math MAMMOTH

Grade 6-A Worktext

Light Blue Series

Review of the basic operations

Expressions and equations

Decimals

Ratios

Percent

By Maria Miller

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Chapter 4: Ratios

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Foreword

Math Mammoth Grade 6-A and Grade 6-B worktexts comprise a complete math curriculum for the sixth grade mathematics studies, aligned to the Common Core Standards.

In sixth grade, students encounter the beginnings of algebra, learning about algebraic expressions, one-variable equations and inequalities, integers, and ratios. We also review and deepen the students' understanding of rational numbers: both fractions and decimals are studied in depth, while percent is a new topic for 6th grade. In geometry, students learn to compute the area of various polygons, and also calculate volume and surface area of various solids. The last major area of study is statistics, where students learn to summarize and describe distributions using both measures of center and variability.

The year starts out with a review of the four operations with whole numbers (including long division), place value, and rounding. Students are also introduced to exponents and do some problem solving.

Chapter 2 starts the study of algebra topics, delving first into expressions and equations. Students practice writing expressions in many different ways, and use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple one-step equations. We also study briefly inequalities and using two variables.

Chapter 3 has to do with decimals. This is a long chapter, as wee review all of decimal arithmetic, just using more decimal digits than in 5th grade. Students also convert measuring units in this chapter.

Ratios is a new topic (chapter 4). Students are already familiar with finding fractional parts from earlier grades, and now it is time to advance that knowledge into the study of ratios, which arise naturally from dividing a quantity into many equal parts. We study such topics as rates, unit rates, equivalent ratios, and problem solving using bar models.

Percent (chapter 5) is an important topic to understand thoroughly, because of it many applications in real life. The goal of this chapter is to develop a basic understanding of percent, to see percentages as decimals, and to learn to calculate discounts.

In part 6-B, students study number theory, fractions, integers, geometry, and statistics.

I wish you success in teaching math!

Maria Miller, the author

Chapter 1: Review of the Basic Operations
Introduction

The goal of the first chapter in grade 6 is to review the four basic operations with whole numbers, place value, and rounding, and to learn about exponents and problem solving.

A lot of this chapter is review, and I hope this provides a gentle start for 6th grade math. In the next chapter, we will delve into some beginning algebra topics.

The Lessons in Chapter 1

Helpful Resources on the Internet

Long division

Snork's Long Division Game
Interactive and guided long division practice that only accepts correct answers and truly guides the student step-by-step through long division problems. In the beginning, choose the highest number you want to work with (the divisor) to be a two-digit number, in order to practice with two-digit divisors.
http://www.kidsnumbers.com/long-division.php

Mr. Martini's Classroom: Long Division
An interactive long division tool.
http://www.thegreatmartinicompany.com/longarithmetic/longdivision.html

Short Division
A page that explains short division in detail. Short division is the same algorithm as long division, but some steps are only done in one's head, not written down.
http://www.themathpage.com/ARITH/divide-whole-numbers.htm

All four operations

Math Mahjong
A Mahjong game where you need to match tiles with the same value. It uses all four operations and has three levels.
http://www.sheppardsoftware.com/mathgames/mixed_mahjong/mahjongMath_Level_1.html

Pop the Balloons
Pop the balloons in the order of their value. You need to use all four operations.
http://www.sheppardsoftware.com/mathgames/numberballoons/BalloonPopMixed.htm

MathCar Racing
Keep ahead of the computer car by thinking logically, and practice the four operations at the same time.
http://www.funbrain.com/cgi-bin/osa.cgi?A1=s&A2=4

Calculator Chaos
Most of the keys have fallen off the calculator but you have to make certain numbers using the keys that are left.
http://www.mathplayground.com/calculator_chaos.html

ArithmeTiles
Use the four operations and numbers on neighboring tiles to make target numbers.
http://www.primarygames.com/math/arithmetiles/index.htm

SpeedMath Deluxe
Create an equation from the four given digits using addition, subtraction, multiplication and division. Make certain that you remember the order of operations. Includes negative numbers sometimes.
http://education.jlab.org/smdeluxe/index.html

Place value

Numbers
Practice place value, comparing numbers, and ordering numbers with this interactive online practice.
http://www.aaamath.com/B/grade6.htm#topic3

Megapenny Project
Visualizes big numbers with pictures of pennies.
http://www.kokogiak.com/megapenny/default.asp

Powers of Ten
A 9-minute movie that illustrates the dramatic changes of scale when zooming in or out by powers of ten (40 powers of ten), starting from a picnic blanket and ending in the universe, and then starting from a hand to the proton inside an atom.
http://www.youtube.com/watch?v=0fKBhvDjuy0

Keep My Place
Fill in the big numbers to this cross-number puzzle.
http://www.counton.org/magnet/kaleidoscope2/Crossnumber/index.html

Estimation at AAA Math
Exercises about rounding whole numbers and decimals, front-end estimation, estimating sums and differences. Each page has an explanation, interactive practice, and games.
http://www.aaamath.com/B/est.htm

Place Value Game
Create the largest possible number from the digits the computer gives you. Unfortunately, the computer will give you each digit one at a time and you won't know what the next number will be.
http://education.jlab.org/placevalue/index.html

Free Exponent Worksheets
Create a variety of customizable, printable worksheets to practice exponents.
http://www.homeschoolmath.net/worksheets/exponents.php

Baseball Exponents
Choose the right answer from three possibilities before the pitched ball comes.
http://www.xpmath.com/forums/arcade.php?do=play&gameid=95

Exponents Quiz from ThatQuiz.org
Ten questions, fairly easy, and not timed. You can change the parameters as you like to include negative bases, square roots, and even logarithms.
http://www.thatquiz.org/tq-2/?-j1-l4-p0

Exponents Jeopardy
The question categories include evaluating exponents, equations with exponents, and exponents with fractional bases.
http://www.math-play.com/Exponents-Jeopardy/Exponents-Jeopardy.html

Pyramid Math
Simple practice of either exponents, roots, LCM, or GCF. Drag the triangle with the right answer to the vase.
http://www.mathnook.com/math/pyramidmath.html

Exponents Battleship
A regular battleship game against the computer. Each time you "hit", you need to answer a math problem involving exponents (and multiplication).
http://www.quia.com/ba/1000.html

Exponent Battle
A card game to practice exponents. I would limit the cards to small numbers, instead of using the whole deck.
http://www.learn-with-math-games.com/exponent-game.html

Pirates Board Game
Steer your boat in pirate waters in this online board game, and evaluate powers.
http://mathgames4children.com/fun-board-games/6th-grade/pirate/exponents-pirate-waters-grade-6-game.html

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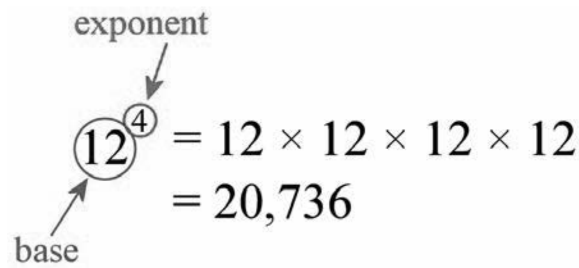
Powers and Exponents

Exponents are a "shorthand" for writing repeated multiplications by the same number.

For example, $2 \times 2 \times 2 \times 2 \times 2$ is written 2^5.
$5 \times 5 \times 5 \times 5 \times 5 \times 5$ is written 5^6.

The tiny raised number is called the *exponent*. It tells us how many times the *base* number is multiplied by itself.



The expression 2^5 is read as "two to the fifth power," "two to the fifth," or "two raised to the fifth power."

Similarly, 7^9 is read as "seven to the ninth power," "seven to the ninth," or "seven raised to the ninth power."

The "powers of 6" are simply expressions where 6 is raised to some power: For example, 6^3, 6^4, 6^{45}, and 6^{99} are powers of 6. What would powers of 10 be?

Expressions with the exponent 2 are usually read as something "**squared.**" For example, 11^2 is read as "**eleven squared.**" That is because it gives us the <u>area of a square</u> with the side length of 11 units.

Similarly, if the exponent is 3, the expression is usually read using the word "**cubed.**" For example, 31^3 is read as "**thirty-one cubed**" because it gives the <u>volume of a cube</u> with the edge length of 31 units.

1. Write the expressions as multiplications, and then solve them using mental math.

a. $3^2 =$ $3 \times 3 = 9$

b. 1^6

c. 4^3

d. 10^4

e. 5^3

f. 10^2

g. 2^3

h. 8^2

i. 0^5

j. 10^5

k. 50^2

l. 100^3

2. Rewrite the expressions using an exponent, then solve them. You may use a calculator.

a. $2 \times 2 \times 2 \times 2 \times 2 \times 2$

b. $8 \times 8 \times 8 \times 8 \times 8$

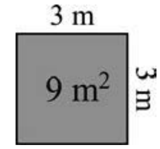
c. 40 squared

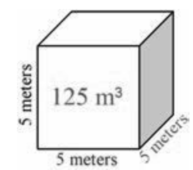
d. $10 \times 10 \times 10 \times 10$

e. nine to the eighth power

f. eleven cubed

You just learned that the expression 7^2 is read “seven *squared*” because it tells us the area of a *square* with a side length of 7 units. Let’s compare that to square meters and other units of area. If the sides of a square are 3 m long, then its area is 3 m × 3 m = 9 m^2 or nine square meters. Notice that the symbol for square meters is $\mathbf{m^2}$. This means “**meter × meter**.” We are, in effect, squaring the unit *meter* (multiplying the unit of length *meter* by itself)! Or, in the expression 9 cm × 9 cm, we multiply 9 by itself, but we also multiply the unit *cm* by itself. That is why the result is **81** $\mathbf{cm^2}$, and the square centimeter (cm^2) comes from multiplying “**centimeter × centimeter**.”
We do the same thing with any other unit of length to form the corresponding unit for area, such as square kilometers or square millimeters. With the customary units of area, such as square inches, square feet, and square miles, people often write “sq. in.”, “sq. ft.”, or “sq. mi.” , instead of $\mathbf{in^2}$, $\mathbf{ft^2}$, and $\mathbf{mi^2}$. Both ways are correct.
In a similar way, to calculate the volume of this cube, we multiply 5 m × 5 m × 5 m = 125 m^3. We not only multiply 5 by itself three times, but also multiply the unit *meter* by itself three times (meter × meter × meter) to get the unit of volume “cubic meter” or m^3.





