

math

MAMMOTH

Grade 4-B Worktext

Division

Geometry

Fractions

Decimals



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Edition 3.0

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Foreword

Math Mammoth Grade 4-A and *Grade 4-B* worktexts comprise a complete math curriculum for the fourth grade mathematics studies, aligned to the Common Core Standards.

In the fourth grade, students focus on multi-digit multiplication and division, learning to use bigger numbers, solving multi-step word problems that involve several operations, and they get started in studying fractions and decimals. This is of course accompanied by studies in geometry and measuring.

The year starts out with a review of addition and subtraction, patterns and graphs. We illustrate word problems with bar diagrams and study finding missing addends, which teaches algebraic thinking. Children also learn addition and subtraction terminology, the order of operations, and statistical graphs.

Next come large numbers—up to millions, and the place value concept. At first the student reviews thousands and some mental math with them. Next are presented numbers up to one million, calculations with them, the concept of place value and comparing. In the end of the chapter we find out more about millions and an introduction to multiples of 10, 100, and 1000.

The third chapter is all about multiplication. After briefly reviewing the concept and the times tables, the focus is on learning multi-digit multiplication (multiplication algorithm). The children also learn why it works when they multiply in parts. We also study the order of operations again, touch on proportional reasoning, and do more money and change related word problems.

The last chapter in part A is about time, temperature, length, weight, and volume. Students will learn to solve more complex problems using various measuring units and to convert between measuring units.

In part B, we first study division. The focus is on learning long division and using division in word problems. In geometry, we first review area and perimeter, and then concentrate on the topic of angles. Students measure and draw angles, solve simple angle problems, and classify triangles according to their angles. They also study parallel and perpendicular lines.

Fractions and decimals are presented last in the school year. These two chapters practice only some of the basic operations with fractions and decimals. The focus is still on conceptual understanding and on building a good foundation towards 5th grade math, where fractions and decimals will be in focus.

When you use these books as your only or main mathematics curriculum, they can be like a “framework”, but you do have some liberty in organizing the study schedule. Chapters 1, 2, and 3 should be studied in this order, but you can be flexible with chapters 4 (Time and Measuring) and 6 (Geometry) and schedule them somewhat earlier or later if you so wish. Chapter 3 (Multiplication) needs to be studied before long division in Chapter 5. Many topics from chapters 7 and 8 (Fractions and Decimals) can also be studied earlier in the school year; however finding parts with division should naturally be studied only after mastering division.

I wish you success in your math teaching!

Maria Miller, the author

Chapter 5: Division

Introduction

The fifth chapter of *Math Mammoth Grade 4* includes lessons on division, long division, remainder, part problems, average, and problem solving. It is a long chapter, as division and long division are “in focus” in fourth grade.

We start out reviewing basic division by single-digit numbers. Then students study division terms and dividing by whole tens and hundreds.

The lesson *Finding Fractional Parts with Division* shows an important relationship between fractions and division. For example, we can find $\frac{3}{4}$ of a number by first finding $\frac{1}{4}$ (divide by 4), then multiplying that result by 3.

Next we briefly study order of operations again, this time including divisions in the problems.

In the lesson *The Remainder, Part 1*, we study the concept of remainder, first using pictures and small numbers. In the second lesson on remainder, we still use small numbers, but students work the problems using the long division symbol or “corner”, as I like to call it. That is of course preparing them for long division.

Next, long division is taught in several small steps over many lessons. We start with the situation where each of the thousands, hundreds, tens, and ones can be divided evenly by the divisor. Then is introduced the remainder in the ones. Next comes the situation where we have a remainder in the tens. Finally, when we have a remainder in the hundreds, and so on. We also have lots of word problems to solve.

After long division is mastered, we study the concept of average and problem solving involving a fractional part of a whole. I have included many bar diagrams and pictorial representations of these problems to help the students.

The last section deals with elementary number theory topics. We study some basic divisibility rules (though not all of them), prime numbers, and find all factors of a given two-digit number.

The Lessons in Chapter 5

	page	span
Review of Division	10	3 pages
Division Terms and Division with Zero	13	2 pages
Dividing with Whole Tens and Hundreds	15	2 pages
Finding Fractional Parts with Division	17	3 pages
Order of Operations and Division.....	20	2 pages
The Remainder, Part 1	22	3 pages
The Remainder, Part 2	25	3 pages
Long Division 1	28	4 pages
Long Division 2	32	3 pages
Long Division 3	35	4 pages

Long Division with 4-Digit Numbers	39	4 pages
More Long Division	43	3 pages
Remainder Problems	46	4 pages
Long Division with Money	50	2 pages
Long Division Crossword Puzzle	52	1 page
Average	53	3 pages
Problems with Fractional Parts	56	2 pages
Problems to Solve	58	3 pages
Divisibility	61	4 pages
Prime Numbers	65	3 pages
Finding Factors	68	2 pages
Mixed Review	70	2 pages
Review	72	2 pages

Helpful Resources on the Internet

Long division

MathFrog Dividerama!

Interactive long division practice. Guided help available optionally.

<http://cemc2.math.uwaterloo.ca/mathfrog/english/kidz/div5.shtml>

Snork's Long Division Game

Interactive and guided long division practice that only accepts correct answers and truly guides the student step-by-step through long division problems.

<http://www.kidsnumbers.com/long-division.php>

Mr. Martini's Classroom: Long Division

An interactive long division tool.

<http://www.thegreatmartinicompany.com/longarithmetic/longdivision.html>

Double-Division.org

Another form of long division algorithm - takes the guesswork away from estimating how many times the divisor goes into what needs to be divided. Also called 1-2-4-8 division.

<http://www.doubledivision.org/>

Short Division

A page that explains short division in detail. Short division is the same algorithm as long division, but some steps are only done in one's head, not written down.

<http://www.themathpage.com/ARITH/divide-whole-numbers.htm>

Factors and primes

Arrays and factors

Drag rectangles to show the factorizations of a given number.

<http://www.shodor.org/interactivate/activities/factors2/index.html>

Factor Game

Interactive game to practice divisibility among numbers 1-100. Play against the computer or a friend.

<http://illuminations.nctm.org/ActivityDetail.aspx?ID=12>

Factor Feeder

Eat factors of the given number, and avoid numbers that are not factors of the given number in this Pacman-style game. Use Arrow Keys to move.

<http://hoodamath.com/games/factorfeeder.php>

Sliding Tile Factorization Game

Slide a number over another to capture it, if it is a factor of the other. Number 1 is only supposed to be used to capture a prime number.

http://www.visualmathlearning.com/Games/sliding_factors.html

Octopus Factors

Move counters up the legs of an octopus but only when the number on the circle is a multiple of the number on the card.

<http://www.counton.org/games/map-numbers/octopus/>

Factors Millionaire Game

A millionaire game where the questions have to do with factors, prime numbers, and the greatest common factor.

<http://www.math-play.com/Factors-Millionaire/Factors-Millionaire.html>

Not a Factor

Choose a number that is NOT a factor of the given number.

http://www.helpingwithmath.com/resources/games/target_factors01/not_factor.html

Factors and Remainders

An interactive animation demonstrating factors and remainders. Choose a number and its possible divisor. The animation shows boxes (as given by the number) arranged into rows of (possible divisor), and you can SEE if there is any remainder.

<http://www.absorblearning.com/media/item.action?quick=ml>

Snake

Eat factors, multiples, and prime numbers in this remake of the classic game.

<http://www.pompuzzle.com/Snake>

Product game

For two players; each selects a factor, computer colors the product - who gets four in row wins.

<http://illuminations.nctm.org/ActivityDetail.aspx?ID=29>

Primes, Factors and Divisibility—Explorer at CountOn.org

Lessons explaining divisibility tests, primes, and factors.

<http://www.counton.org/explorer/primes>

The following games can be used to practice basic division facts, if the student hasn't mastered them.

A+ math games

Practice all four basic operations with math bingo (matho), hidden picture games, or concentration games.
<http://www.aplusmath.com/games/>

Math Magician games

Flashcard problems in all 4 operations. Answer 20 questions in 1 minute.
<http://www.oswego.org/ocsd-web/games/Mathmagician/cathymath.html>

Division Practice at AAAMath

Learn or practice basic division facts, and more.
<http://www.aaastudy.com/div39hx3.htm>

Cross the Swamp

Help Little Ron move from log to log across the swamp and practice multiplication/division or addition/subtraction.
<http://www.bbc.co.uk/schools/starship/maths/crosstheswamp.shtml>

Math Car Racing

Keep ahead of the computer car by thinking logically, and practice any of the four operations.
<http://www.funbrain.com/osa/index.html>

Arithmetic Game

Find numbers to fit an equation that may use all four operations.
<http://www.primarygames.com/math/arithmeticgame/index.htm>

Primary Games

A collection of games. The following links open the evaluation versions of some division-related games. The game collections themselves are sold at
<http://www.primarygames.co.uk/>

- **Eggs on Legs**
<http://www.primarygames.co.uk/PG5/Eggs/Div/eggsdiv.html>
- **DiviPods**
<http://www.primarygames.co.uk/pg4/Divipods/divipods.html>
- **Division Divers**
<http://www.primarygames.co.uk/pg3/ddivers/ddivers.html>
- **Sum Sense - Division**
<http://www.primarygames.co.uk/pg2/sumsense/sumdiv.html>

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Long Division 1

Divide hundreds, tens, and ones separately.

