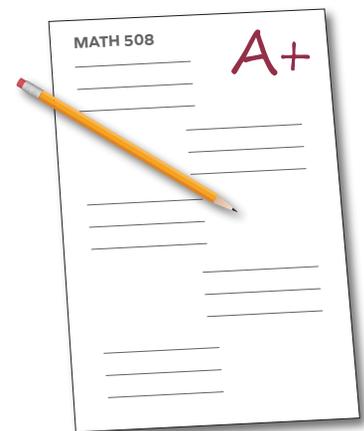
**Example:**

What is the likelihood that you will get an A on your next math test?

**Solution:**

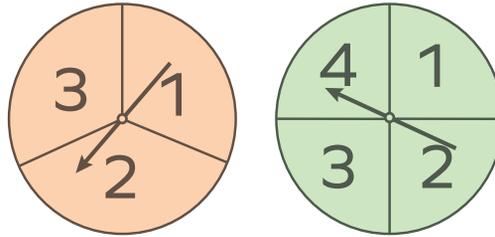
For this event, it depends on you. If you are confident in your math skills and you understand the concepts being taught in class, it is likely you will do well on the next test. If you are not too sure about the concepts being taught, it is unlikely that you'll do well.

For this type of event, there are many factors to consider. For instance, some students understand the math, but just don't do well on tests.



**3.27** If the two spinners below are spun, what is the probability that the numbers will match?

- a.  $\frac{1}{12}$   
 b.  $\frac{1}{6}$   
 c.  $\frac{1}{4}$   
 d.  $\frac{1}{3}$

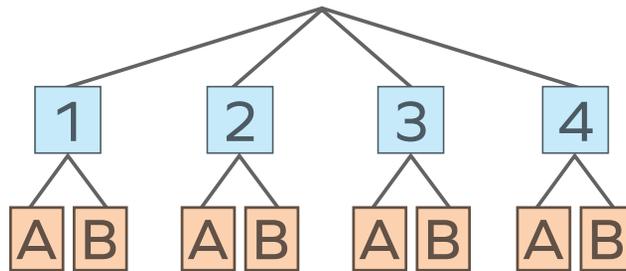


**3.28** If the two spinners from Activity 3.27 are spun, what is the probability that the numbers will add to more than 4?

- a.  $\frac{1}{8}$                       b.  $\frac{1}{6}$                       c.  $\frac{1}{4}$                       d.  $\frac{1}{2}$

**3.29** Here is a tree diagram showing the sample space for two independent events. How many outcomes are there for the first event?

- a. 1  
 b. 2  
 c. 4  
 d. 8



**3.30** Use the tree diagram from Activity 3.29. How many outcomes are there for the second event?

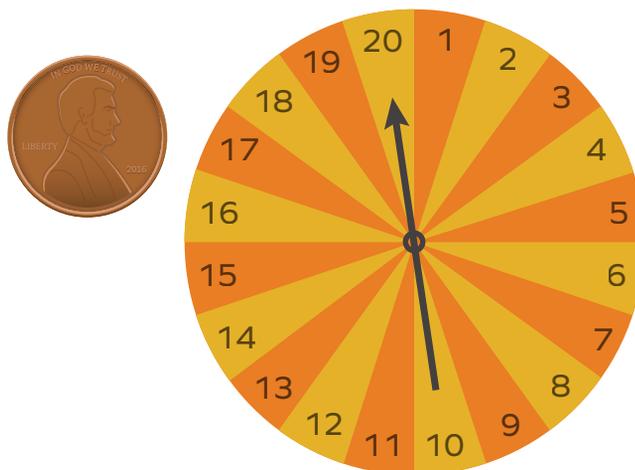
- a. 1                      b. 2                      c. 4                      d. 8

**3.31** Use the tree diagram from Activity 3.29. How many outcomes are there for the *compound* event?

- a. 2                      b. 4                      c. 6                      d. 8

**3.32** If a coin is flipped and then a twenty-part spinner is spun, how many possible outcomes are in the sample space for the compound event?