3. Express the area (A) as a multiplication, and solve.

a. A square with a side of 12 kilometers: A = 12 km × 12 km = ______________	**b.** A square with sides 6 m long: A = ________________________________
c. A square with a side length of 6 inches: A = ________________________________	**d.** A square with a side with a length of 12 ft: A = ________________________________

4. Express the volume (V) as a multiplication, and solve.

a. A cube with a side of 2 cm: V = 2 cm × 2 cm × 2 cm = ____________	**b.** A cube with sides each 10 inches long: V = ________________________________
c. A cube with sides 1 ft in length: V = ________________________________	**d.** A cube with edges that are all 5 m long: V = ________________________________

5. **a.** The perimeter of a square is 40 cm. What is its area?

b. The volume of a cube is 64 cubic inches. How long is its edge?

c. The area of a square is 121 m^2. What is its perimeter?

d. The volume of a cube is 27 cm^3. What is its edge length?

The powers of 10 are very special —and very easy! Notice that the exponent tells us *how many zeros* there are in the answer.	$10^1 = 10$ $10^2 = 10 \times 10 = 100$ $10^3 = 10 \times 10 \times 10 = 1{,}000$	$10^4 = 10{,}000$ $10^5 = 100{,}000$ $10^6 = 1{,}000{,}000$

6. Fill in the patterns. In part (d), choose your own number to be the base.
   Use a calculator in parts (c) and (d).

a.	**b.**	**c.**	**d.**
$2^1 =$	$3^1 =$	$5^1 =$	
$2^2 =$	$3^2 =$	$5^2 =$	
$2^3 =$	$3^3 =$	$5^3 =$	
$2^4 =$	$3^4 =$	$5^4 =$	
$2^5 =$	$3^5 =$	$5^5 =$	
$2^6 =$	$3^6 =$	$5^6 =$	

7. Look at the patterns above. Think carefully how each step comes from the previous one. Then answer.

   a. If $3^7 = 2{,}187$, how can you use that result to find 3^8 ?

   b. Now find 3^8 without a calculator.

   c. If $2^{45} = 35{,}184{,}372{,}088{,}832$, use that to find 2^{46} without a calculator.

8. Fill in.

   a. 17^2 gives us the _____________ of a _______________ with a side length of ______ units.

   b. 101^3 gives us the _____________ of a _______________ with an edge length of ______ units.

   c. 2×6^2 gives us the ____________ of two _______________ with a side length of ______ units.

   d. 4×10^3 gives us the ____________ of ____ _______________ with an edge length of ______ units.

Puzzle Corner

Make a pattern, called a *sequence*, with the powers of 2, starting with 2^6 and going *backwards* to 2^0. At each step, *divide* by 2. What is the logical (though surprising) value for 2^0 from this method?

Make another, similar, sequence for the powers of 10. Start with 10^6 and divide by 10 until you reach 10^0. What value do you calculate for 10^0?

Try this same pattern for at least one other base number, *n*. What value do you calculate for n^0?
Do you think it will come out this way for every base number?
Why or why not?

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Chapter 2: Expressions and Equations
Introduction

In this chapter we concentrate on two important concepts: expressions and equations. We also touch on inequalities and graphing on a very introductory level. In order to make the learning of these concepts easier, the expressions and equations in this chapter do not involve negative numbers (as they typically do when studied in pre-algebra and algebra). The study of negative numbers is in part 6-B.

We start out by learning some basic vocabulary used to describe mathematical expressions verbally—terms such as the sum, the difference, the product, the quotient, and the quantity. Next, we study the order of operations once again. A lot of this lesson is review. The lesson *Multiplying and Dividing in Parts* is also partially review and is leading up to the lesson on distributive property that follows later.

Then, we get into studying expressions in definite terms: students encounter the exact definition of an expression, a variable, and a formula, and practice writing expressions in many different ways.

The concepts of equivalent expressions and simplifying expressions are important. If you can simplify an expression in some way, the new expression you get is equivalent to the first. We study these ideas first using lengths— it is a concrete example, and hopefully easy to grasp.

In the lesson *More On Writing and Simplifying Expressions* students encounter more terminology: term, coefficient, and constant. In exercise #3, they write an expression for the perimeter of some shapes in two ways. This exercise is once again preparing them to understand the distributive property.

Next, students write and simplify expressions for the area of rectangles and rectangular shapes. Then we study the distributive property, concentrating on the symbolic aspect and tying it in with area models.

The next topic is equations. Students learn some basics, such as, the solutions of an equation are the values of the variables that make the equation true. They use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple one-step equations. I have also included a few two-step equations as an optional topic.

Lastly, in this chapter students get to solve and graph simple inequalities, and study the usage of two variables and graphing.

The Lessons in Chapter 2

Helpful Resources on the Internet

Calculator Chaos
Most of the keys have fallen off the calculator but you have to make certain numbers using the keys that are left.
http://www.mathplayground.com/calculator_chaos.html

ArithmeTiles
Use the four operations and numbers on neighboring tiles to make target numbers.
http://www.primarygames.com/math/arithmetiles/index.htm

Choose Math Operation
Choose the mathematical operation(s) so that the number sentence is true. Practice the role of zero and one in basic operations or operations with negative numbers. Helps develop number sense and logical thinking.
http://www.homeschoolmath.net/operation-game.php

Order of Operations Quiz
A 10-question online quiz that includes two different operations and possibly parenthesis in each question. You can also modify the quiz parameters yourself.
http://www.thatquiz.org/tq-1/?-j8f-la

The Order of Operations Millionaire
Answer multiple-choice questions that have to do with the order of operations, and win a million. Can be played alone or in two teams.
http://www.math-play.com/Order-of-Operations-Millionaire/order-of-operations-millionaire.html

Exploring Order of Operations (Object Interactive)
The program shows an expression, and you click on the correct operation (either +, —, ×, ÷ or exponent) to be done first. The program then solves that operation, and you click on the *next* operation to be performed, etc., until it is solved. Lastly the resource includes a game where you click on the falling blocks in the order that order of operations would dictate.
http://www.learnalberta.ca/content/mejhm/html/object_interactives/order_of_operations/use_it.html

Order of Operations Practice
A simple online quiz of 10 questions. Uses parenthesis and the four operations.
http://www.onlinemathlearning.com/order-of-operations-practice.html

Fill and Pour
Fill and pour liquid with two containers until you get the target amount. A logical thinking puzzle.
http://nlvm.usu.edu/en/nav/frames_asid_273_g_2_t_4.html

Balance Beam Activity
A virtual balance that provides balance puzzles where student is to find the weights of various figures, practicing algebraic thinking. Includes three levels.
http://mste.illinois.edu/users/pavel/java/balance/index.html

Algebraic Expressions Millionaire
For each question you have to identify the correct mathematical expression that models a given word expression.
http://www.math-play.com/Algebraic-Expressions-Millionaire/algebraic-expressions-millionaire.html

Escape Planet
Choose the equation that matches the words.
http://www.harcourtschool.com/activity/escape_planet_6/

BuzzMath Practice - Algebraic Expressions
Online practice for simplifying different kinds of algebraic expressions.
http://www.mathplayground.com/practice.php?topic=algebraic-expressions

Expressions : Expressions and Variables Quiz
Choose an equation to match the word problem or situation.
http://www.softschools.com/quizzes/math/expressions_and_variables/quiz815.html

Equation Match
Playing on level 1, you need to match simple equations based on them having the same solution.
http://www.bbc.co.uk/schools/mathsfile/shockwave/games/equationmatch.html

Algebraic Reasoning
Find the value of an object based on two scales.
http://www.mathplayground.com/algebraic_reasoning.html

Algebra Puzzle
Find the value of each of the three objects presented in the puzzle. The numbers given represent the sum of the objects in each row or column.
http://www.mathplayground.com/algebra_puzzle.html

Battleship
Choose the right solution for a 1-step equation every time you hit the enemy's ship. Some of the equations involve negative solutions; however since the game is interesting, some students might be willing to play it anyway (you can always guess at the right solution since it is a multiple choice game).
http://www.quia.com/ba/36544.html

Algebra Meltdown
Solve simple equations using function machines to guide atoms through the reactor. But don't keep the scientists waiting too long or they blow their tops. Again, includes negative numbers.
http://www.mangahigh.com/en/games/algebrameltdown

Words into Equations Battleship Game
Practice expressions such as quotient, difference, product, and sum.
http://www.quia.com/ba/210997.html

Balance when Adding and Subtracting Game
The interactive balance illustrates simple equations. Your task is to add or subtract x's, and add or subtract 1's until you have x alone on one side.
http://www.mathsisfun.com/algebra/add-subtract-balance.html

Algebra Balance Scales
Similar to the one above, but you need to first put the x's and 1's in the balance to match the given equation.
http://nlvm.usu.edu/en/nav/frames_asid_201_g_4_t_2.html — only positive numbers
http://nlvm.usu.edu/en/nav/frames_asid_324_g_4_t_2.html — includes negative numbers

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The Distributive Property

The **distributive property** states that $a(b + c) = ab + ac$

It may look like a meaningless or difficult equation to you now, but do not worry, it will become clearer!