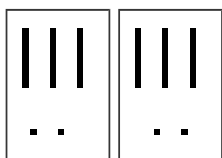
Write the dividend inside the long division “corner”, and the quotient on top.

$$\underline{64 \div 2 = ?}$$

Divide tens and ones separately:

$$6 \text{ tens} \div 2 = 3 \text{ tens (t)}$$

$$4 \text{ ones} \div 2 = 2 \text{ ones (o)}$$



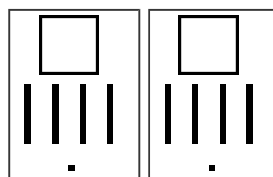
$$\begin{array}{r} \text{t o} \\ 32 \\ 2 \overline{) 64} \end{array}$$

$$\underline{282 \div 2 = ?}$$

$$2 \text{ hundreds} \div 2 = 1 \text{ hundred (h)}$$

$$8 \text{ tens} \div 2 = 4 \text{ tens (t)}$$

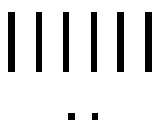
$$2 \div 2 = 1. \text{ (o)}$$



$$\begin{array}{r} \text{h t o} \\ 141 \\ 2 \overline{) 282} \end{array}$$

1. Make groups. Divide. Write the dividend inside the “corner” if it is missing.

a. Make 2 groups



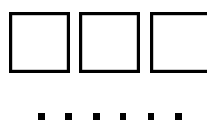
$$2 \overline{) 62}$$

b. Make 3 groups



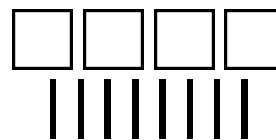
$$3 \overline{) \quad}$$

c. Make 3 groups



$$3 \overline{) \quad}$$

d. Make 4 groups



$$4 \overline{) \quad}$$

2. Divide thousands, hundreds, tens, and ones separately.

a. $4 \overline{) 84}$

b. $3 \overline{) 393}$

c. $3 \overline{) 660}$

d. $4 \overline{) 8040}$

e. $3 \overline{) 66}$

f. $6 \overline{) 6036}$

g. $3 \overline{) 330}$

h. $4 \overline{) 4804}$

<div style="text-align: center;"> h t o 0 4 $\overline{) 248}$ </div> <p>4 does not go into 2. You can put zero in the quotient in the hundreds place or omit it. But 4 does go into 24, six times. Put 6 in the quotient.</p>	<div style="text-align: center;"> h t o 0 6 2 4 $\overline{) 248}$ </div>	<div style="text-align: center;"> th h t o 0 5 $\overline{) 3505}$ </div>	<div style="text-align: center;"> th h t o 0 7 0 1 5 $\overline{) 3505}$ </div> <p>5 does not go into 3. You can put zero in the quotient. But 5 does go into 35, seven times.</p>
<p style="text-align: center;"><u>Explanation:</u></p> <p>The 2 of 248 is of course 200 in reality. If you divided 200 by 4, the result would be less than 100, so that is why the quotient won't have any whole hundreds.</p> <p>But then you combine the 2 hundreds with the 4 tens. That makes 24 tens, and you CAN divide 24 tens by 4. The result 6 tens goes as part of the quotient.</p> <p>Check the final answer: $4 \times 62 = 248$.</p>		<p style="text-align: center;"><u>Explanation:</u></p> <p>$3,000 \div 5$ will not give any whole thousands to the quotient because the answer is less than 1,000.</p> <p>But 3 thousands and 5 hundreds make 35 hundreds together. You can divide $3,500 \div 5 = 700$, and place 7 as part of the quotient in the hundreds place.</p> <p>Check the final answer: $5 \times 701 = 3,505$.</p>	
<p>If the divisor does not “go into” the first digit of the dividend, look at the <u>first two digits</u> of the dividend.</p>			

3. Divide. Check your answer by multiplying the quotient and the divisor.

a. $3 \overline{) 123}$

b. $4 \overline{) 284}$

c. $6 \overline{) 360}$

d. $8 \overline{) 248}$

e. $2 \overline{) 184}$

f. $7 \overline{) 427}$

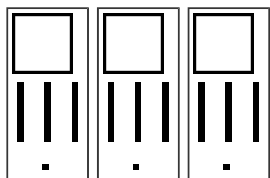
g. $3 \overline{) 1833}$

h. $4 \overline{) 2404}$

i. $7 \overline{) 4970}$

j. $5 \overline{) 4505}$

Ones division is not even. There is a remainder.



$$\begin{array}{r} \text{h} \quad \text{t} \quad \text{o} \\ 1 \quad 3 \\ 3 \overline{) 395} \end{array}$$

$$\begin{array}{r} \text{h} \quad \text{t} \quad \text{o} \\ 1 \quad 3 \quad 1 \text{ R}2 \\ 3 \overline{) 395} \end{array}$$

$395 \div 3 = 131 \text{ R}2$

3 goes into 3 one time.
3 goes into 9 three times.

3 goes into 5 one time, but not evenly.
Write the remainder 2 after the quotient.

$$\begin{array}{r} \text{h} \quad \text{t} \quad \text{o} \\ 0 \quad 4 \quad 1 \text{ R}1 \\ 4 \overline{) 165} \end{array}$$

4 does not go into 1 (hundred). So combine the 1 hundred with the 6 tens (160).

4 goes into 16 four times.

4 goes into 5 once, leaving a remainder of 1.

$$\begin{array}{r} \text{th} \quad \text{h} \quad \text{t} \quad \text{o} \\ 0 \quad 4 \quad 0 \quad 0 \text{ R}7 \\ 8 \overline{) 3207} \end{array}$$

8 does not go into 3 of the thousands. So combine the 3 thousands with the 2 hundreds (3,200).

8 goes into 32 four times ($3,200 \div 8 = 400$)

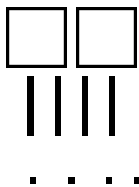
8 goes into 0 zero times (tens).

8 goes into 7 zero times, and leaves a remainder of 7.

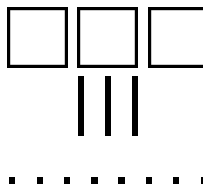
4. Divide into groups. Find the remainder.



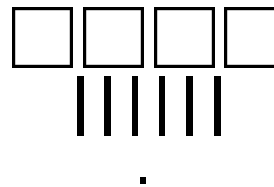
a. $2 \overline{) 63}$



b. $2 \overline{) \quad}$



c. $3 \overline{) \quad}$



d. $2 \overline{) \quad}$

5. Divide. Indicate the remainder if any.

a. $4 \overline{) 847}$

b. $2 \overline{) 69}$

c. $3 \overline{) 367}$

d. $4 \overline{) 89}$

e. $2 \overline{) 121}$

f. $6 \overline{) 1805}$

g. $7 \overline{) 215}$

h. $8 \overline{) 2482}$

In the problems before, you just wrote down the remainder of the ones. Usually, we write down the subtraction that actually finds the remainder. Look carefully:

$$\begin{array}{r} \text{h} \quad \text{t} \quad \text{o} \\ 0 \quad 6 \quad 1 \\ 4 \overline{) 247} \\ \underline{-4} \\ 3 \end{array}$$

When dividing the ones, 4 goes into 7 one time. Multiply $1 \times 4 = 4$, write that four under the 7, and subtract. This finds us the remainder of 3.

Check: $4 \times 61 + 3 = 247$

$$\begin{array}{r} \text{th} \quad \text{h} \quad \text{t} \quad \text{o} \\ 0 \quad 4 \quad 0 \quad 2 \\ 4 \overline{) 1609} \\ \underline{-8} \\ 1 \end{array}$$

When dividing the ones, 4 goes into 9 two times. Multiply $2 \times 4 = 8$, write that eight under the 9, and subtract. This finds us the remainder of 1.