- a. 20  
 b. 40  
 c. 80  
 d. 400



# MATH 509

## GEOMETRY

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## Classifying Angles as Right, Acute, or Obtuse

We can classify any angle using the three angles we've discussed so far:  $0^\circ$ ,  $90^\circ$ , and  $180^\circ$ . All angles are **acute**, right, **obtuse**, or straight. They are named for the way they relate to  $90^\circ$ .

*Acute angles:* from  $0^\circ$  to  $< 90^\circ$

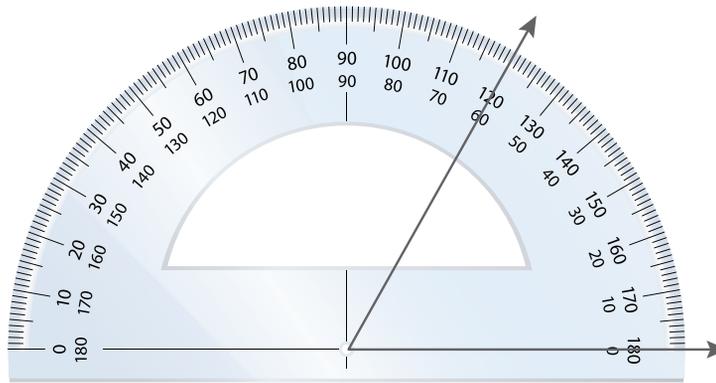
*Right angles:*  $= 90^\circ$

*Obtuse angles:* from  $> 90^\circ$  to  $< 180^\circ$

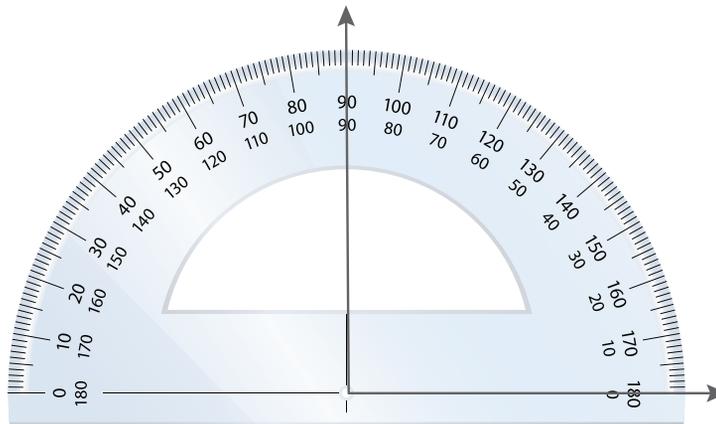
*Straight angles:*  $= 180^\circ$

By comparing any angle to  $0^\circ$ ,  $90^\circ$ , and  $180^\circ$ , we can determine what types of angle it is.