The equation $\boldsymbol{a(b + c) = ab + ac}$ means that you can *distribute* the multiplication (by a) over the sum ($b + c$) so that you multiply the numbers b and c separately by a, and lastly, add.

You have already used it! For example, think of $3 \cdot 84$ as $3 \cdot (80 + 4)$. You can then multiply 80 and 4 *separately by 3*, and lastly, add: $3 \cdot 80 + 3 \cdot 4 = 240 + 12 = 252$. We called this the partial products or multiplying in parts.

Example 1. Using the distributive property, we can write the product $2(x + 1)$ as $2x + 2 \cdot 1$, which simplifies to $2x + 2$.

Notice what happens: Each term in the sum ($x + 1$) gets multiplied by the factor 2! Graphically:

$$2(x + 1) = \underline{2x} + \underline{2 \cdot 1}$$

Example 2. To multiply $s \cdot (3 + t)$ using the distributive property, we need to multiply **<u>both</u>** 3 and t by s:

$s \cdot (3 + t) = s \cdot 3 + s \cdot t$, which simplifies to $3s + st$.

1. Multiply using the distributive property.

a. $3(90 + 5) = 3 \cdot$ ___ $+ 3 \cdot$ ___ $=$	**b.** $7(50 + 6) = 7 \cdot$ ___ $+ 7 \cdot$ ___ $=$
c. $4(a + b) = 4 \cdot$ ___ $+ 4 \cdot$ ___ $=$	**d.** $2(x + 6) = 2 \cdot$ ___ $+ 2 \cdot$ ___ $=$
e. $7(y + 3) =$	**f.** $10(s + 4) =$
g. $s(6 + x) =$	**h.** $x(y + 3) =$
i. $8(5 + b) =$	**j.** $9(5 + c) =$

Example 3. We can use the distributive property also when the sum has three or more terms. Simply multiply **each term** in the sum by the factor in front of the parentheses:

$5(x + y + 6) = 5 \cdot x + 5 \cdot y + 5 \cdot 6$, which simplifies to $5x + 5y + 30$

2. Multiply using the distributive property.

a. $3(a + b + 5) =$	**b.** $8(5 + y + r) =$
c. $4(s + 5 + 8) =$	**d.** $3(10 + c + d + 2) =$

Example 4. Now, one of the terms in the sum has a coefficient (the 2 in $2x$):

$$6(2x + 3) = 6 \cdot 2x + 6 \cdot 3 = 12x + 18$$

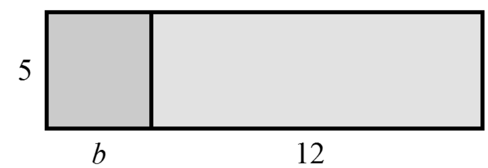
3. Multiply using the distributive property.

a. $2(3x + 5) =$	**b.** $7(7a + 6) =$
c. $5(4a + 8b) =$	**d.** $2(4x + 3y) =$
e. $3(9 + 10z) =$	**f.** $6(3x + 4 + 2y) =$
g. $11(2c + 7a) =$	**h.** $8(5 + 2a + 3b) =$

To understand even better why the the distributive property works, let's look at an area model (this, too, you have seen before!).

The area of the whole rectangle is 5 times $(b + 12)$.

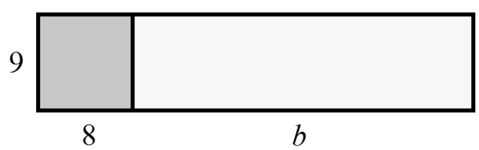
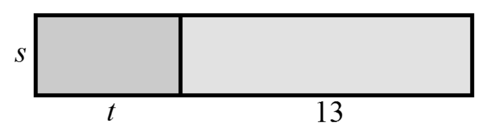
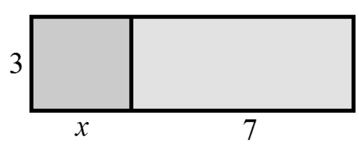
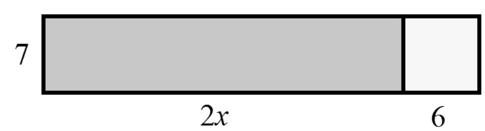
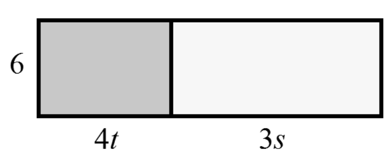
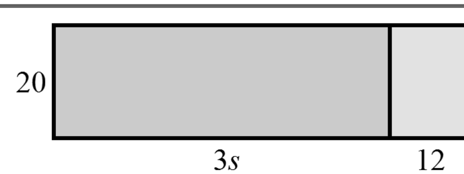
But, if we think of it as *two* rectangles, the area of the first rectangle is $5b$, and of the second, $5 \cdot 12$.



Of course, these two expressions have to be equal:

$$5 \cdot (b + 12) = 5b + 5 \cdot 12 = 5b + 60$$

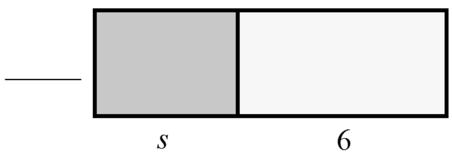
4. Write an expression for the area in two ways, thinking of one rectangle or two.

 **a.** 9(_____ + _____) and 9 · _____ + 9 · _____ =	 **b.** *s*(_____ + _____) and *s* · _____ + *s* · _____ =
 **c.** ____ (_____ + _____) and	 **d.**
 **e.**	 **f.**

5. Find the missing number or variable in these area models.

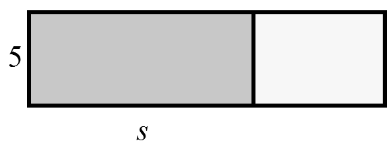
a. ____ $(x + 2) = 3x + 6$

b. _____ $(t + 8) = 7t + 56$

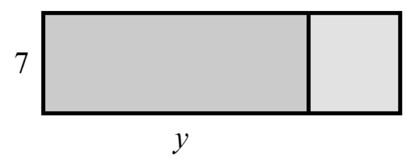


c. The total area is $9s + 54$.

d. $4($ ____ $+ 5) = 4z + 20$



e. $5(s +$ ____ $) = 5s + 30$



f. The total area is $7y + 42$.

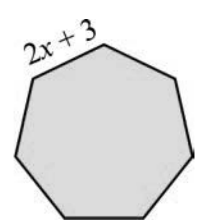
6. Find the missing number in the equations.

a. ____ $(x + 5) = 6x + 30$	**b.** $10(y +$ ____ $) = 10y + 30$
c. $6($ ____ $+ z) = 12 + 6z$	**d.** $8(r +$ ____ $) = 8r + 24$

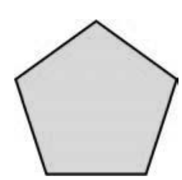
7. Find the missing number in the equations. These are just a little bit trickier!

a. ____ $(2x + 5) = 6x + 15$	**b.** ____ $(3w + 5) = 21w + 35$
c. ____ $(6y + 4) = 12y + 8$	**d.** ____ $(10s + 3) = 50s + 15$
e. $2($ _____ $+ 9) = 4x + 18$	**f.** $4($ ______ $+ 3) = 12x + 12$
g. $5($ _____ $+ 3) = 20y + 15$	**h.** $8($ ____ $+$ ____ $+ 7) = 40t + 8s + 56$

8. Write an expression for the perimeter of this regular heptagon, as a <u>product</u>.
Then, multiply the expression using the distributive property



9. The perimeter of a regular pentagon is $15x + 5$. How long is one of its sides?



When we use the distributive property “backwards”, and write a sum as a product, it is called **factoring**.
Example 5. The sum $5x + 5$ can be written as $5(x + 1)$. We took the SUM $5x + 5$ and wrote it as a PRODUCT— something times something, in this case 5 times the quantity $(x + 1)$.
Example 6. The sum $24x + 16$ can be written as the product $8(3x + 2)$.   *Notice* that the numbers 24 and 16 are *both divisible by 8*! That is why we write 8 as one of the factors.

10. Think of the distributive property “backwards”, and factor these sums. Think of divisibility!

a. $6x + 6$ = ____ (x + 1)	**b.** $8y + 16$ = 8(____ + ____)
c. $15x + 45$ = ____ (x + ____)	**d.** $4w + 40$ = ____ (w + ____)
e. $6x + 30$ = ____ (____ + ____)	**f.** $8x + 16y + 48$ = ____ (____ + ____ + ____)

11. Factor these sums (writing them as products). Think of divisibility!

a. $8x + 4$ = ____ ($2x$ + ____)	**b.** $15x + 10$ = ____ ($3x$ + ____)
c. $24y + 8$ = ____ (____ + ____)	**d.** $6x + 3$ = ____ (____ + ____)
e. $42y + 14$ = ____ (____ + ____)	**f.** $32x + 24$ = ____ (____ + ____)
g. $27y + 9$ = ____ (____ + ____)	**h.** $55x + 22$ = ____ (____ + ____)
i. $36y + 12$ = ____ (____ + ____)	**j.** $36x + 9z + 27$ = ____ (____ + ____ + ____)

12. The perimeter of a square is $48x + 16$. How long is its side?

Puzzle Corner

As a storekeeper, you need to purchase 1,000 items to get a wholesale (cheaper) price of \$8 per item, so you do. You figure you might sell 600 of them. You also want to advertise a \$3 discount to your customers. What should the non-discounted selling price be for you to actually earn a \$500 profit from the sale of these items?