Check: $4 \times 402 + 1 = 1,609$

6. Practice some more. Subtract to find the remainder in the ones. Check your answer by multiplying the divisor times the quotient, and then adding the remainder. You should get the dividend.

a. $3 \overline{) 128}$

b. $3 \overline{) 95}$

c. $6 \overline{) 4267}$

d. $4 \overline{) 2845}$

e. $5 \overline{) 5507}$

f. $2 \overline{) 8063}$

7. Divide these numbers mentally. Remember, you can always check by multiplying!

a. $440 \div 4 =$

$820 \div 2 =$

b. $3600 \div 400 =$

$369 \div 3 =$

c. $824 \div 2 =$

$560 \div 90 =$

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Divisibility

A number a is **divisible** by another number b if the division $a \div b$ is exact (no remainder).

For example, $18 \div 3 = 6$. So, 18 is divisible by 3. Also, 18 is divisible by 6, because we can write the other division $18 \div 6 = 3$. So, 18 is divisible by both 6 and 3.

We say 6 and 3 are **divisors** or **factors** of 18.

You can use long division to check if a number is divisible by another.

$67 \div 4 = 16, R3$. There is a remainder, so 67 is not divisible by 4.

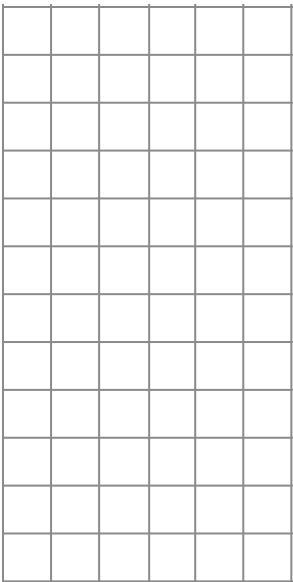
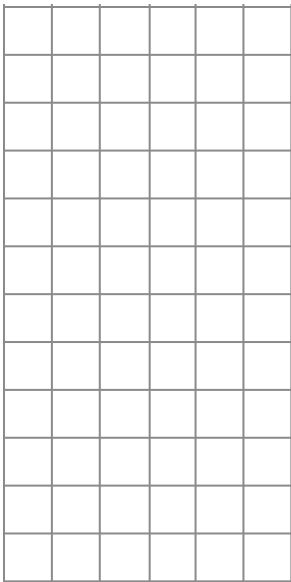
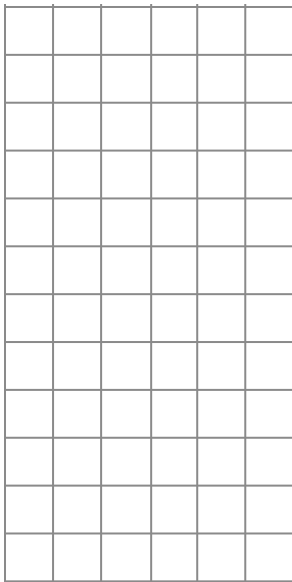
Also, from this we learn that neither 4 nor 16 is a factor (divisor) of 67.

$$\begin{array}{r} 16 \\ 4 \overline{) 67} \\ \underline{-4} \\ 27 \\ \underline{-24} \\ 3 \end{array}$$

1. Divide and determine if the numbers are divisible by the given number.

a. $21 \div 3 = \underline{\hspace{2cm}}$ Is 21 divisible by 3?	b. $40 \div 6 = \underline{\hspace{2cm}}$ Is 40 divisible by 6?	c. $17 \div 5 = \underline{\hspace{2cm}}$ Is 5 a divisor of 17?	d. $84 \div 7 = \underline{\hspace{2cm}}$ Is 7 a factor of 84?
---	---	---	--

2. Answer the questions. You may need long division.

a. Is 98 divisible by 4? 	b. Is 603 divisible by 7? 	c. Is 3 a factor of 1,256? 
--	---	--

In any multiplication, the numbers that are multiplied are called **factors** and the result is called a **product**.

factor		factor		product
7	\times	6	$=$	42

So, since $6 \times 7 = 42$, 6 and 7 are **factors** of 42.

From this multiplication fact we can write two divisions: $42 \div 6 = 7$ and $42 \div 7 = 6$. So, this also means that 42 is divisible by both 6 and 7.

Yet one more new word that ties in with all of this: **multiple**.

We say **42 is a multiple of 6**, because 42 is some number times 6, namely 7×6 .

And of course 42 is also a multiple of 7, because it is some number times 7!

3. Fill in.

Here's a multiplication fact: $8 \times 9 = 72$. So, 8 is a _____ of 72, and so is 9.

Also, 72 is a _____ of 8, and also 72 is a _____ of 9.

And, 72 is _____ by 8 and also by 9.

4. Fill in.

a. Is 5 a factor of 55? Yes, because ____ \times ____ = _____.	b. Is 8 a divisor of 45? No, because ____ \div ____ = _____.
c. Is 36 a multiple of 6? _____, because ____ \times ____ = _____.	d. Is 34 a multiple of 7? _____, because ____ \div ____ = _____.
e. Is 7 a factor of 46? _____, because _____.	f. Is 63 a multiple of 9? _____, because _____.

Multiples of 6 are all those numbers we get when we multiply 6 by other numbers. For example, we can multiply 0×6 , 7×6 , 11×6 , 109×6 , and so on, and the resulting numbers are all multiples of six.

In fact, the skip-counting pattern of 6 gives us a list of multiples of 6:

0, 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72, 78, 84, and so on.

5. **a.** Make a list of multiples of 11, starting at 0 and at least till 154.

b. Make a list of multiples of 111, starting at 0. Make it as long as you can in this space!

Divisibility by 2

Numbers that are divisible by 2 are called **even** numbers.
Numbers that are NOT divisible by 2 are called **odd** numbers.

Even numbers end in 0, 2, 4, 6, or 8. Every second number is even.

Divisibility by 5

Numbers that end in 0 and 5 are divisible by 5.

For example, 10, 35, 720, and 3,675 are such numbers.

6. Mark with “x” if the numbers are divisible by 2 or 5.

number	divisible		number	divisible		number	divisible		number	divisible	
	by 2	by 5		by 2	by 5		by 2	by 5		by 2	by 5
750			755			760			765		
751			756			761			766		
752			757			762			767		
753			758			763			768		
754			759			764			769		

Divisibility by 10

Numbers that end in 0 are divisible by 10.

For example, 10, 60, 340, and 2,570 are such numbers.

7. Mark an “x” if the numbers are divisible by 2 or 5 or 10.

number	divisible			number	divisible			number	divisible		
	by 2	by 5	by 10		by 2	by 5	by 10		by 2	by 5	by 10
860				865				870			
861				866				871			
862				867				872			
863				868				873			
864				869				874			

If a number is divisible by 10, it ends in zero, so it is ALSO divisible by ____ and ____.

8. **a.** Write a list of numbers divisible by 2, from 0 to 60.

This is also a list of _____ of 2.

- b.** In the list above, *underline* those numbers that are divisible by 4.
What do you notice?
- c.** In the list above, *color* those numbers that are divisible by 6.
What do you notice?
- d.** Which numbers are divisible by both 4 and 6?

9. **a.** Write a list of numbers divisible by 3, from 0 to 60.

This is also a list of _____ of 3.

- b.** In the list above, *underline* those numbers that are divisible by 6.
What do you notice?
- c.** In the list above, *color* those numbers that are divisible by 9.
What do you notice?

10. Use the lists you made in (7) and (8). Find numbers that are divisible by *both* 2 and 9.

11. What number is a factor of every number?

12. Twenty is a multiple of 4. It is also a multiple of 5. It is also a multiple of four other numbers.
Which ones?

Who am I?

(Hint: I am less than 50.)

Mystery Number
38 25 19
47 101 9

Divided by 9, I leave a remainder of 6.
Divided by 4, I leave a remainder of 1.
Divided by 10, I leave a remainder of 3.

Who am I?

(Hint: I am less than 100.)

Mystery Number
38 25 19
47 101 9

I am a multiple of 3, 4, 5, and 6.
I am a factor of 120.
Divided by 7, I leave a remainder of 4.

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Chapter 6: Geometry

Introduction

We start fourth grade geometry by reviewing the concepts of area and the perimeter of rectangles (from third grade). Students get to apply these concepts in problem solving, including problems where they write simple equations and explore possible perimeters for a given fixed area.

The focus of this chapter is angles. Students learn about lines, rays and angles, and about acute, right, obtuse, and straight angles. Next they learn how to measure and draw angles with a protractor. We also study angle problems where students write simple equations, and estimate some common angles.

The lesson *Parallel and Perpendicular Lines* ties in with the topic of angles, because perpendicular means to be at a right angle. Next we study parallelograms and other quadrilaterals in more detail, paying attention to the angles and side lengths in them.

We study triangles, and classify them according to the angles. Classifying triangles according to their sides (equilateral vs. isosceles triangles) is left for the 5th grade. The last topic for this chapter (an easy one) is line symmetry.

The study of geometry is full of strange-sounding words to learn. I encourage you to let the student(s) keep a *geometry notebook*, where they will write every new concept or term, and draw a picture or pictures and text to explain the term. The students could also do the drawing exercises from this chapter in this notebook. It will then become their very own geometry book, and while working with it, it helps them to learn and remember the terms and concepts better.