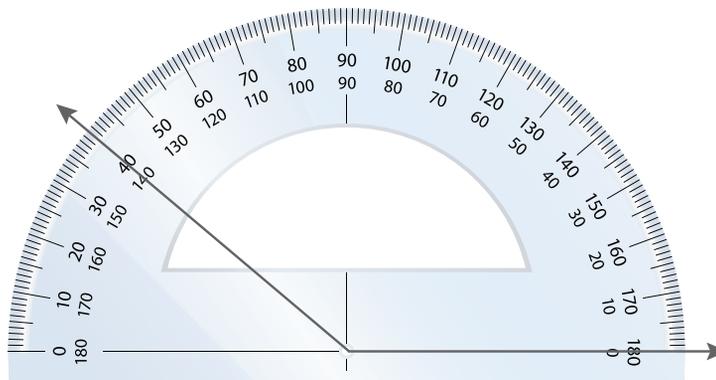
acute



right

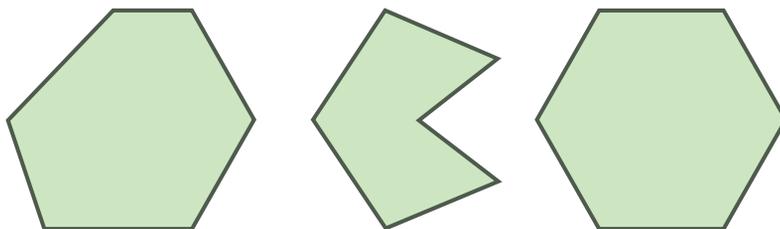


obtuse



**Example:**

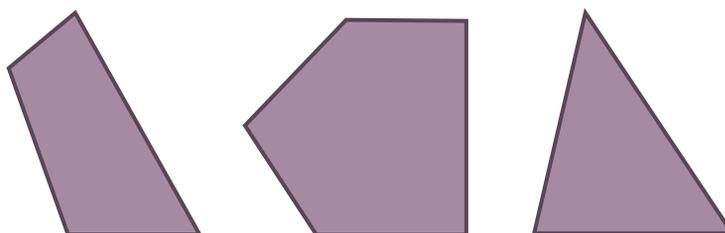
What do these polygons have in common?

**Solution:**

Looking at the characteristics of each polygon, we can see that only one is a regular polygon. Counting the sides though, we can see that each polygon has six sides, so these are all hexagons.

**Example:**

What do these polygons have in common?

**Solution:**

Looking at the characteristics of each polygon, we can see that the polygons do not have the same number of sides (4, 5, and 3). Also, none of the polygons are regular. So, each shape is called an **irregular polygon**.

If we look at all nine of the polygons in the three previous examples, we could sort them by the name of the polygon and whether it is regular or irregular. All of the polygons fit two categories. Can you see why?

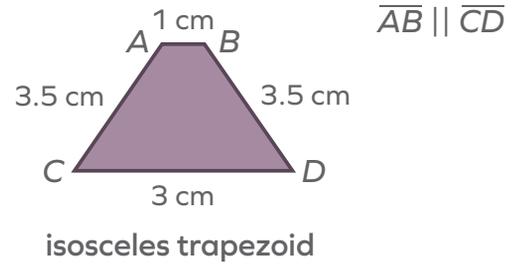
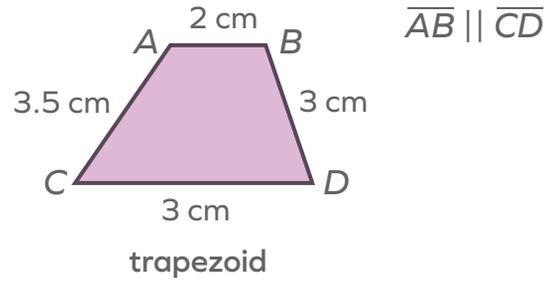
	TRIANGLES	QUADRILATERALS	PENTAGONS	HEXAGONS
REGULAR POLYGONS				
IRREGULAR POLYGONS				

If a quadrilateral has one set of parallel sides, it is called a **trapezoid**. The parallel sides (opposite sides) will not be the same length.

If the pair of sides in a trapezoid that are not parallel are the same length, it is called an **isosceles trapezoid**.

**S-T-R-E-T-C-H**

Can you see how an isosceles trapezoid is similar to an isosceles triangle, and why they have similar names?



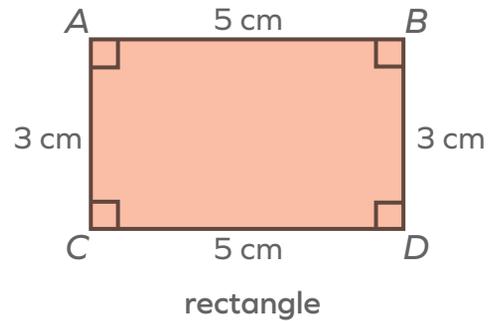
**Classifying Quadrilaterals by Their Angles**

**Angles:** When we look at the angles of a quadrilateral as an attribute, we are looking at whether the quadrilateral has  $90^\circ$  angles (right angles) or not.

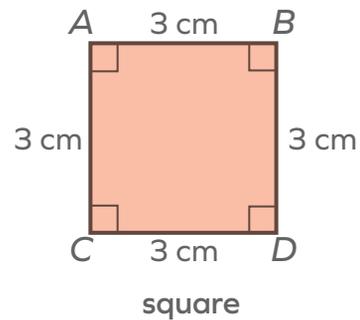
If a parallelogram has four right angles, it is called a **rectangle**.