Epilogue: It may be hard to see now where distributive property or factoring might be useful, but it IS extremely necessary later in algebra, when solving equations.

To solve the problem above, you *can* figure it out without algebra, but it becomes fairly straightforward if we write an equation for it. Let p be the non-discounted price. We get

$$600(p - \$3) = 1{,}000 \cdot \$8 + \$500$$

To solve this equation, one needs to use the distributive property in the very first step:

$$600p - \$1800 = \$8{,}500$$

$$600p = \$10{,}300$$

(Can you solve this last step yourself?)

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Chapter 3: Decimals
Introduction

In this chapter we study all four operations of decimals, the metric system, and using decimals in measuring units. Most of the topics here have already been studied in 5th grade, but in 5th grade, we were using numbers with a maximum of three decimal digits. This time, there is no such restriction and the decimals used have many more decimal digits than that.

However, since the topics are the same, if the student has a good grasp of decimals already, consider assigning only 1/3 - 1/2 of the problems because the student should be able to go through this chapter quickly.

We study place value with decimals and comparing decimals, up to six decimal digits. The next several lessons contain a lot of review, just using longer decimals than in 5th grade: adding and subtracting decimals, rounding decimals, multiplying and dividing decimals, fractions and decimals, and multiplying and dividing decimals by the powers of ten.

In the lessons about dividing decimals by decimals, I have tried to explain the principle behind the common shortcut ("Move the decimal point in both the divisor and the dividend enough steps that the divisor becomes a whole number"). This shortcut actually has to do with the principle that when you multiply the divisor and the dividend by the same number (*any* number), the value of the quotient does not change. This even ties in with equivalent fractions. Many school books never explain this principle in connection with decimal division.

The last lessons in this chapter deal with measuring units and the metric system, and nicely round up our study of decimals.

The Lessons in Chapter 3

Helpful Resources on the Internet

Place Value Strategy
Place the 3 or 4 digits given by the spinner to make the largest number possible.
www.decimalsquares.com/dsGames/games/placevalue.html

Decimal Darts
Try to pop balloons with darts by estimating the balloons' height.
www.decimalsquares.com/dsGames/games/darts.html

Decimal Challenge
Try to guess a decimal number between 0 and 10. Each time feedback tells you whether your guess was too high or too low.
www.interactivestuff.org/sums4fun/decchall.html

Beat the Clock
Type in the decimal number for the part of a square that is shaded in this timed game.
www.decimalsquares.com/dsGames/games/beatclock.html

Scales
Move the pointer to match the decimal number given to you. Refresh the page from your browser to get another problem to solve.
www.interactivestuff.org/sums4fun/scales.html

Switch
Put the sequence of decimal numbers into ascending order by switching them around. Refresh the page from your browser to get another problem to solve.
www.interactivestuff.org/sums4fun/switch.html

Smaller and Smaller Maze
Practice ordering decimal numbers to find your way through the maze.
http://www.counton.org/magnet/kaleidoscope/smaller/index.html

Decimal and Whole Number Jeopardy
Review place value and comparing and rounding numbers. Also, practice number patterns.
http://www.quia.com/cb/8142.html

Decimals in Space
An Asteroids-style game where you first answer a question about the smallest decimal and then get to shoot asteroids, earning points based on the numbers on them.
http://themathgames.com/arithmetic-games/place-value/decimal-place-value-math-game.php

Sock
Push the green blocks into the holes to make the target number.
http://www.interactivestuff.org/sums4fun/sock.html

Decimal Squares Blackjack
Play cards with decimals, trying to get as close to 2 as possible without going over.
http://www.decimalsquares.com/dsGames/games/blackjack.html

A Decimal Puzzle
Make every circle add up to 3.
http://nlvm.usu.edu/en/nav/frames_asid_187_g_2_t_1.htmlsopen=instructions&from=category_g_2_t_1.html

FunBrain Decimal Power Football
Simple games for addition, subtraction, multiplication, and division of decimals, including some with a missing factor or divisor. Solve a problem, and the football player moves down the field.
http://www.funbrain.com/cgi-bin/getskill.cgi?A1=choices&A2=fb&A3=6&A4=0&A7=0

Exploring Division of Decimals
Use a square to explore the products of two numbers with one decimal digit. The product is shown as an area.
http://www.hbschool.com/activity/elab2004/gr6/1.html

Decimal Speedway
Practice decimal multiplication in this fun car-racing game.
http://www.decimalsquares.com/dsGames/games/speedway.html

The Metric Units Tutorial—Metric Number line
A tutorial of the common metric unit prefixes, and a way to convert between metric units using a "metric unit number line," which visually shows you how many steps you need to move the decimal point.
http://www.dmacc.edu/medmath1/METRIC/Metric%20Number%20Line%20HTML/sld001.htm

Fractions - Decimals calculator
Convert fractions to decimals, or decimals to fractions, including repeating (recurring) decimals to any decimal places, which normal calculators do not do.
http://www.maths.surrey.ac.uk/hosted-sites/R.Knott/Fractions/FractionsCalc.html

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Fractions and Decimals

You already know how to change decimals to fractions. The number of decimal digits tells you the denominator —it is always a power of ten with as many zeros as you have decimal digits. For the numerator, just copy all the digits from the number. **Example:** $3.0928 = \dfrac{30{,}928}{10{,}000}$	You can also write this as a mixed number, in which case you take the whole number part from the decimal, and the actual decimal digits form the numerator: $15.30599 = \dfrac{1{,}530{,}599}{100{,}000} = 15\dfrac{30{,}599}{100{,}000}$

1. Write as fractions.

a. 0.09	**b.** 0.005	**c.** 0.045
d. 0.00371	**e.** 0.02381	**f.** 0.0000031

2. Write as fractions, *and also* as mixed numbers.

a. 2.9302	**b.** 2.003814
c. 5.3925012	**d.** 3.0078
e. 3.294819	**f.** 45.00032

When changing a **<u>fraction into a decimal</u>**, we have several tools in our "toolbox." **<u>Tool 1.</u>** If the denominator of a fraction is already a power of ten, there is not much to do but to write it as a decimal. The number of zeros in the power of ten tells you the number of decimal digits you need.	$\dfrac{3}{10} = 0.3$	$\dfrac{451{,}593}{10{,}000} = 45.1593$

3. Write as decimals.

a. $\dfrac{36}{100}$	**b.** $\dfrac{5{,}009}{1000}$	**c.** $1\dfrac{45}{1000}$
d. $\dfrac{3908}{10{,}000}$	**e.** $2\dfrac{593}{100{,}000}$	**f.** $\dfrac{5903}{1{,}000{,}000}$
g. $\dfrac{45{,}039{,}034}{1{,}000{,}000}$	**h.** $\dfrac{435{,}112}{10{,}000}$	**i.** $\dfrac{450{,}683}{100{,}000}$

Tool 2. With some fractions, you can find an equivalent fraction with a denominator of 10, 100, 1000, etc. and then write the fraction as a decimal.	$\dfrac{27}{30} \underset{\div 3}{\overset{\div 3}{=}} \dfrac{9}{10} = 0.9$	$\dfrac{66}{200} \underset{\div 2}{\overset{\div 2}{=}} \dfrac{33}{100} = 0.33$	$\dfrac{3}{8} \underset{\times 125}{\overset{\times 125}{=}} \dfrac{375}{1{,}000} = 0.375$

4. Write as decimals. Think of the equivalent fraction that has a denominator of 10, 100, or 1000.

a. $\frac{1}{5}$	**b.** $\frac{1}{8}$	**c.** $1\frac{1}{20}$
d. $3\frac{9}{25}$	**e.** $\frac{12}{200}$	**f.** $8\frac{3}{4}$
g. $4\frac{3}{5}$	**h.** $\frac{13}{20}$	**i.** $\frac{7}{8}$
j. $\frac{11}{125}$	**k.** $\frac{24}{400}$	**l.** $\frac{95}{500}$

5. In these problems, you see both fractions and decimals. Either change the decimal into a fraction, or vice versa. You decide which way is easier! Then, calculate mentally.

a. $0.2 + \frac{1}{4}$	**b.** $0.34 + 1\frac{1}{5}$	**c.** $2\frac{3}{5} + 1.3$	**d.** $\frac{5}{8} - 0.09$
e. $0.02 + \frac{3}{4}$	**f.** $1.9 + 3\frac{1}{8}$	**g.** $\frac{14}{20} - 0.23$	**h.** $\frac{18}{25} + 0.07$

Tool 3. Most of the time, in order to change a fraction to a decimal, you simply treat the fraction as a division problem and divide (with a calculator or long division).	$\frac{5}{6} = 5 \div 6 = 0.83333.... \approx 0.83$

6. Use long division in your notebook to write these fractions as decimals. Give your answers to three decimal digits.

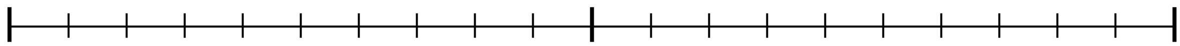
a. $\frac{2}{9} =$	**b.** $\frac{3}{7} =$	**c.** $\frac{7}{16} =$

7. Use a calculator to write these fractions as decimals. Give your answers to three decimal digits.

a. $\frac{1}{11} =$	**b.** $\frac{3}{23} =$	**c.** $\frac{47}{56} =$

8. Mark the following numbers on this number line that starts at 0 and ends at 2.

$$0.2,\quad \frac{1}{4},\quad 0.65,\quad 1\frac{1}{3},\quad 0.04,\quad \frac{2}{5},\quad 1.22,\quad 1\frac{3}{4},\quad 1.95,\quad 1\frac{4}{5}$$



9. A bag of milk powder contains 900 g of milk powder.
Another contains 3/4 kg of milk powder.
What is the combined weight of the two?