The Lessons in Chapter 6

	page	span
Review: Area of Rectangles	79	5 pages
Review: Area and Perimeter	84	4 pages
Lines, Rays, and Angles	88	5 pages
Measuring Angles	93	5 pages
Drawing Angles	98	2 pages
Angle Problems	100	5 pages
Estimating Angles	105	5 pages
Parallel and Perpendicular Lines	110	5 pages
Parallelograms	115	3 pages
Triangles	118	4 pages
Line Symmetry	122	3 pages
Mixed Review	125	2 pages
Review	127	4 pages

Helpful Resources on the Internet

Area and perimeter

Shape explorer

Find the perimeter and area of odd shapes on a rectangular grid.

<http://www.shodor.org/interactivate/activities/perimeter/index.html>

Math Playground: Measuring the Area and Perimeter of Rectangles

Amy and her brother, Ben, explain how to find the area and perimeter of rectangles and show you how changing the perimeter of a rectangle affects its area. After the lesson, you will use an interactive ruler to measure the length and width of 10 rectangles, and to calculate the perimeter and area of each.

http://www.mathplayground.com/area_perimeter.html

Math Playground: Party Designer

You need to design areas for the party, such as crafts table, food table, seesaw, and so on, so that they have the given perimeters and areas.

<http://www.mathplayground.com/PartyDesigner/PartyDesigner.html>

Geometry Area/Perimeter Quiz from ThatQuiz.org

An online quiz, about the area and perimeter of rectangles, triangles, and trapezoids. You can modify the quiz parameters to your liking, for example to omit a certain shape, or instead of solving for perimeter/area, you solve for an unknown side when perimeter/area is given.

<http://www.thatquiz.org/tq-4/?-j1200b-lc-p0>

Perimeter Game from Cyram.org

A simple online quiz for finding the perimeter of rectangles, triangles, or compound rectangles where not all side lengths are given.

<http://www.cyram.org/Projects/perimetergame/index.html>

FunBrain: Shape Surveyor Geometry Game

A simple & easy game that practices finding either the perimeter or area of rectangles.

<http://www.funbrain.com/poly/index.html>

Angles

Turtle Pond

Guide a turtle to a pond using commands, which include turning him in certain angles, or moving him a specific distance.

<http://illuminations.nctm.org/ActivityDetail.aspx?ID=83>

Banana hunt at Primary Games

Help the monkey to find bananas and learn to estimate angles.

<http://www.primarygames.co.uk/pg2/bhunt/bhunt.html>

Ladybug Leaf

Guide the ladybug by giving her commands to turn 90° or 45° , right or left, or to move forward/backward.

http://nlvm.usu.edu/en/nav/frames_asid_287_g_2_t_3.html

LadyBug Mazes

Similar to the Ladybug Leaf, but this time you guide the ladybug through the maze.

http://nlvm.usu.edu/en/nav/frames_asid_141_g_2_t_3.html

Shapes/Polygons

Interactive Quadrilaterals

See all the different kinds of quadrilaterals “in action”. You can drag the corners, see how the angles change, and observe what properties do not change.

<http://www.mathsisfun.com/geometry/quadrilaterals-interactive.html>

Dynamic Rectangle and Parallelogram

Drag the sides of a dynamic parallelogram or a rectangle to explore these concepts.

<http://standards.nctm.org/document/eexamples/chap5/5.3/index.htm>

Polygon Matching Game

Learn all the common polygons by playing this fun, timed matching game.

http://www.mathplayground.com/matching_shapes.html

Polygon Vocabulary

A matching game.

<http://www.quia.com/cc/2758.html>

Shapes Identification Quiz from ThatQuiz.org

An online quiz in a multiple-choice format, asking to identify common two-dimensional shapes. You can modify the quiz parameters to your liking.

<http://www.thatquiz.org/tq-f/math/shapes/>

General

Interactivate! Tessellate

An online, interactive tool for creating your own tessellations. Choose a shape, then edit its corners or edges. The program automatically changes the shape so that it will tessellate (tile) the plane. Then push the tessellate button to see your creation!

<http://www.shodor.org/interactivate/activities/Tessellate>

Patch Tool

An online activity where the student designs a pattern using geometric shapes.

<http://illuminations.nctm.org/ActivityDetail.aspx?ID=27>

Polygon Playground

Drag various colorful polygons to the work area to make your own creations!

<http://www.mathcats.com/explore/polygons.html>

Interactive Tangram Puzzle

Place the tangram pieces so they form the given shape.

http://nlvm.usu.edu/en/nav/frames_asid_112_g_2_t_1.html

Tangram set

Cut out your Tangram set by folding paper

<http://tangrams.ca/fold-set>

Logic Tangram game

Note: this uses four pieces only. Use logic and spatial reasoning skills to assemble the four pieces into the given shape.

<http://www.mathplayground.com/tangrams.html>

Geometry worksheets & quizzes

Worksheets about complementary and supplementary angles, parallel, perpendicular, and intersecting lines, types of angles, basic shapes, area & perimeter of rectangles, and parts of a circle.

[http://www.dadsworksheets.com/v1/Worksheets/Basic Geometry.html](http://www.dadsworksheets.com/v1/Worksheets/Basic%20Geometry.html)

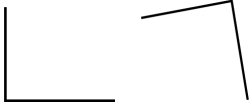
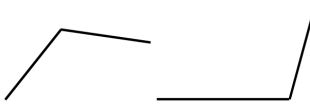
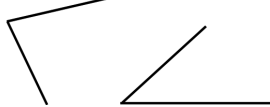
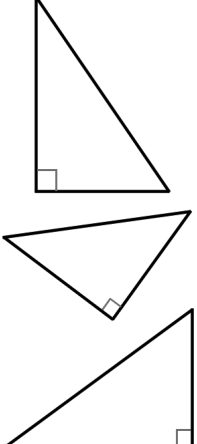
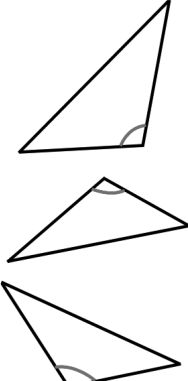
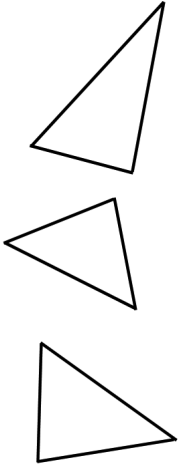
Geometry worksheets & quizzes

A bunch of PDF worksheets on geometry topics for elementary level, plus online quizzes.

<http://www.math4children.com/Topics/Geometry>

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Triangles

 <p>right angles (exactly 90°)</p>	 <p>obtuse angles (more than 90°, less than 180°)</p>	 <p>acute angles (less than 90°)</p>
 <p>Right triangles have exactly one right angle.</p>	 <p>Obtuse triangles have exactly one obtuse angle.</p>	 <p>Acute triangles have three acute angles. In other words, ALL the angles are acute.</p>

1. **a.** Draw a right *angle*. Then make it into a right *triangle* by drawing in the third side.
- b.** Draw another, different right triangle.
- c.** A right triangle has one right angle. Are the other two angles in a right triangle acute, right, or obtuse?



A right triangle has one right angle. The other two angles are _____.

2. **a.** Draw an obtuse angle.
Then make it into an obtuse triangle by drawing in the third side.



- b.** Draw another, different obtuse triangle.

- c.** An obtuse triangle has one obtuse angle. Are the other two angles in a obtuse triangle acute, right, or obtuse?



An obtuse triangle has one obtuse angle. The other two angles are _____.

3. **a.** Draw an acute triangle.
The side lengths can be any.

- b.** Measure its angles.

They measure _____ $^{\circ}$,
_____ $^{\circ}$, and _____ $^{\circ}$.



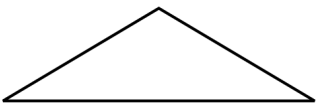
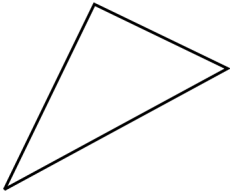
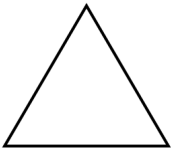
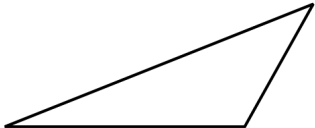
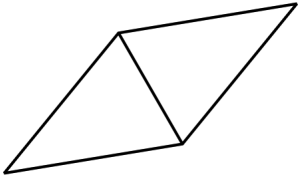
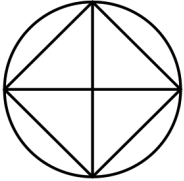
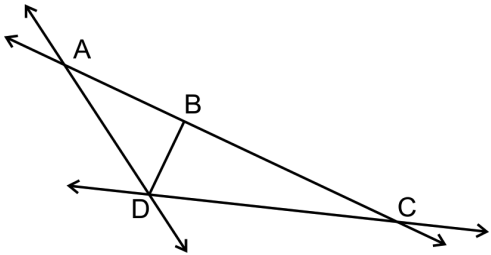
4. Observe all you have done thus far in this lesson, and fill in.