If all four sides of a rectangle are the same length, it is called a **square**. A square can also be called a regular quadrilateral.

$$\overline{AB} \parallel \overline{CD}, \overline{AC} \parallel \overline{BD}$$

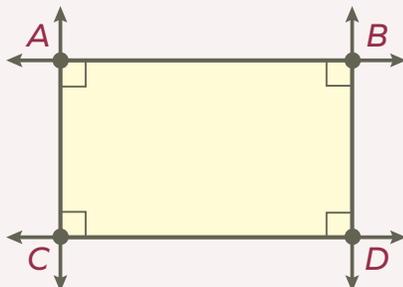


$$\overline{AB} \parallel \overline{CD}, \overline{AC} \parallel \overline{BD}$$



**Key point!**

Remember,  $90^\circ$  angles are formed when two lines are perpendicular to each other. So, in a rectangle, since the sides form  $90^\circ$  angles, they are like line segments of perpendicular lines.



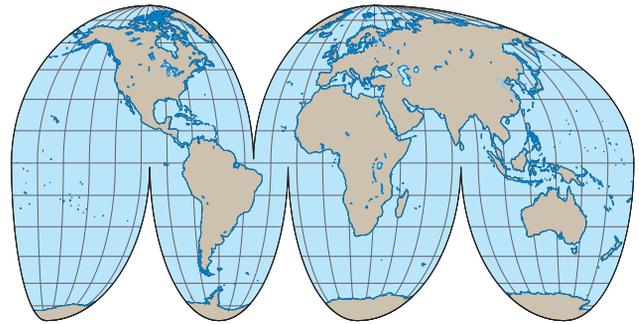
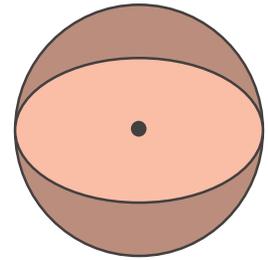
There is one more three-dimensional figure that involves circles. Imagine if you took a circle and spun it along its diameter ... you would have a sphere.

Whereas a circle is defined as all the points in a plane that are a given distance from the center, a sphere is defined as all the points in *space* that are a given distance from the center.

It is very difficult to draw the net of a sphere because all its surfaces are curved. There are no lateral surfaces, edges, or vertices.

Have you ever tried to peel an orange in one piece? Even if you can do this, you can't flatten the peel without tearing it. The problem of a sphere net has been hard to solve for world mapmakers. Translating a curved surface to a flat surface is very difficult and distorts the sphere. So, in geometry we don't use the net of a sphere.

Many everyday objects use cylinders, cones, and spheres. An ice cream cone, with ice cream, is a cone with a sphere inside.



### Example:

What solid figure(s) is/are used in each of these objects?



### Solution:

The tennis ball can is a cylinder with two circular bases. The balls are spheres.

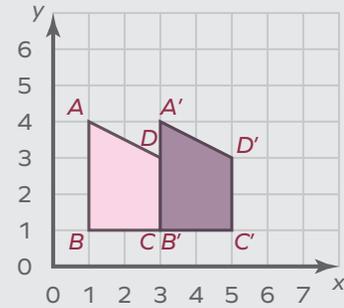
The capsules are a combination of cylinders, with the halves of spheres on either end.

The party hat is a cone with the circular base removed.



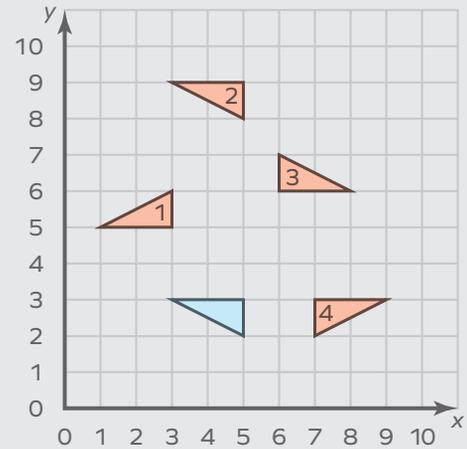
17. What transformation is shown here?