10. A puzzle measures 14 3/8 inches by 20 3/8 inches.

a. Write these mixed numbers as decimals.

b. Calculate the area of the puzzle, in square inches (as a decimal).

11. Flax seed costs $11.45 per kilogram. Sally bought 1 3/4 kg of it.
Calculate the total price of Sally's purchase (in dollars and cents).

12. Explain two different ways to calculate the price of 3/8 of a liter of oil, if one liter costs $12.95.
(You do not have to calculate the price; just explain two ways *how* to do it.)

13. Give your answers to the following problems as both a fraction and as a decimal.

a. 0.3 × 5/8

b. 3/4 × 1.5

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Convert Metric Measuring Units

The metric system has one basic unit for each thing we might measure: For length, the unit is the **meter**. For weight, it is the **gram**. And for volume, it is the **liter**.

All of the other units for measuring length, weight, or volume are *derived* from the basic units using *prefixes*. The prefixes tell us what multiple of the basic unit the *derived unit* is.

For example, centiliter is 1/100 part of a liter (*centi* means 1/100).

Prefix	Abbreviated	Meaning
kilo-	k	1,000
hecto-	h	100
deka-	da	10
-	-	(the basic unit)
deci-	d	1/10
centi-	c	1/100
milli-	m	1/1000

Unit	Abbr	Meaning
kilometer	km	1,000 meters
hectometer	hm	100 meters
dekameter	dam	10 meters
meter	m	(the basic unit)
decimeter	dm	1/10 meter
centimeter	cm	1/100 meter
millimeter	mm	1/1000 meter

Unit	Abbr	Meaning
kilogram	kg	1,000 grams
hectogram	hg	100 grams
dekagram	dag	10 grams
gram	g	(the basic unit)
decigram	dg	1/10 gram
centigram	cg	1/100 gram
milligram	mg	1/1000 gram

Unit	Abbr	Meaning
kiloliter	kl	1,000 liters
hectoliter	hl	100 liters
dekaliter	dal	10 liters
liter	L	(the basic unit)
deciliter	dl	1/10 liter
centiliter	cl	1/100 liter
milliliter	ml	1/1000 liter

1. Write these amounts using the basic units (meters, grams, or liters) by "translating" the prefixes. Use both fractions and decimals, like this: 3 cm = 3/100 m = 0.03 m (since "centi" means "hundredth part").

a. 3 cm = *3/100 m* = *0.03 m*	**b.** 2 cg = ___________ g = ___________ g
5 mm = ___________ m = ___________ m	6 ml = ___________ L = ___________ L
7 dl = ___________ L = ___________ L	1 dg = ___________ g = ___________ g

2. Write the amounts in basic units (meters, grams, or liters) by "translating" the prefixes.

a. 3 kl = __________ L	**b.** 2 dam = __________ m	**c.** 70 km = __________ m
8 dag = ___________ g	9 hl = ___________ L	5 hg = ___________ g
6 hm = ___________ m	7 kg = ___________ g	8 dal = ___________ L

3. Write the amounts with derived units (units with prefixes), and using a single-digit numbers.

a. 3,000 g = __3__ __kg__	**b.** 0.01 m = _________ ______	**c.** 0.04 L = _________ ______
800 L = __8__ ______	0.2 L = _________ ______	0.8 m = _________ ______
60 m = __6__ ______	0.005 g = _________ ______	0.007 L = _________ ______

4. Write using prefixed units.

a. 0.04 meters = 4 cm **b.** 0.005 grams = 5 ________ **c.** 0.037 meters = 37 ________

d. 400 liters = 4 _______ **e.** 0.6 meters = 6 ________ **f.** 2,000 meters = 2 ________

g. 0.206 liters = 206 ________ **h.** 20 meters = 2 ________ **i.** 0.9 grams = 9 ________

5. Change into the basic unit (either meter, liter, or gram). Think of the meaning of the prefix.

a. 45 cm = *0.45 m* **b.** 65 mg = **c.** 2 dm =

d. 81 km = **e.** 6 ml = **f.** 758 mg =

g. 2 kl = **h.** 8 dl = **i.** 9 dag =

Example. Convert 2.5 cg to grams.

					2.	5
kg	hg	dag	g	dg	cg	mg

→

			0.	0	2	5
kg	hg	dag	g	dg	cg	mg

Write 2.5 in the chart so that "2", which is in the ones place, is placed in the centigrams place.

Move the decimal point just after the grams place. Add necessary zeros. Answer: 0.025 g.

6. Write the measurements in the place value charts.

a. 12.3 m

km	hm	dam	m	dm	cm	mm

c. 56 cl

kl	hl	dal	l	dl	cl	ml

b. 78 mm

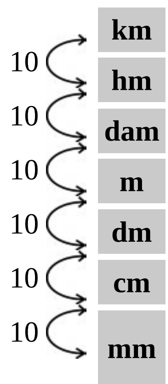
km	hm	dam	m	dm	cm	mm

d. 9.83 hg

kg	hg	dag	g	dg	cg	mg

7. Convert the measurements to the given units, using the charts above.

	m	**dm**	**cm**	**mm**
a. 12.3 m	12.3			
b. 78 mm				78 mm
	L	**dl**	**cl**	**ml**
c. 56 cl				
	g	**dg**	**cg**	**mg**
d. 9.83 hg				



You can also convert measurements by thinking of how many steps apart the two units are in the chart, and then multiplying or dividing by the corresponding power of ten.

Example 1. Convert 2.4 km into centimeters.

There are five steps from kilometers to centimeters. That means we would multiply 2.4 by 10, five times—or multiply 2.4 by 10^5.

2.4 × 100,000 = 240,000, so 2.4 km = 240,000 cm.

Example 2. Convert 2,900 cg into hectograms.

“Centi” and “hecto” are four steps apart, so we will divide by 10^4 = 1000.
2,900 ÷ 10,000 = 0.29, so therefore 2,900 cg = 0.29 hg.

8. Convert the measurements. You can write the numbers in the place value charts, or count the steps.

a. 560 cl = __________ L

b. 0.493 kg = __________ dag

c. 24.5 hm = __________ cm

d. 491 cm = __________ m

e. 35,200 mg = __________ g

f. 32 dal = __________ cl

g. 0.483 km = __________ dm

h. 0.0056 km = __________ cm

i. 1.98 hl = __________ dl

j. 9.5 dl = __________ L

kl	hl	dal	l	dl	cl	ml

kg	hg	dag	g	dg	cg	mg

km	hm	dam	m	dm	cm	mm

9. Each measurement has a flub, either in the unit or in the decimal point. Correct them.

a. The length of a pencil: 13 m

b. The length of an eraser: 45 cm

c. Length of Dad’s waist: 9.2 m

d. The height of a room: 0.24 m

e. Jack’s height: 1.70 mm

f. Jenny’s height: 1.34 cm

10. Find the total …

a. … weight of books that weigh individually:
1.2 kg, 1.04 kg, 520 g, and 128 g.

b. … volume of containers whose individual volumes are:
1.4 L, 2.25 L, 550 ml, 240 ml, and 4 dl.

11. A dropper measures 4 ml. How many full droppers can you get from a 2-dl bottle?

12. A nurse has to give 3 mg of medicine for each kilogram of body weight, and do that once a day.
The patient weighs 70 kg. In how many days will the patient have received 2 g of medicine?

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Chapter 4: Ratios
Introduction

In this chapter we concentrate on the concept of ratio and various applications involving ratios and rates.

The chapter starts out with the basic concepts of ratio, rate, and unit rate. The lesson *Equivalent Rates* allows students to solve a variety of word problems involving ratios and rates. We also connect the concept of rates (specifically, tables of equivalent rates) with ordered pairs, use equations (such as $y = 3x$) to describe these tables, and plot the ordered pairs in the coordinate plane.

Next, we study various kinds of word problems involving ratios, and use a bar model to solve these problems in two separate lessons. These lessons tie ratios in with the student's previous knowledge of bar models as a tool for problem solving.

Then, students encounter the concept of aspect ratio, which is simply the ratio of a rectangle's width to its height, and solve a variety of problems involving aspect ratio.

Lastly, students learn how rates can be used to convert measurement units. This method is in addition to the methods for converting measurement units that were explained in the decimals chapter. It does not mean that students should "change over", and forget what they learned earlier—it is simply a different method for doing the conversions. Some students may choose one method over another; some may be able to accommodate all the methods. Most will probably choose one method they prefer for doing these conversions.