Right triangles have exactly 1 _____, _____,
and the other two angles are _____.


Obtuse triangles have exactly 1 _____, _____,
and the other two angles are _____.

Acute triangles have _____ angles.

5. Label the triangles in the pictures as right, acute, or obtuse.

<p>a.</p> 	<p>b.</p> 	<p>c.</p> 	<p>d.</p> 
<p>e.</p> 	<p>f.</p> 	<p>g.</p> 	

6. Label the triangles in the pictures as right, acute, or obtuse.

<p>a.</p>  <p>National Museum of Marine Corps Photo by Ron Cogswell</p> <p>_____</p>	<p>b.</p>  <p>The City Center of Woburn, MA Photo by Randy Robertson</p> <p>_____</p>	<p>c.</p>  <p>The White House Photo by Glyn Lower</p> <p>_____</p>
<p>d.</p>  <p>Waterlily Photo by Francis Chung</p> <p>_____</p>	<p>e.</p>  <p>Warning of a steep hill ahead Photo by www.flickr.com/photos/shirokazan/</p> <p>The black triangle is _____</p> <p>The red triangle is _____</p>	

When you go outside, find acute, obtuse, and right triangles in nature, in buildings, in signs, and so on!

7. **a.** Draw a triangle with 85° and 40° angles.

Hint: First draw a 85° angle. Then, mark a point anywhere on one side of that angle to be the second vertex of the triangle. Use that point as a vertex for the 40° angle, and draw the 40° angle.

- b.** Measure the third angle.
It is _____ degrees.
- c.** What kind of triangle is it?
(acute, right, obtuse)
- d.** What is the angle sum?



8. **a.** Draw a triangle with 125° and 40° angles.

- b.** Measure the third angle.
It is _____ degrees.
- c.** What kind of triangle is it?
(acute, right, obtuse)
- d.** What is the angle sum?



9. **a.** Draw a triangle with 55° and 35° angles.

- b.** Measure the third angle.
It is _____ degrees.
- c.** What kind of triangle is it?
(acute, right, obtuse)
- d.** What is the angle sum?



New Terms	
• <i>an acute triangle</i>	• <i>an obtuse triangle</i>
• <i>a right triangle</i>	• <i>a diagonal</i>

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Chapter 7: Fractions

Introduction

In the third grade, children studied the concept of a fraction, equivalent fractions, and compared some easy fractions. In fourth grade, it is time to expand the fraction topics. We study

- mixed numbers
- adding and subtracting like fractions and mixed numbers with like fractional parts (the denominators are the same)
- equivalent fractions
- comparing fractions
- multiplying a fraction by a whole number

Then in fifth grade, students tackle *all* of the four operations with fractions. Our studies here are still laying groundwork for that, emphasizing conceptual understanding and using visual models a lot.

These lessons are also important because they are the basis for understanding decimal numbers, the topic of the next chapter. Decimals are just another way of writing fractions with denominators 10, 100, 1,000 etc.

The topics in this chapter are studied with the help of visual models in order to emphasize the concepts. We must avoid presenting fraction math as a list of computational rules. Children easily confuse the various fraction rules, because there are so many, such as:

- a rule for converting a mixed number to a fraction, and vice versa
- a rule for adding like fractions
- a rule for finding a common denominator
- a rule for changing fractions to like fractions
- a rule for adding unlike fractions
- a rule for simplifying fractions
- a rule for finding equivalent fractions
- a rule for multiplying fractions
- a rule for dividing fractions
- a few rules for doing the four operations with mixed numbers

There is a place for the rules, as *shortcuts* for ideas that are already understood, but we do not start with them. The goal is to let the big ideas sink in conceptually first, followed by some shortcuts.

The Lessons in Chapter 7

	page	span
One Whole and its Fractional Parts	135	3 pages
Mixed Numbers	138	4 pages
Adding Fractions and Mixed Numbers 1	142	4 pages
Adding Fractions and Mixed Numbers 2	146	3 pages
Equivalent Fractions	149	5 pages
Subtracting Fractions and Mixed Numbers	154	3 pages
Comparing Fractions	157	4 pages
Multiplying Fractions by Whole Numbers	161	3 pages
Practicing With Fractions	164	2 pages
Mixed Review	166	2 pages
Review	168	2 pages

Helpful Resources and Games on the Internet

General

Visual Fractions

Great site for studying all aspects of fractions: identifying, renaming, comparing, addition, subtraction, multiplication, division. Each topic is illustrated by either a number line or a circle with a Java applet. Also a couple of games, for example: make cookies for Grampy.

<http://www.visualfractions.com/>

Conceptua Math Fraction Tools

Free and interactive fraction tools for identifying fractions, adding and subtracting, estimating, comparing, equivalent fractions, finding common denominators and more. Each activity uses several fraction models such as fraction circles, horizontal and vertical bars, number lines, etc. that allow students to develop conceptual understanding of fractions. Free registration required.

<https://www.conceptuamath.com/app/tool-library>

Fraction Games at Sheppard Software

Games for addition & subtraction of fractions, simplifying fractions, equivalent fractions, and a fraction of a set.

<http://www.sheppardsoftware.com/math.htm#fractions>

Who Wants pizza?

This site explains the concept of fractions, addition, and multiplication with a pizza example, then has some interactive exercises.

<http://math.rice.edu/~lanius/fractions/index.html>

Fractioncity

Make “fraction streets” and help children with comparing fractions, equivalent fractions, addition of fractions of like and unlike denominators while they drive toy cars on the streets. This is not an online activity but has instructions of how to do it at home or at school.

<http://www.teachnet.com/lesson/math/fractioncity.html>

Fraction Worksheets: Equivalent Fractions, Simplifying, Convert to Mixed Numbers

Create custom-made worksheets for some other fraction operations.

<http://www.homeschoolmath.net/worksheets/fraction-b.php>

Fractions and mixed numbers

Identifying Fractions at Conceptua Fractions

A tool that shows fractions or mixed numbers using a pie, a bar, dots, and a number line. A free registration required.

<https://www.conceptuamath.com/app/tool/identifying-fractions>

Visualizing Fractions

The computer shows a fraction, and you divide the pie and color the pieces.

http://nlvm.usu.edu/en/nav/frames_asid_103_g_2_t_1.html

Pattern Blocks—Parts as Wholes

Click on the “Activities” in the top menu, and click on arrows until you find Parts as Wholes activity.

http://nlvm.usu.edu/en/nav/frames_asid_170_g_2_t_3.html

Fraction Model

Adjust the the numerator and the denominator, and the applet shows the fraction as a pie/rectangle/set model, as a decimal and as a percent.

<http://illuminations.nctm.org/ActivityDetail.aspx?ID=44>

Clara Fraction's Ice Cream Shop

Convert improper fractions to mixed numbers and scoop the right amount of ice cream flavors onto the cone.

<http://www.mrnussbaum.com/icecream/index.html>

Addition and subtraction

MathSplat

Click on the right answer for addition problems or the bug splats on your windshield!

<http://fen.com/studentactivities/MathSplat/mathsplat.htm>

Action Fraction

A racing game with several levels where you answer questions about adding and subtraction fractions. The levels advance from using like fractions to using unlike fractions and eventually subtraction.

http://funschool.kaboose.com/formula-fusion/number-fun/games/game_action_fraction.html

Fraction Worksheets: Addition and Subtraction

Create custom-made worksheets for the four operations with fractions and mixed numbers. Choose “Like Fractions” for this level.

<http://www.homeschoolmath.net/worksheets/fraction.php>

Comparing Fractions

Comparison Shoot Out

Choose level 2 or 3 to compare fractions and shoot the soccer ball to the goal.

<http://www.fuelthebrain.com/Game/play.php?ID=47>

Comparing Fractions—XP Math

Simple timed practice with comparing two fractions.

<http://xpmath.com/forums/arcade.php?do=play&gameid=8>

Ordering Fractions at Conceptua Fractions

An interactive tool where students place numbers, visual models, and decimals on a number line.

<http://www.conceptuamath.com/fractions.html#OrderingFractions>

Fractional Hi Lo

The computer has selected a fraction. You guess and it tells you if your guess was too high or too low.