- a. translation
- b. reflection
- c. rotation
- d. can't be determined



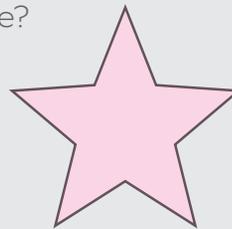
18. Which triangle is a reflection of the blue triangle?

- a.  $\triangle 1$
- b.  $\triangle 2$
- c.  $\triangle 3$
- d.  $\triangle 4$

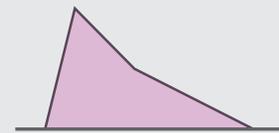


19. How many lines of symmetry does this figure have?

- a. 2
- b. 3
- c. 5
- d. 10



20. If the half-figure shown here is reflected over the line of symmetry, what will the completed figure look like?

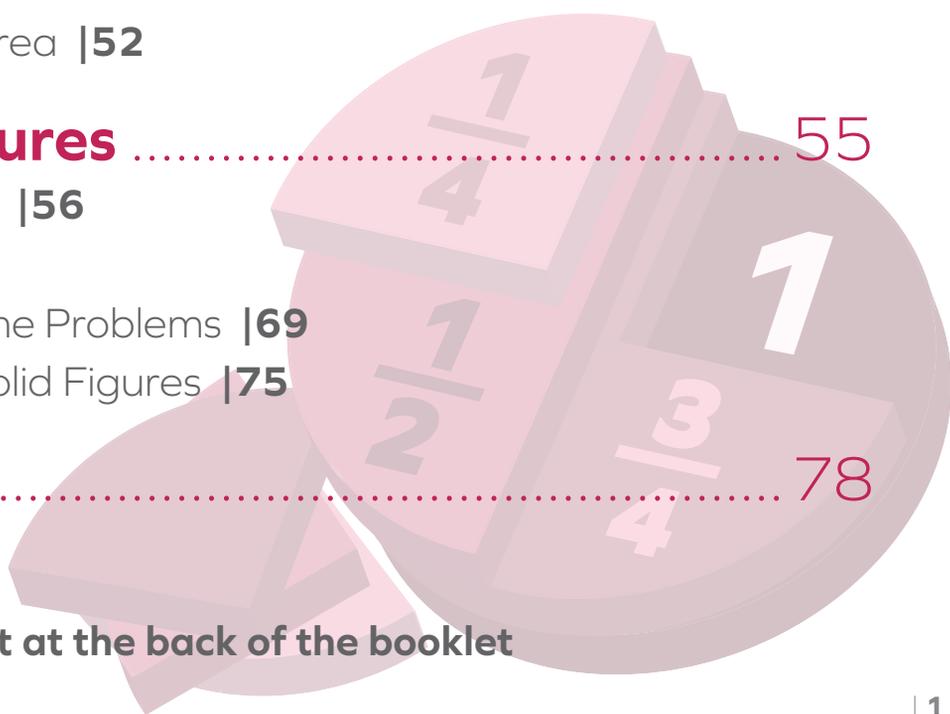


- a.
- b.
- c.
- d.

# MATH 510

## PERIMETER, AREA, AND VOLUME

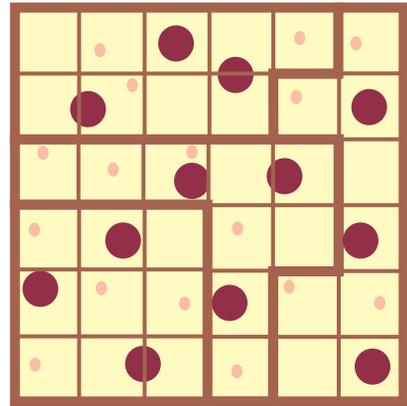
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Test	Pull-out at the back of the booklet



Let's take a look at the pizza from the start of the lesson. We'll add a grid to make it easier to find the area.