The Lessons in Chapter 4

Helpful Resources on the Internet

Practice with Ratios
An online quiz from Regents Exam Prep Center that includes both simple and challenging questions and word problems concerning ratios.
http://www.regentsprep.org/Regents/math/ALGEBRA/AO3/pracRatio.htm

Ratio Pairs Matching Game
Match cards representing equivalent ratios.
Easy: **http://nrich.maths.org/4824** Challenge: **http://nrich.maths.org/4821**

Equivalent Ratios Workout
10 online practice problems.
http://www.math.com/school/subject1/practice/S1U2L1/S1U2L1Pract.html

Ratio Stadium
A multi-player online racing game for matching equivalent ratios. The student with the fastest rate of correct answers will win the race.
http://www.arcademicskillbuilders.com/games/ratio-stadium/

Dirt Bike Proportions
A racing game where you need to find the unknown in a simple proportion. This game would actually work equally well for practicing equivalent fractions, because the proportions are quite simple.
http://www.arcademicskillbuilders.com/games/dirt-bike-proportions/dirt-bike-proportions.html

All About Ratios - Quizzes
Online quizzes about same and different ratios.
http://math.rice.edu/~lanius/proportions/index.html

Free Ride
An interactive activity about bicycle gear ratios. Choose the front and back gears, which determines the gear ratio. Then choose a route, pedal forward, and make sure you land exactly on the five flags.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=178

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Using Equivalent Ratios

Example. If Jake can ride his bike to a town that is 21 miles away in 45 minutes, how far can he ride in 1 hour?

Let's form some equivalent rates, starting with 21 miles per 45 minutes, and hoping to arrive at so many miles per 60 minutes.

However, it is not easy to go directly from 45 minutes to 60 minutes (1 hour). So, let's first figure the rate for <u>*15 minutes*</u>, which *is* easy.

Why? Because to get from 45 minutes to 15 minutes you simply divide both terms of the rate by 3.

$$\frac{21\text{ miles}}{45\text{ minutes}} \underset{\div 3}{\overset{\div 3}{=}} \frac{\boxed{7}\text{ miles}}{15\text{ minutes}} \underset{\times 4}{\overset{\times 4}{=}} \frac{\boxed{28}\text{ miles}}{60\text{ minutes}}$$

Then from 15 minutes, we can easily get to 60 minutes: Just multiply both terms by 4. We find that he can ride 28 miles in one hour.

1. Write the equivalent rates.

a. $\frac{15\text{ km}}{3\text{ hr}} = \frac{\underline{\qquad}}{1\text{ hr}} = \frac{\underline{\qquad}}{15\text{ min}} = \frac{\underline{\qquad}}{45\text{ min}}$	**b.** $\frac{\$6}{45\text{ min}} = \frac{\underline{\qquad}}{15\text{ min}} = \frac{\underline{\qquad}}{1\text{ hr}} = \frac{\underline{\qquad}}{1\text{ hr }45\text{ min}}$
c. $\frac{3\text{ in}}{8\text{ ft}} = \frac{\underline{\qquad}}{2\text{ ft}} = \frac{\underline{\qquad}}{12\text{ ft}} = \frac{\underline{\qquad}}{20\text{ ft}}$	**d.** $\frac{115\text{ words}}{2\text{ min}} = \frac{\underline{\qquad}}{1\text{ min}} = \frac{\underline{\qquad}}{3\text{ min}}$

2. **a.** Jake can ride 8 miles in 14 minutes. How long will it take him to ride 36 miles? Use the equivalent rates.

$$\frac{8\text{ miles}}{14\text{ minutes}} = \frac{4\text{ miles}}{\boxed{}\text{ minutes}} = \frac{36\text{ miles}}{\boxed{}\text{ minutes}}$$

b. How many miles can Jake ride in 35 minutes?

3. A car can go 50 miles on 2 gallons of gasoline.

a. How many gallons of gasoline would the car need for a trip of 60 miles? Use the equivalent rates below.

$$\frac{50\text{ miles}}{2\text{ gallons}} = \frac{5\text{ miles}}{\boxed{}\text{ gallons}} = \frac{60\text{ miles}}{\boxed{}\text{ gallons}}$$

b. How far can the car travel on 15 gallons of gasoline?

Example. You get 20 erasers for \$1.80.
How much would 22 erasers cost?

Cost (C)			\$0.90	\$1.80	
Erasers (E)	1	2	10	20	22

You can solve this problem in many ways.
Let’s use a table of rates this time.

First, find the cost for *10 erasers*, and then the cost for 2. After that, you can get the cost for 22 by adding.

Ten erasers will cost half of \$1.80. Two erasers will cost one-fifth of that (divide by 5 to find it!).
Lastly, add the cost of 20 erasers to the cost of 2 erasers to get the cost for 22 erasers.

Note 1: *Each* pair of numbers in the table is a rate. For example, *\$1.80 for 20 erasers* (or \$1.80/20 erasers) is a rate, and so is \$0.90 for 10 erasers.

Note 2: We can write an equation relating the Cost (C) and the number of Erasers (E). You will find that easily from the unit rate (price for one): C = 0.09E. In other words, the cost is 0.09 times the number of erasers.

4. Finish solving the problem in the example above.

5. How many erasers would you get with \$1.35?

6. On average, Scott makes a basket nine times out of twelve shots when he is practicing. How many baskets can he expect to make when he tries 200 shots? Fill in a table of rates to solve this.

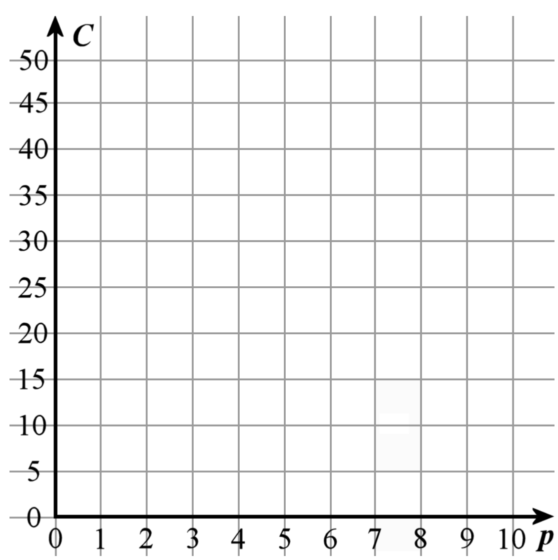
baskets						
shots						

7. **a.** Three pairs of socks cost \$9. Fill in the table of rates.
The variable *C* stands for cost, and *p* for pairs of socks.

C			9							
p	1	2	3	4	5	6	7	8	9	10

b. Each number pair in the table *is* a rate, but we can also view them as points with two coordinates. Plot the number pairs in the coordinate grid.

c. Write an equation relating the cost (C) and the number of pairs of socks (*p*).



8. **a.** You get 30 pencils for \$4.50. How much would 52 pencils cost?

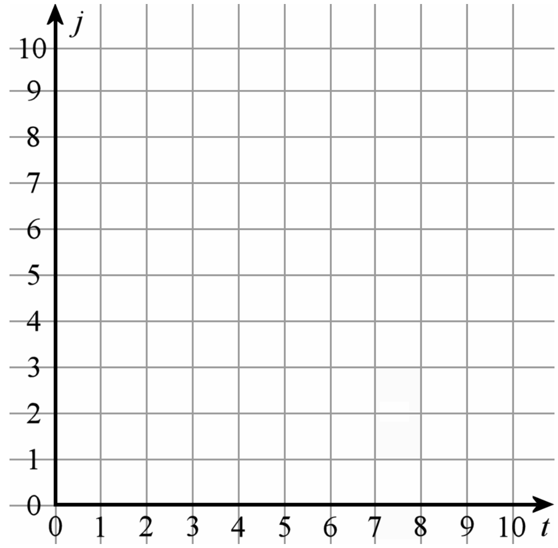
Cost						
Pencils						

b. Write an equation relating the cost (C) and the number of pencils (P).

9. When Kate makes 4 liters of tea (a pot full), she needs five jars for the tea. From this, we get the rate of 4 liters / 5 jars.

a. Fill in the table. The variable *t* stands for the amount of tea, and *j* for the number of jars.

t					4					
j	1	2	3	4	5	6	7	8	9	10



b. Plot the number pairs from the table in this coordinate grid.

c. How many jars will Kate need for 20 liters of tea?

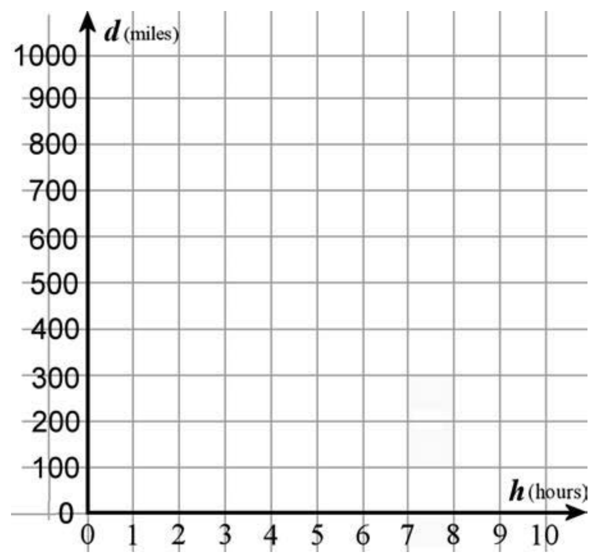
d. If Kate has 16 jars full of tea, how many liters of tea is in them?

10. **a.** A train travels at a constant speed of 80 miles per hour. Fill in the table of rates.

d										
h	1	2	3	4	5	6	7	8	9	10

b. Write an equation relating the distance (*d*) and the number of hours (*h*).

c. Plot the points in the grid on the right. The variable *h* stands for hours, and *d* for distance.