<http://www.theproblemsite.com/games/hilo.asp>

Equivalent fractions

Equivalent Fractions from National Library of Virtual Manipulatives (NLVM)

See the equivalency of two fractions as the applet divides the whole into more pieces.

http://nlvm.usu.edu/en/nav/frames_asid_105_g_2_t_1.html

Equivalent Fractions

Draw two equivalent fractions for the given fraction. Choose either a square or a circle for the shape.

<http://illuminations.nctm.org/ActivityDetail.aspx?ID=80>

Fraction Frenzy

Click on pairs of equivalent fractions, as fast as you can. See how many levels you can get!

<http://www.learningplanet.com/sam/ff/index.asp>

Fresh Baked Fractions

Practice equivalent fractions by clicking on a fraction that is not equal to others.

<http://www.funbrain.com/fract/index.html>

Free Equivalent Fractions Worksheets

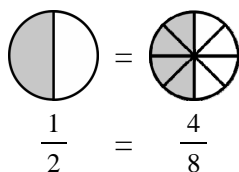
Create custom-made worksheets for equivalent fractions that can either include pie images or not.

http://www.homeschoolmath.net/worksheets/equivalent_fractions.php

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Equivalent Fractions

If you eat half of a pizza, or $\frac{4}{8}$ of a pizza, you have eaten the same amount.



$\frac{1}{2}$ and $\frac{4}{8}$ are **equivalent fractions**.



The two fraction strips show an equal amount. So, we can write an equal sign between the two mixed numbers:

$$1\frac{1}{5} = 1\frac{2}{10}$$

1. Color the first fraction. Shade the same *amount of pie* in the second picture. Write the second fraction.

<p>a. = =</p> <p>$\frac{1}{2} =$</p>	<p>b. = =</p> <p>$\frac{3}{4} =$</p>	<p>c. = =</p> <p>$\frac{6}{10} =$</p>	<p>d. = =</p> <p>$\frac{8}{12} =$</p>
<p>e. = =</p> <p>$\frac{1}{3} =$</p>	<p>f. = =</p> <p>$1\frac{2}{3} =$</p>		<p>g. = =</p> <p>$1\frac{10}{12} =$</p>

2. Write the fractions that have thirds using sixths instead. You can shade parts in the pictures.

	a. $\frac{3}{3} =$	b. $\frac{4}{3} =$	c. $\frac{7}{3} =$
	d. $2\frac{1}{3} =$	e. $1\frac{2}{3} =$	f. $2\frac{2}{3} =$

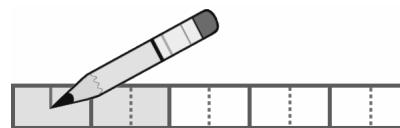
3. Mark the equivalent fractions on the number lines.

<p>a. = =</p> <p>$\frac{4}{5} =$</p>	<p>b. = =</p> <p>$\frac{3}{9} =$</p>
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The fraction strip illustrates $\frac{2}{5}$. If you split each piece (both the colored and white pieces) into *two* new pieces, what fraction do you get?

You get $\frac{4}{10}$ – four colored pieces, and ten pieces total.

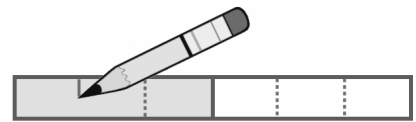
You have *two* times as many colored pieces, and *two* times as many total pieces as before.



4. Split both the colored and white pieces as instructed. Write the fraction after you change it.

<p>a. Split all the pieces into two new ones.</p> <div style="border: 1px solid black; width: 100px; height: 20px; margin: 10px 0; position: relative;"> <div style="background-color: gray; width: 40%; position: absolute; left: 0;"></div> </div> <div style="display: flex; align-items: center; justify-content: center; margin-top: 10px;"> $\frac{1}{2} =$ <div style="border: 1px solid black; width: 40px; height: 40px; position: relative; margin: 0 10px;"> <div style="background-color: gray; width: 50%; height: 50%; position: absolute; top: 0; left: 0;"></div> </div> </div>	<p>b. Split all the pieces into four new ones.</p> <div style="border: 1px solid black; width: 100px; height: 20px; margin: 10px 0; position: relative;"> <div style="background-color: gray; width: 40%; position: absolute; left: 0;"></div> </div> <div style="display: flex; align-items: center; justify-content: center; margin-top: 10px;"> $\frac{1}{2} =$ <div style="border: 1px solid black; width: 40px; height: 40px; position: relative; margin: 0 10px;"> <div style="background-color: gray; width: 50%; height: 50%; position: absolute; top: 0; left: 0;"></div> </div> </div>	<p>c. Split all the pieces into three new ones.</p> <div style="border: 1px solid black; width: 100px; height: 20px; margin: 10px 0; position: relative;"> <div style="background-color: gray; width: 20%; position: absolute; left: 0;"></div> </div> <div style="display: flex; align-items: center; justify-content: center; margin-top: 10px;"> $\frac{1}{4} =$ <div style="border: 1px solid black; width: 40px; height: 40px; position: relative; margin: 0 10px;"> <div style="background-color: gray; width: 50%; height: 50%; position: absolute; top: 0; left: 0;"></div> </div> </div>
<p>d. Split all the pieces into three new ones.</p> <div style="border: 1px solid black; width: 100px; height: 20px; margin: 10px 0; position: relative;"> <div style="background-color: gray; width: 33%; position: absolute; left: 0;"></div> </div> <div style="display: flex; align-items: center; justify-content: center; margin-top: 10px;"> $\frac{1}{3} =$ <div style="border: 1px solid black; width: 40px; height: 40px; position: relative; margin: 0 10px;"> <div style="background-color: gray; width: 50%; height: 50%; position: absolute; top: 0; left: 0;"></div> </div> </div>	<p>e. Split all the pieces into two new ones.</p> <div style="border: 1px solid black; width: 100px; height: 20px; margin: 10px 0; position: relative;"> <div style="background-color: gray; width: 83%; position: absolute; left: 0;"></div> </div> <div style="display: flex; align-items: center; justify-content: center; margin-top: 10px;"> $\frac{5}{6} =$ <div style="border: 1px solid black; width: 40px; height: 40px; position: relative; margin: 0 10px;"> <div style="background-color: gray; width: 50%; height: 50%; position: absolute; top: 0; left: 0;"></div> </div> </div>	<p>f. Split all the pieces into three new ones.</p> <div style="border: 1px solid black; width: 100px; height: 20px; margin: 10px 0; position: relative;"> <div style="background-color: gray; width: 66%; position: absolute; left: 0;"></div> </div> <div style="display: flex; align-items: center; justify-content: center; margin-top: 10px;"> $\frac{2}{5} =$ <div style="border: 1px solid black; width: 40px; height: 40px; position: relative; margin: 0 10px;"> <div style="background-color: gray; width: 50%; height: 50%; position: absolute; top: 0; left: 0;"></div> </div> </div>
<p>Can you notice a <i>shortcut</i> for finding the second fraction without using a picture?</p>		
<p>g. Split all the pieces into four new ones.</p> <div style="border: 1px solid black; width: 100px; height: 20px; margin: 10px 0; position: relative;"> <div style="background-color: gray; width: 66%; position: absolute; left: 0;"></div> </div> <div style="display: flex; align-items: center; justify-content: center; margin-top: 10px;"> <div style="border: 1px solid black; width: 40px; height: 40px; position: relative; margin: 0 10px;"> <div style="background-color: gray; width: 50%; height: 50%; position: absolute; top: 0; left: 0;"></div> </div> $=$ <div style="border: 1px solid black; width: 40px; height: 40px; position: relative; margin: 0 10px;"> <div style="background-color: gray; width: 50%; height: 50%; position: absolute; top: 0; left: 0;"></div> </div> </div>	<p>h. Split all the pieces into two new ones.</p> <div style="border: 1px solid black; width: 100px; height: 20px; margin: 10px 0; position: relative;"> <div style="background-color: gray; width: 80%; position: absolute; left: 0;"></div> </div> <div style="display: flex; align-items: center; justify-content: center; margin-top: 10px;"> <div style="border: 1px solid black; width: 40px; height: 40px; position: relative; margin: 0 10px;"> <div style="background-color: gray; width: 50%; height: 50%; position: absolute; top: 0; left: 0;"></div> </div> $=$ <div style="border: 1px solid black; width: 40px; height: 40px; position: relative; margin: 0 10px;"> <div style="background-color: gray; width: 50%; height: 50%; position: absolute; top: 0; left: 0;"></div> </div> </div>	<p>i. Split all the pieces into three new ones.</p> <div style="border: 1px solid black; width: 100px; height: 20px; margin: 10px 0; position: relative;"> <div style="background-color: gray; width: 80%; position: absolute; left: 0;"></div> </div> <div style="display: flex; align-items: center; justify-content: center; margin-top: 10px;"> <div style="border: 1px solid black; width: 40px; height: 40px; position: relative; margin: 0 10px;"> <div style="background-color: gray; width: 50%; height: 50%; position: absolute; top: 0; left: 0;"></div> </div> $=$ <div style="border: 1px solid black; width: 40px; height: 40px; position: relative; margin: 0 10px;"> <div style="background-color: gray; width: 50%; height: 50%; position: absolute; top: 0; left: 0;"></div> </div> </div>
<p><u>If</u> you found the shortcut, explain how it works in these problems:</p>	<p>Split all the pieces into three new ones.</p> <div style="display: flex; align-items: center; justify-content: center; margin-top: 10px;"> $\frac{1}{3} =$ <div style="border: 1px solid black; width: 40px; height: 40px; position: relative; margin: 0 10px;"> <div style="background-color: gray; width: 50%; height: 50%; position: absolute; top: 0; left: 0;"></div> </div> </div>	<p>Split all the pieces into two new ones.</p> <div style="display: flex; align-items: center; justify-content: center; margin-top: 10px;"> $\frac{3}{5} =$ <div style="border: 1px solid black; width: 40px; height: 40px; position: relative; margin: 0 10px;"> <div style="background-color: gray; width: 50%; height: 50%; position: absolute; top: 0; left: 0;"></div> </div> </div>