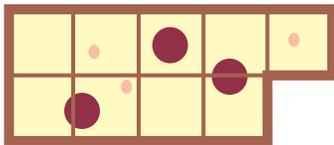
**Example:**

Which piece of pizza is the largest?

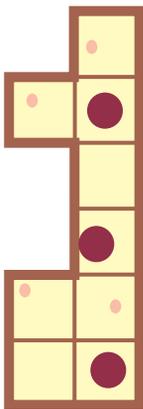


**Solution:**

To find the largest piece, we'll find the area of each piece by counting the number of square units.



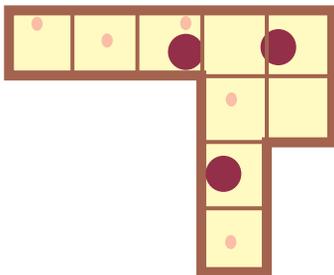
Counting the squares inside the figure, we can see that there are 9. So the area of the piece is 9 square units.



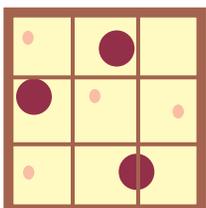
The area of this piece of pizza is also 9 square units, because there are 9 squares inside.

**Key point!**

Different shapes can have the same area. The shape of a figure can be deceptive and does not necessarily give us an idea of its area.



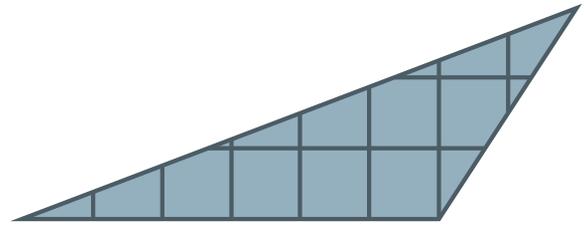
This piece of the pizza is also 9 square units. Counting the squares inside the figure, we can see that there are 9.



So, all four of the pieces have the same area. It doesn't matter which piece you choose!

## Triangles

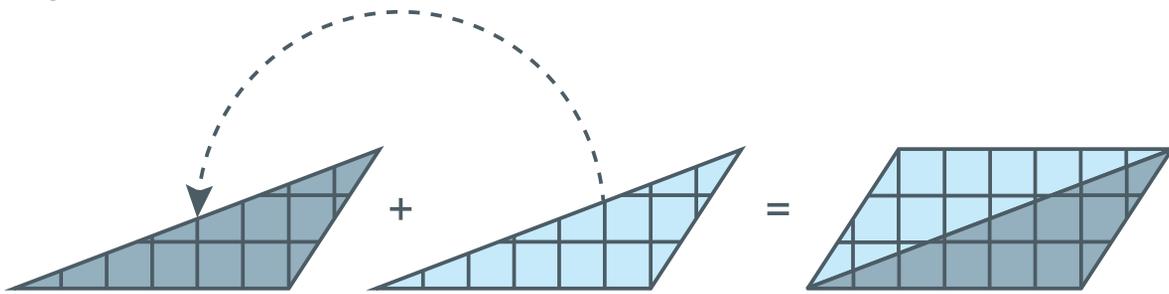
How would you find the area of this triangle? You know how to find the area of a plane figure by counting the number of square units inside of the figure. However, that would be very difficult to do with this triangle because there are many partial squares.



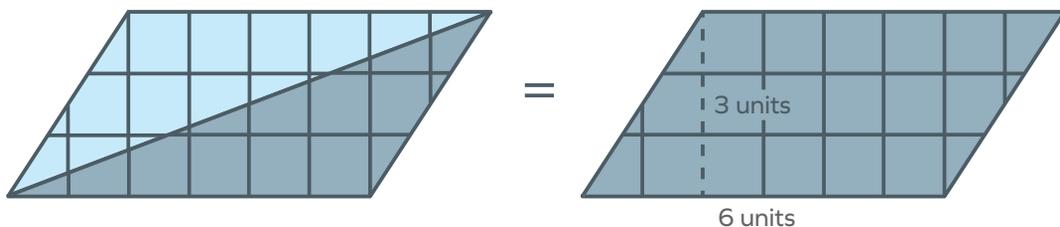
Surprisingly, this triangle (or any triangle) is related to a parallelogram. We know how to find the area of a parallelogram, so this is a clue to finding the area of a triangle. In this lesson, we will learn how to find the area of a triangle based on the area of a parallelogram.