11. Another train travels at the constant speed of 60 miles per hour. Fill in the table of rates. Then, plot the points in the same coordinate grid as for the train in #10.

d					
h	1	2	3	4	5

d					
h	6	7	8	9	10

12. *How* can you see <u>from the graph</u> which train travels faster?

13. The plot shows the walking speeds for two persons (t is in minutes, d is in miles). Your task is to fill in the two ratio tables below. To make that easier, first find dots that are placed exactly on the lines, so that you can easily read the coordinates.
(Hint: For some of the points, you will need to use fractions and mixed numbers.)

Person 1 (red dot)

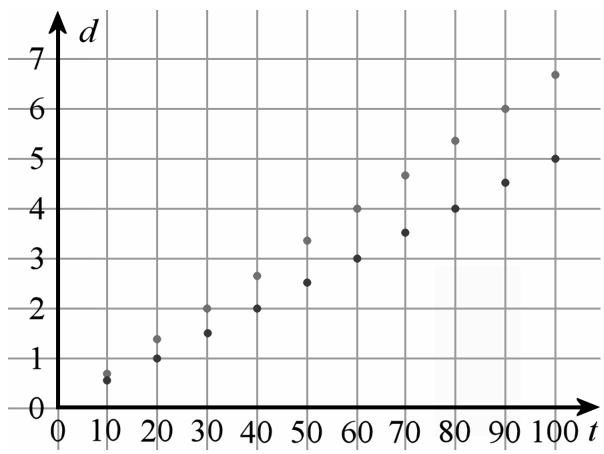
d (miles)										
t (minutes)										

Person 2 (blue dot)

d (miles)										
t (minutes)										

a. What is the speed of the first person in miles per hour?

b. What is the speed of the second person in miles per hour?



14. Train 1 travels at a constant speed of 240 miles in three hours.
Train 2 travels 490 miles in seven hours. Which train is faster?

15. Find which is a better deal by comparing the unit rates: $45 for eight bottles of shampoo, or $34 for six bottles of shampoo?

16. In a poll of 1,000 people, 640 said they liked blue.

a. Simplify this ratio to the lowest terms:

640 people *out of* 1000 people = _______ people *out of* ______ people

b. Assuming the same ratio holds true in another group of 100 people, how many of those people can we expect to like blue?

c. Assuming the same ratio holds true in another group of 225 people, how many of those people can we expect to like blue?

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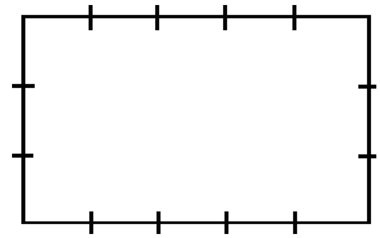
Aspect Ratio

You might have heard about the **aspect ratio** of the screens of televisions, computer monitors, and other monitors. The aspect ratio is simply **the ratio** of a **rectangle's** <u>**width to its height**</u>.

Example. A rectangle's width and height are in a ratio of 5:3. This means the aspect ratio is 5:3. If the rectangle's perimeter is 64 cm, what are its width and its height?



Let's draw the rectangle. Working from the 5:3 aspect ratio, let's divide the sides into "parts," or the same-sized segments, 5 for the width, and 3 for the height. We can see in the picture that perimeter is made up of 16 of these "parts." Since 64 ÷ 16 = 4, each part is 4 cm long.

Therefore, the rectangle's width is 5 × 4 cm = 20 cm, and its length is 3 × 4 cm = 12 cm.

1. The width and height of a rectangle are in a ratio of 9:2.

   a. Draw the rectangle, and divide its width and length into parts according to its aspect ratio.

   b. If the rectangle's perimeter is 220 cm, find its width and its height.

2. A rectangle's width is three times its height, and its perimeter is 120 mm. Find the rectangle's width and its height.

3. Find the aspect ratio of each rectangle:

   a. a rectangle whose height is 2/5 of its width

   b. a rectangle whose height is five times its width

   c. a square

4. The door of a fridge is 4/9 as wide as it is tall.

   a. What is the ratio of the door's width to height?

   b. If the door is 54 cm wide, how tall is it?

5. Little Mary drew a picture on a rectangular piece of paper that was 6 inches wide and 9 inches high.

a. Write the aspect ratio, and simplify it to the lowest terms.

b. If this picture were enlarged to be 20 inches *wide*, how high would it be? Use equivalent ratios.

6. Mr. Miller is ordering custom-made windows for his new house. He is considering windows of these sizes: 70 cm × 90 cm, 80 cm × 100 cm, 90 cm × 110 cm, and 100 cm × 120 cm.

a. Write the aspect ratios of all the windows and simplify them to the lowest terms.

b. Do any of the windows share the exact same aspect ratio when simplified? If so, then which ones? (That would mean they would have the exact same shape.)

7. A sandbox for children is two times as wide as it is long.

a. What is its aspect ratio?

b. The perimeter of the sandbox is 15 ft. Find its length and width.

c. Find its area.

Don't confuse area with perimeter.

The aspect ratio pertains to the *length* and *width*, not to the area. However, once you know the length and the width, you can calculate the area.

8. Two television sets have the same perimeter, 150 cm. The aspect ratio of one is 16 : 9, and the aspect ratio of the other is 4 : 3.

a. Find the length and width of each television.

b. Which television has the larger area?

9. The area of a square is 49 sq. in. If two of these squares are put side by side, we get a rectangle.

a. Find the aspect ratio of that rectangle.

b. Find the perimeter of the rectangle.

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Chapter 5: Percent
Introduction

The concept of percent builds on the student's understanding of fractions and decimals. Specifically, students should be very familiar with the idea of finding a fractional part of a whole (such as finding 3/4 of $240). Students who have used Math Mammoth have been practicing that concept since 4th grade. One reason why I have emphasized finding a fractional part of a whole so much in the earlier grades is specifically to lay a groundwork for the concept of percent. Assuming the student has mastered how to find a fractional part a whole, and can easily convert fractions to decimals, then studying the concept of percent should not be difficult.

The first lesson, *Percent*, practices the concept of percent as a hundredth part, and how to write fractions and decimals as percentages. Next, we study how to find a percentage, when the part and the whole are given (for example, if 15 out of 25 club members are girls, what percentage of them are girls?).

The following two lessons have to do with finding a certain percentage of a given number or quantity. First, we study how to do that using mental math techniques. For example, students find 10% of $400 by dividing $400 by 10. Next, students find a percentage of a quantity using decimal multiplication, including using a calculator. For example, students find 17% of 45 km by multiplying 0.17 × 45 km.

I prefer teaching students to calculate percentages of quantities using decimals, instead of using percent proportion or some other method (such as changing 17% into the fraction 17/100 for calculations). That is because using decimals is simpler: we simply change the percentage into a decimal, and multiply, instead of having to build a proportion or use fractions. Also, decimals will be so much easier to use later on, when solving word problems that require the usage of equations.

Next is a lesson about discounts, which is an important application in everyday life. Then, we go on to the lesson *Practice with Percent*, which contrasts the two types of problems students have already studied: questions that ask for a certain percentage of a number (the percentage is given), and questions that ask for the percentage. For example, the first type of question could be "*What is 70% of $380?*", and the second type could be "*What percentage is $70 of $380?*"

The last lesson lets students find the total when the percentage and the partial amount are known. For example: "Three-hundred twenty students, which is 40% of all students, take PE. How many students are there in total?" We solve these with the help of bar models.

I have made several videos to match these lessons. You can watch them here:
http://www.mathmammoth.com/videos/percent.php

The Lessons in Chapter 5

Helpful Resources on the Internet

Percent videos by Maria
Videos on percent-related topics that match the lessons in this chapter!
http://www.mathmammoth.com/videos/percent.php

Games & Tools

Virtual Manipulative: Percentages
Interactive tool where you fill in any two of the three 'boxes' (whole, part, and percent) and it will calculate the missing part and show the result visually in two ways.
http://nlvm.usu.edu/en/nav/frames_asid_160_g_2_t_1.html

Mission: Magnetite
Hacker tries to drop magnetite on Motherboard. To stop him, match up percentages, fractions, and images showing fractional parts.
http://pbskids.org/cyberchase/media/games/percent/index.html

Fractions and Percent Matching Game
A simple matching game: match fractions and percentages.
http://www.mathplayground.com/matching_fraction_percent.html

Fraction/Decimal/Percent Jeopardy
Answer the questions correctly, changing between fractions, decimals, and percentages.
http://www.quia.com/cb/34887.html

Flower Power
Grow flowers and harvest them to make money in this addictive order-'em-up game. Practice ordering decimals, fractions, and percentages. The game starts with ordering decimals (daisies), and proceeds into fractions (tulips or roses).
http://www.mangahigh.com/en/games/flowerpower

Percent Shopping
Choose toys to purchase. In level 1, you find the sale price when the original price and percent discount are known. In level 2, you find the percent discount when the original price and the sale price are known.
http://www.mathplayground.com/percent_shopping.html

Penguin Waiter
Simple game where you calculate the correct tip to leave the penguin waiter.
http://www.funbrain.com/penguin/

Worksheets

Percent worksheets
Create an unlimited number of free customizable percent worksheets to print.
www.homeschoolmath.net/worksheets/percent-decimal.php
www.homeschoolmath.net/worksheets/percent-of-number.php
www.homeschoolmath.net/worksheets/percentages-words.php

Worksheets & quizzes for percentages, ratios, and proportions
Several online quizzes and a few PDF worksheets for these topics.
www.math4children.com/Topics/Percentages