The fraction strip illustrates $\frac{1}{2}$. If we split each piece (both the colored and the white piece) into *three* new pieces, we get $\frac{3}{6}$.



We now have *three* times as many colored pieces, and *three* times as many total pieces as before. We can show this in writing this way:

$$\frac{1}{2} = \frac{3}{6}$$

$\times 3$
 \swarrow
 \searrow
 $\times 3$

We multiply both the top and bottom number in a fraction by 3. We get an equivalent fraction—it is the **same amount**, just cut into more pieces. *This does not mean we multiply the whole fraction by 3.*

5. Split the pieces. Fill in the missing parts.



a. This is $\frac{3}{4}$. Make it $\frac{9}{12}$.

Each piece is split into ____ new ones.

$$\frac{3}{4} = \frac{9}{12}$$

\times ____
 \swarrow
 \searrow
 \times ____



b. This is $\frac{1}{3}$. Make it $\frac{4}{12}$.

Each piece is split into ____ new ones.

$$\frac{1}{3} = \frac{4}{12}$$

\times ____
 \swarrow
 \searrow
 \times ____



c. This is $\frac{1}{2}$. Make it $\frac{5}{10}$.

Each piece is split into ____ new ones.

$$\frac{1}{2} = \frac{5}{10}$$

\times ____
 \swarrow
 \searrow
 \times ____



d. This is $\frac{1}{4}$. Make it $\frac{4}{16}$.

$$\frac{1}{4} = \frac{4}{16}$$

\times ____
 \swarrow
 \searrow
 \times ____



e. This is $\frac{2}{3}$. Make it $\frac{6}{9}$.

$$\frac{2}{3} = \frac{6}{9}$$

\times ____
 \swarrow
 \searrow
 \times ____



f. This is $\frac{2}{3}$. Make it $\frac{8}{12}$.

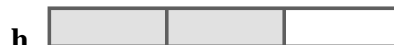
$$\frac{2}{3} = \frac{8}{12}$$

\times ____
 \swarrow
 \searrow
 \times ____



$$\frac{4}{5} = \frac{\boxed{}}{10}$$

\times ____
 \swarrow
 \searrow
 \times ____



$$\frac{2}{3} = \frac{\boxed{}}{15}$$

\times ____
 \swarrow
 \searrow
 \times ____



$$\frac{2}{5} = \frac{\boxed{}}{15}$$

\times ____
 \swarrow
 \searrow
 \times ____

6. Write the equivalent fraction. Use multiplication.

a. Split all the pieces into three new ones. $\frac{5}{6} = \frac{\square}{\square}$	b. Split all the pieces into five new ones. $\frac{3}{4} = \frac{\square}{\square}$	c. Split all the pieces into four new ones. $\frac{2}{5} = \frac{\square}{\square}$	d. Split all the pieces into ten new ones. $\frac{9}{10} = \frac{\square}{\square}$
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7. Figure out how many new pieces the existing pieces were split into. Fill in the missing parts.

a. Pieces were split into ____ new ones. $\frac{1}{2} = \frac{\square}{6}$	b. Pieces were split into ____ new ones. $\frac{3}{10} = \frac{30}{\square}$	c. Pieces were split into ____ new ones. $\frac{2}{5} = \frac{\square}{30}$	d. Pieces were split into ____ new ones. $\frac{7}{8} = \frac{35}{\square}$
e. $\frac{2}{3} = \frac{\square}{6}$	f. $\frac{3}{5} = \frac{9}{\square}$	g. $\frac{5}{6} = \frac{\square}{12}$	h. $\frac{1}{3} = \frac{\square}{9}$

8. Write the fractions that have tenths with hundredths instead.

a. $\frac{1}{10} = \frac{\square}{100}$	b. $\frac{3}{10} = \frac{\square}{\square}$	c. $\frac{6}{10} = \frac{\square}{\square}$	d. $\frac{4}{10} = \frac{\square}{\square}$	e. $\frac{13}{10} = \frac{\square}{\square}$
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9. Connect the equivalent fractions with a line.

a.	$\frac{2}{3}$	$\frac{1}{3}$
	$\frac{1}{4}$	$\frac{1}{2}$
	$\frac{5}{10}$	$\frac{2}{8}$
	$\frac{2}{6}$	$\frac{6}{9}$

b.	$\frac{1}{2}$	$\frac{2}{10}$
	$\frac{3}{4}$	$\frac{1}{3}$
	$\frac{1}{5}$	$\frac{6}{12}$
	$\frac{4}{12}$	$\frac{9}{12}$

c.	$\frac{3}{6}$	$\frac{3}{12}$
	$\frac{1}{4}$	$\frac{1}{2}$
	$\frac{1}{3}$	$\frac{8}{12}$
	$\frac{2}{3}$	$\frac{4}{12}$

10. Write chains of equivalent fractions!

a. $\frac{1}{2} = \frac{\square}{4} = \frac{\square}{6} = \frac{\square}{8} = \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square}$	b. $\frac{1}{3} = \frac{\square}{6} = \frac{\square}{9} = \frac{\square}{12} = \frac{\square}{\square}$
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We can use equivalent fractions to add fractions that have different denominators.

Example. Add $\frac{2}{10} + \frac{17}{100}$. First, write $\frac{2}{10}$ as $\frac{20}{100}$ (an equivalent fraction).

Then you can add, because the fractions now have the same denominator: $\frac{20}{100} + \frac{17}{100} = \frac{37}{100}$.

11. Add.

a. $\frac{1}{10} + \frac{8}{100}$ $\downarrow \quad \downarrow$ $\frac{\square}{100} + \frac{8}{100} =$	b. $\frac{7}{10} + \frac{3}{100}$ $\downarrow \quad \downarrow$ $\frac{\square}{100} + \frac{\square}{100} =$	c. $\frac{45}{100} + \frac{3}{10}$
d. $\frac{9}{10} + \frac{9}{100}$	e. $\frac{7}{10} + \frac{23}{100}$	f. $\frac{24}{100} + \frac{9}{10}$
g. $\frac{7}{100} + 1\frac{4}{10}$	h. $2\frac{28}{100} + 1\frac{5}{10}$	i. $\frac{6}{10} + \frac{35}{100} + \frac{7}{100}$

12. Draw a picture showing that $\frac{1}{3}$ and $\frac{4}{12}$ are equivalent fractions.

Puzzle Corner

Add. This is challenging. *Hint: You cannot simply add the top numbers and the bottom numbers. Use equivalent fractions.*

a. $\frac{3}{4} + \frac{1}{2}$	b. $\frac{1}{5} + \frac{3}{10}$	c. $\frac{2}{3} + \frac{2}{9}$
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Chapter 8: Decimals

Introduction

In fourth grade, we study decimal numbers with one or two decimal digits, and add and subtract them. It is important that the student grasps these simple topics well, because we are laying a groundwork towards fifth and sixth grade, where decimal operations and using decimals take a “center stage.”

For now, the focus is first of all, understanding the fact that decimals are simply fractions with a denominator 10 or 100. Then with that in mind (decimals are fractions), we study comparing, adding, and subtracting them.

Notice:

- In the addition problem $0.5 + 0.9$, we get 14 tenths, which is 1.4. A common student misconception is to add $0.5 + 0.9 = 0.14$.
- In a problem such as $0.5 + 0.11$, a common student misconception is to get 0.16. Such students are thinking of the decimal parts as if they were “whole numbers.” To solve $0.5 + 0.11$ correctly, students can rewrite 0.5 as 0.50, and then the problem becomes $0.50 + 0.11 = 0.61$.

