Integer Operations Mat

Activity Guide

Contents

What is the Integer Operations Mat?

The **Integer Operations Mat** helps students visualize addition, subtraction, multiplication, and division with integers. Students use two-color counters, with one color representing negative numbers and another color representing positive numbers. In this guide, red (negative) and yellow (positive) counters are used. Red counters are placed on the left side of the mat to represent negative numbers, and yellow counters are placed on the right side of the mat to represent positive numbers. Work with integers on this mat helps students begin to think algebraically.

The –10 to +10 number line below the work space is an additional tool for visualizing operations with integers and can be used to confirm and reinforce students' work with the two-color counters.

The mat is write-on/wipe-off, enabling students to write the equation above the "T" at the top of the mat, to show their work on the number line, and also to circle groups of counters if they choose to show their thinking that way.

Vocabulary

Students should be familiar with the following terms:

Integers: The set of counting numbers, their opposites, and zero.

Absolute Value: The value of a number regardless of the sign. Also the distance from a number (point) on the number line to 0. **Example:** The absolute value of –4 is 4.

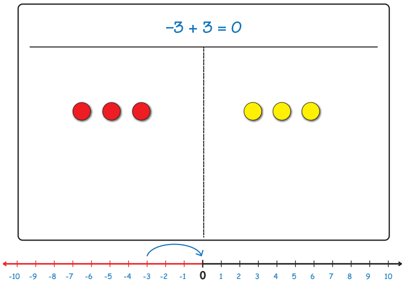
Opposites: Two numbers that have the same absolute value but opposite signs. Opposite numbers sum to zero. **Example:** +3, –3.

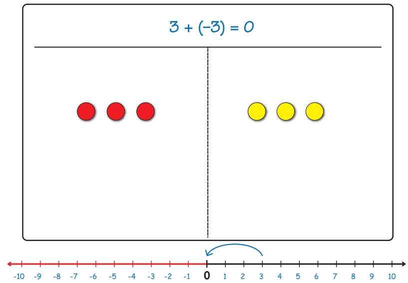
Zero Pairs: Pairing a positive number with a negative number, or a red counter with a yellow counter, to make zero. **Examples:** –1, 1; –3, 3.

Making Zero on the Integer Operations Mat

Using two-color counters on the **Integer Operations Mat** makes it easy to visualize why two opposite numbers, in this case, –3 and 3, sum to zero. Each yellow counter pairs with a red counter to make a zero pair. The equation can be written as $-3 + 3 = 0$ or $3 + (-3) = 0$. The number line below each equation shows that the pairing of –3 and 3 results in 0, for both equations. Note that if the second number (addend) is positive, you move in the positive direction, right, on the number line. If the second number is negative, you move in the negative direction, left, on the number line.

© Didax





Addition on the Integer Operations Mat

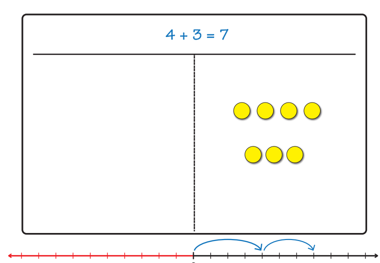
Several rules and shortcuts apply for adding, subtracting, multiplying, and positive and negative numbers. For example, when adding, the rules are:

- The sum is positive if the signs of both integers are positive.
- The sum is negative if the signs of both integers are negative.
- The sum is positive if the positive integer is greater than the absolute value of the negative integer.
- The sum is negative if the absolute value of the negative integer is greater than the positive integer.

Modeling addition problems with two-color counters on the **Integer Operations Mat** will help students visualize why these rules work before they are asked to memorize them. Remind students to use zero pairs as they model the following addition problems on the mat.

4 + 3 = ?

The first addend is **4:** Take 4 yellow (positive) counters and place them on the right side of the mat.



The second addend is **3:** Take 3 more yellow counters and place them on the right side of the mat.