### Finding the Area of Triangles

How are triangles related to parallelograms? Two congruent triangles will combine to form a parallelogram.



If two congruent triangles combine to form a parallelogram, then the area of the parallelogram must be two times the area of the triangle. Another way to say this is that the area of the triangle is half of the area of the parallelogram.



To find the area of the parallelogram, we multiply the base by the height:

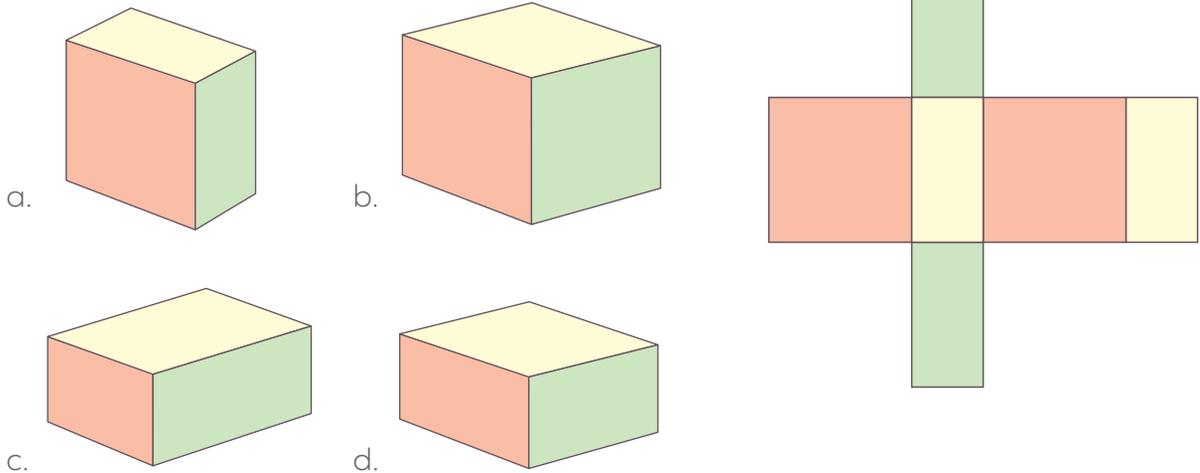
$$6 \text{ units} \times 3 \text{ units} = 18 \text{ square units}$$

Because the area of the triangle is half of the area of the parallelogram, we can divide the area by two:

$$18 \text{ square units} \div 2 = 9 \text{ square units}$$

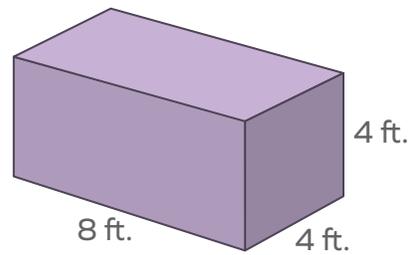
The area of the triangle is 9 square units.

**3.4** Which rectangular prism matches the net shown here?



**3.5** What is the surface area of the rectangular prism shown here?

- a.  $64 \text{ ft}^2$
- b.  $128 \text{ ft}^2$
- c.  $160 \text{ ft}^2$
- d.  $256 \text{ ft}^2$

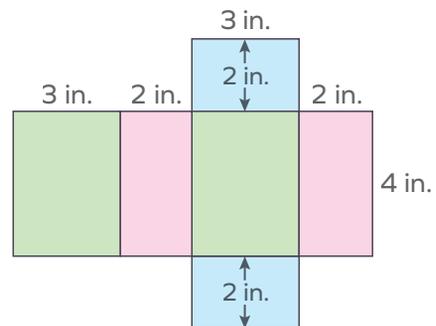


**3.6** The area of the front face of a rectangular prism is  $12 \text{ cm}^2$ . The area of the top face is  $16 \text{ cm}^2$ , and the area of the right side face is  $20 \text{ cm}^2$ . What is the surface area of the rectangular prism?