In the lesson Using Decimal Numbers, students use decimals with some metric measuring units, including converting between units. This topic will also be studied further in 5th grade.

The Lessons in Chapter 8

	page	span
Decimal Numbers—Tenths.....	172	2 <i>pages</i>
Adding and Subtracting with Tenths	174	2 <i>pages</i>
Two Decimal Digits—Hundredths	176	4 <i>pages</i>
Adding and Subtracting Hundredths	180	4 <i>pages</i>
Adding and Subtracting Decimals in Columns	184	3 <i>pages</i>
Using Decimals with Measuring Units	187	2 <i>pages</i>
Mixed Review	189	2 <i>pages</i>
Review	191	2 <i>pages</i>

Helpful Resources on the Internet

Mathematical Interactivities

<http://mathematics.hellam.net/>

Find several games related to fractions and decimals in the **Number Puzzles** section, including:

- **Decimal Challenge** - Guess the decimal number between 0 and 10.
Each time feedback tells whether your guess was too high or too low.
<http://www.interactivestuff.org/sums4fun/decchall.html>
- **Switch** - Put the sequence of decimal numbers into ascending order by switching them around.
Refresh the page from your browser to get another problem to solve.
<http://www.interactivestuff.org/sums4fun/switch.html>
- **Scales** - Move the pointer to match the decimal number given to you.
Refresh the page from your browser to get another problem to solve.
<http://www.interactivestuff.org/sums4fun/scales.html>

A Decimal Puzzle

Make every circle add up to 3.

http://nlvm.usu.edu/en/nav/frames_asid_187_g_2_t_1.html?open=instructions&from=category_g_2_t_1.html

Fraction/Decimal Worksheets

Change fractions to decimal numbers or decimal numbers to fractions.

<http://www.homeschoolmath.net/worksheets/fraction-decimal.php>

Modeling Decimals (Area and Grid Models)

An interactive “gizmo” for modeling decimals in a grid or on a number. By subscription, but you can try the gizmo for free for 5 minutes.

<http://www.explorelearning.com/index.cfm?method=cResource.dspDetail&ResourceID=1007>

Adding Decimals (Base 10 Blocks)

An interactive “gizmo” for modeling decimal addition with regrouping. By subscription, but you can try the gizmo for free for 5 minutes.

<http://www.explorelearning.com/index.cfm?method=cResource.dspDetail&ResourceID=1023>

Subtracting Decimals (Base 10 Blocks)

An interactive “gizmo” for modeling decimal subtraction with regrouping. By subscription, but you can try the gizmo for free for 5 minutes.

<http://www.explorelearning.com/index.cfm?method=cResource.dspDetail&ResourceID=1030>

Beat the Clock

Type in the decimal to show much of the square is shaded in this timed game.

<http://www.decimalsquares.com/dsGames/games/beatclock.html>

Decimal Darts

Try to pop the balloons with darts by estimating at which height the balloons are.

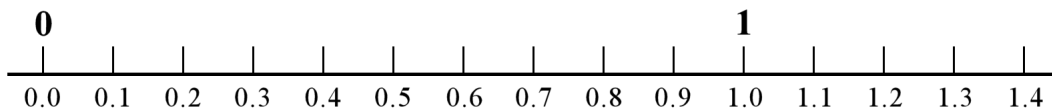
<http://www.decimalsquares.com/dsGames/games/darts.html>

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Adding and Subtracting with Tenths

<p>You <i>already</i> know how to add or subtract decimals with tenths. They are just fractions with a denominator of 10.</p> <p>Compare these additions that are written with decimals or fractions.</p>	$0.1 + 0.5 = 0.6$ $\frac{1}{10} + \frac{5}{10} = \frac{6}{10}$	$8.4 - 2.3 = 6.1$ $8\frac{4}{10} - 2\frac{3}{10} = 6\frac{1}{10}$
<p>There is one tricky part though: $0.6 + 0.7$ is <u>NOT</u> 0.13 !!</p> <p>To see why, add the fractions. Notice that six tenths and seven tenths makes more than one whole!</p>	$0.6 + 0.7 = 1.3$ $\frac{6}{10} + \frac{7}{10} = \frac{13}{10} = 1\frac{3}{10}$	$1.5 + 0.9 = 2.4$ $1\frac{5}{10} + \frac{9}{10} = 2\frac{4}{10}$

1. Write an addition *or* subtraction sentence for each “number line jump.”



- a. You're at 0.7, and you jump *five tenths* to the right. _____
- b. You're at 0.6, and you jump *eight tenths* to the right. _____
- c. You're at 1.1, and you jump *eight tenths* to the left. _____
- d. You're at 1.3, and you jump *four tenths* to the left. _____
- e. You're at 0.2, and you jump *eleven tenths* to the right. _____

2. Solve the fraction additions, and then write them using decimals.

<p>a. $\frac{2}{10} + \frac{7}{10} =$</p> <p>$0.2 +$</p>	<p>b. $\frac{5}{10} + \frac{6}{10} =$</p>	<p>c. $\frac{9}{10} + \frac{8}{10} =$</p>
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3. Add and subtract.

a.	b.	c.	d.
$0.9 + 0.2 =$ _____	$0.5 + 0.7 =$ _____	$0.8 + 0.7 =$ _____	$1.8 - 0.9 =$ _____
$1.9 + 0.2 =$ _____	$3.5 + 0.7 =$ _____	$0.8 + 2.7 =$ _____	$5.8 - 0.9 =$ _____

4. Fill in the missing parts.

a. $2.3 + 0.9 = \underline{\hspace{2cm}}$	b. $1.5 + 0.7 = \underline{\hspace{2cm}}$	c. $6.6 - 0.5 = \underline{\hspace{2cm}}$	d. $4.7 - 1.7 = \underline{\hspace{2cm}}$
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5. Write the numbers.

a. 3 tenths, 5 ones	d. Write the numbers in order. 9 8.9 9.1 9.0 9.9 1.9
b. 7 tens, 8 ones, 4 tenths	
c. 4 tenths, 3 ones, 6 tens	

6. Continue the patterns by adding or subtracting the same number repeatedly.

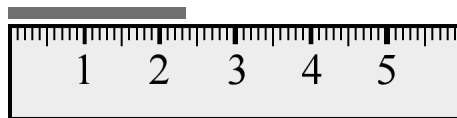
a. 0.1 $+ 0.2 = \underline{\hspace{2cm}}$ $+ 0.2 = \underline{\hspace{2cm}}$ $+ 0.2 = \underline{\hspace{2cm}}$ $+ 0.2 = \underline{\hspace{2cm}}$ $+ 0.2 = \underline{\hspace{2cm}}$ $+ 0.2 = \underline{\hspace{2cm}}$	b. 1.1 $+ 0.5 = \underline{\hspace{2cm}}$ $+ 0.5 = \underline{\hspace{2cm}}$ $+ 0.5 = \underline{\hspace{2cm}}$ $+ 0.5 = \underline{\hspace{2cm}}$ $+ 0.5 = \underline{\hspace{2cm}}$ $+ 0.5 = \underline{\hspace{2cm}}$	c. 2.5 $+ 0.3 = \underline{\hspace{2cm}}$ $+ 0.3 = \underline{\hspace{2cm}}$ $+ 0.3 = \underline{\hspace{2cm}}$ $+ 0.3 = \underline{\hspace{2cm}}$ $+ 0.3 = \underline{\hspace{2cm}}$ $+ 0.3 = \underline{\hspace{2cm}}$	d. 3.6 $- 0.4 = \underline{\hspace{2cm}}$ $- 0.4 = \underline{\hspace{2cm}}$ $- 0.4 = \underline{\hspace{2cm}}$ $- 0.4 = \underline{\hspace{2cm}}$ $- 0.4 = \underline{\hspace{2cm}}$ $- 0.4 = \underline{\hspace{2cm}}$
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7. Remember? **1 millimeter is one-tenth of a centimeter.** Or, $1 \text{ mm} = 0.1 \text{ cm}$.

a. Draw a line that is 4.7 cm long.



b. Measure the line in centimeters.
Use a decimal.



8. Convert. In (c), add and give your answer in centimeters.

a. $0.5 \text{ cm} = \underline{\hspace{2cm}} \text{ mm}$

b. $7 \text{ mm} = \underline{\hspace{2cm}} \text{ cm}$

c. $5 \text{ mm} + 0.9 \text{ cm} = \underline{\hspace{2cm}} \text{ cm}$

$1.2 \text{ cm} = \underline{\hspace{2cm}} \text{ mm}$

$35 \text{ mm} = \underline{\hspace{2cm}} \text{ cm}$

$4 \text{ cm} + 3.4 \text{ cm} = \underline{\hspace{2cm}} \text{ cm}$

9. The two sides of a rectangle measure 6.5 cm and 3.6 cm.

Draw the rectangle on blank paper. What is its perimeter?