Tutorials

Percentages of Something
See simple percentages illustrated in different ways.
http://www.bbc.co.uk/skillswise/game/ma16perc-game-percentages-of-something

A Conceptual Model for Solving Percent Problems
Explanation of how to use a 10 x 10 grid to explain basic concept of percent, AND solve various types of percent problems.
http://illuminations.nctm.org/LessonDetail.aspx?id=L249

Meaning of Percent -- Writing Fractions as Percents
Free percent lessons from Math Goodies.
http://www.mathgoodies.com/lessons/vol4/meaning_percent.html
http://www.mathgoodies.com/lessons/vol4/fractions_to_percents.html

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Percent of a Number (Mental Math)

100% of something means all of it.   1% of something means 1/100 of it.
To calculate a percentage of some quantity is calculating a fractional part of that quantity because *percent* simply means a <u>hundredth part</u>. Therefore, percentages *are* simply fractions!
How much is 1% of 200 kg? This means how much is 1/100 of 200 kg? It is simply 2 kg. **To find 1% of something (1/100 of something), divide by 100.** Do you remember how to divide by 100 mentally? Just move the decimal point two places to the left. For example, 1% of 540 is 5.4, or 1% of 8.30 is 0.083.
To find 2% of some quantity, first find 1% of it, and double that. For example, let's find 2% of \$6. Since 1% of 6 is \$0.06, then 2% of 6 is \$0.12.
To find 10% of some quantity, divide by 10. Why does that work? It is because 10% is 10/100, which equals 1/10. So, 10% is 1/10 of the quantity! For example, 10% of \$780 is \$78. Or, 10% of \$6.50 is \$0.65. (To divide by 10 mentally, just move the decimal point one place to the left.)
Can you think of a way to find 20% of a number? (Hint: Start with finding 10% of the number.) ____

1. Find 10% of these numbers.

a. 700 _______ **b.** 321 _______ **c.** 60 _______ **d.** 7 _______

2. Find 1% of these numbers.

a. 700 _______ **b.** 321 _______ **c.** 60 _______ **d.** 7 _______

3. One percent of Mom's paycheck is \$22. How much is her total paycheck?

4. Fill in the table. Use mental math.

percentage / number	**1,200**	**80**	**29**	**9**	**5.7**
1% of the number					
2% of the number					
10% of the number					
20% of the number					

5. Fill in this guide for using mental math with percentages:

Mental Math and Percentage of a Number	
50% is $\frac{1}{2}$. To find 50% of a number, divide by _______. **10% is $\frac{1}{\square}$. To find 10% of a number, divide by ________.** **1% is $\frac{1}{\square}$. To find 1% of a number, divide by ________.**	50% of 244 is __________. 10% of 47 is __________. 1% of 530 is __________ .
To find 20%, 30%, 40%, 60%, 70%, 80%, or 90% of a number, • **First find __________% of the number, and** • **then multiply by 2, 3, 4, 6, 7, 8, or 9.**	10% of 120 is __________. 30 % of 120 is __________. 60 % of 120 is __________.

6. Find the percentages. Use mental math.

a. 10% of 60 kg __________ 20% of 60 kg __________	**b.** 10% of \$14 __________ 30% of \$14 __________	**c.** 10% of 5 mi __________ 40% of 5 mi __________
d. 1% of \$60 __________ 4% of \$60 __________	**e.** 10% of 110 cm __________ 70% of 110 cm __________	**f.** 1% of \$1,330 __________ 3% of \$1,330 __________

7. David pays a 20% income tax on his \$2,100 salary.

a. How many dollars is the tax?

b. How much money does he have left after paying the tax?

c. What percentage of his salary does he have left?

8. Nancy pays 30% of her \$3,100 salary in taxes. How much money does she have left after paying the tax?

9. Identify the errors that these children made. Then find the correct answers.

a. Find 90% of \$55. Peter's solution: 10% of \$55 is \$5.50 So, I subtract 100% − \$5.50 = \$94.50	**b.** Find 6% of \$1,400. Patricia's solution: 1% of \$1,400 is \$1.40. So, 6% is six times that, or \$8.40.

Some more mental math “tricks”	
90% of a quantity First find 10% of the quantity and then subtract that from 100% of it.	**25% of a quantity** 25% is the same as 1/4. So, to find 25% of a quantity, divide it by 4.
12% of a quantity First find 10% of it. Then find 1% of it, and use that 1% to find 2% of it. Then, add the 10% and the 2%.	**75% of a quantity** 75% is 3/4. First find 1/4 of the quantity and multiply that by 3.

10. Find percentages of the quantities.

a. 50% of 26 in. ____________	**b.** 25% of 40 ft ____________	**c.** 80% of 45 m ____________
d. 75% of $4.40 ____________	**e.** 90% of 1.2 m ____________	**f.** 25% of 120 lb ____________

11. Fill in the mental math method for finding 12% of $65.

10% of $65 is $__________. 1% of $65 is $___________. 2% of $65 is $___________.

Now, add to get 12% of $54: $___________ + $___________ = $___________

12. Fill in the mental math shortcut for finding 24% of 44 kg.

25% of 44 kg is __________ kg. 1% of 44 kg is ___________ kg.

Subtract __________ kg − __________ kg = __________ kg

13. From her cell phone bill, Hannah sees that of the 340 text messages she sent last month, 15% were sent during the night with a cheaper rate. How many messages did Hannah send during the night? During the day?

14. A herd of 40 horses had some bay, some chestnut, and some white horses. Thirty percent of them are bay, and 45% are chestnut. How many horses are white?

15. A college has 1,500 students, and 12% of them ride the bus. Another 25% walk to the college. How many students do not do either?

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Discounts

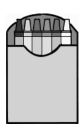
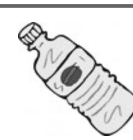
Other than figuring sales tax, the area of life in which you will probably most often need to use percentages is in calculating discounts.
A laptop that costs $600 is 20% off. What is the sale price? **<u>Method 1.</u>** We calculate 20% of $600. That is the discounted amount in *dollars*. Then we subtract that from the original price, $600. 20% of $600 is $120. And $600 − $120 = $480. So, the sale price is $480. **<u>Method 2.</u>** Since 20% of the price has been removed, 80% of the price is <u>*left*</u>. By calculating 80% of the original price, you will get the new discounted price: 0.8 × $600 = $480
Two methods for calculating the discounted price: 1. Calculate the discount amount as a percentage of the original price. Then subtract. 2. Find what percentage of the price is left. Then calculate that percentage of the normal price.

1. All of these items are on sale. Calculate the discount in dollars and the resulting sale price.

a.  Price: $90 20% off Discount amount: $ 18 Sale price: $_______________	**b.**  Price: $5 40% off Discount amount: $___________ Sale price: $_______________	**c.**  Price: $15 30% off Discount amount: $___________ Sale price: $_____________

2. A $25 swimsuit was on sale for 20% off.
   Monica calculated the discounted price this way: $25 − $20 = $5.
   What went wrong? Find the correct discounted price.

3. All the items are on sale. Find the discounted price.

a. Price: $1.20 25% off  Discount amount: $___________ Discounted price: $__________	**b.** Price: $18 25% off  Discount amount: $___________ Discounted price: $__________	**c.** Price: $150 30% off  Discount amount: $___________ Discounted price: $__________
d. Price: $20 40% off  Discount amount: $___________ Discounted price: $___________	**e.** Price: $2.20 10% off  Discount amount: $___________ Discounted price: $__________	**f.** Price: $1.30 50% off  Discount amount: $___________ Discounted price: $___________

You can often use <u>*estimation*</u> when calculating the discounted price.
Example. A $198.95 bicycle is discounted by 25%. What is the discounted price? To estimate, round the original price of the bicycle to $200. Then, 25% of $200 is $50 (it is 1/4 of it). So, the discounted price is about $150.
Example. A $425.90 laptop is discounted by 28%. What is the discounted price? Round the discount percentage to 30%, and the price of the laptop to $430. 10% of $430 is $43. 30% of $430 is three times that much, or $129. Subtract using rounded numbers: $430 − $130 = $300.

4. *Estimate* the discounted price.

a. 30% off of a $39.90 book

b. 17% off of a $12.50 block of cheese

c. 75% off of a $75.50 pair of shoes

5. Which is a better deal? Estimate using rounded numbers and mental math.

a. 75% off of a $199 brand-name mp3 player
OR an off-brand mp3 player for $44.99

b. 40% off of a new, $89 textbook
OR a used copy of the same textbook for $39.90.

6. A company sells a computer program for $39.99. They estimate they would sell 50 copies of it in a week, with that price. If they discount the price by 25%, they think they could sell 100 copies.
Estimate which way would they earn the most money.

Example. A pair of shoes costing $50 is discounted, and now costs only $35. What is the discount percent? Think what *fraction* of the price "disappeared." Then, write that fraction as a percent. We see that $15 of the price "went away." The <u>fraction</u> of the price that was taken off is 15/50. Now, we simply write 15/50 as 30/100, and from that, as the percentage 30%. It was discounted by 30%.

7. Find the discount percent.

a. jeans - original price $50, discounted price $45

b. phone - original price $40, discounted price $30

c. haircut - original price $25, discounted price $20

8. Which of these methods work for calculating the discounted price for 25% off of $46?

$0.25 \times \$46$	$0.75 \times \$46$	$\$46 - \frac{\$46}{25}$	$\$46 - \frac{\$46}{4}$	$\frac{\$46}{4}$	$\frac{\$46}{4} \times 3$