- a.  $56 \text{ cm}^2$
- b.  $48 \text{ cm}^2$
- c.  $60 \text{ cm}^2$
- d.  $96 \text{ cm}^2$

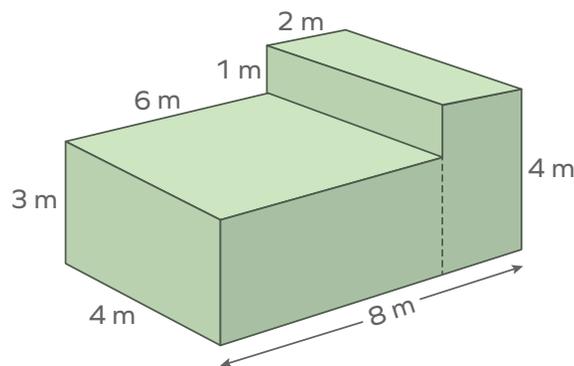
**3.7** Given the net shown here, what is the surface area of the rectangular prism?

- a.  $24 \text{ in}^2$
- b.  $40 \text{ in}^2$
- c.  $52 \text{ in}^2$
- d.  $60 \text{ in}^2$



**3.22** In the solid figure shown here, what is the volume of the left section of the figure?

- a.  $12 \text{ m}^3$
- b.  $72 \text{ m}^3$
- c.  $84 \text{ m}^3$
- d.  $96 \text{ m}^3$



**3.23** In the solid figure from Activity 3.22, what is the volume of the right section of the figure?

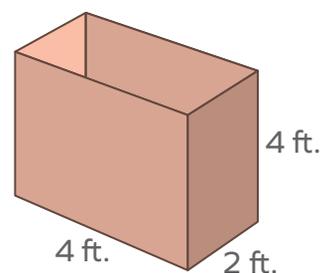
- a.  $8 \text{ m}^3$
- b.  $16 \text{ m}^3$
- c.  $24 \text{ m}^3$
- d.  $32 \text{ m}^3$

**3.24** What is the volume of the solid figure from Activity 3.22?

- a.  $28 \text{ m}^3$
- b.  $96 \text{ m}^3$
- c.  $104 \text{ m}^3$
- d.  $128 \text{ m}^3$

**3.25** Doug digs a hole, shown here, for the foundation of a wall. How much dirt was removed to make the hole?

- a.  $10 \text{ ft}^3$
- b.  $16 \text{ ft}^3$
- c.  $32 \text{ ft}^3$
- d.  $64 \text{ ft}^3$



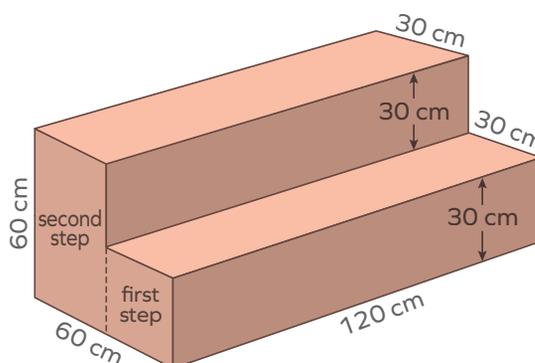
**3.26** If a semi truck's trailer is 40 feet long, 8 feet wide, and has a height of 10 feet, what is its volume?



- a.  $400 \text{ ft}^2$
- b.  $480 \text{ ft}^3$
- c.  $3,200 \text{ ft}^3$
- d.  $32,000 \text{ ft}^3$

**3.27** Linda is building steps for her porch. What is the volume of the first step?

- a.  $9,000 \text{ cm}^3$
- b.  $36,000 \text{ cm}^3$
- c.  $72,000 \text{ cm}^3$
- d.  $108,000 \text{ cm}^3$





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