The total number of yellow counters is **4 + 3 = 7.**

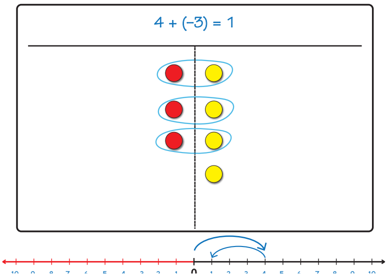
Show the addition on the number line at the bottom of the mat.

4 + (– 3) = ?

The first addend is **4:** Take 4 yellow (positive) counters and place them on the right side of the mat.

The second addend is **–3:** Take 3 red (negative) counters and place them on the left side of the mat.

Form as many zero pairs as you can, in this case 3 zero pairs. One yellow counter is left over. The solution is 4 + (– 3) = 1.



Show the addition on the number line at the bottom of the mat.

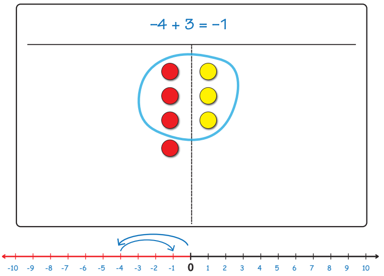
Students may notice that 4 + (–3) = 1 can also be written as the subtraction equation 4 –3 = 1.

–4 + 3 = ?

The first addend is **–4:** Take 4 red (negative) counters and place them on the left side of the mat.

The second addend is **3:** Take 3 yellow (positive) counters and place them on the right side of the mat.

Form as many zero pairs as you can, in this case 3 zero pairs, with one red counter left over. The solution is –4 + 3 = –1.



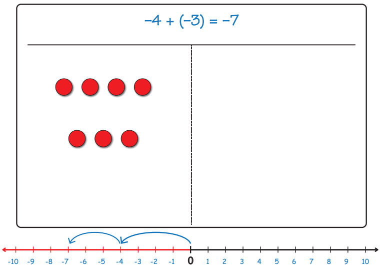
On the number line, start at –4 and count 3 spaces to the right to –1.

–4 + (–3) = ?

The first addend is **–4:** Take 4 red (negative) counters and place them on the left side of the mat.

The second addend is **–3:** Take 3 more red counters and place them on the left side of the mat.

The total number of red counters is 7, so –4 + (– 3) = –7.



Show the subtraction on the number line at the bottom of the mat by counting left 3 spaces from –4.

© Didax

Addition Practice on the Integer Operations Mat

Give students the following addition problems to solve on their mats:

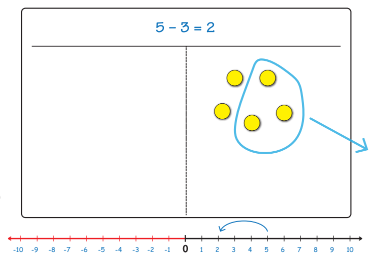
5 + 2 = ?	**–3 + (– 2) = ?**	**–10 + 6 = ?**
–4 + 6 = ?	**8 + (– 3) = ?**	**2 + (– 9) = ?**

Subtraction on the Integer Operations Mat

Subtraction is the inverse operation of addition. It is helpful for students to remember that we *add* counters to the **Integer Operations Mat** when we model addition and *take away* counters from the mat when we model subtraction.

5 – 3 = ?

When we rewrite this equation as 5 – (+3) = ? we know what to do. The first number is 5, so take 5 yellow (positive) counters and place them on the right side of the mat. Since we are asked to subtract 3 yellow (positive) counters from 5, we simply remove (take away) 3 of the 5 yellow counters, leaving 2. So 5 –3 = 2.

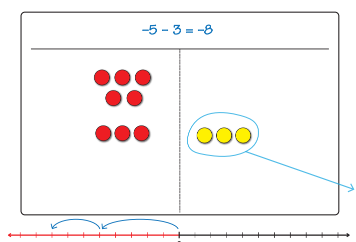


We show the subtraction on the number line by starting at 5 and moving 3 spaces to the left.

– 5 – 3 = ?

In this equation, we are asked to subtract positive 3 from –5. The equation can also be written as –5 – (+3) = ?

The first number is –5, so take 5 red (negative) counters and place them on the left side of the mat.



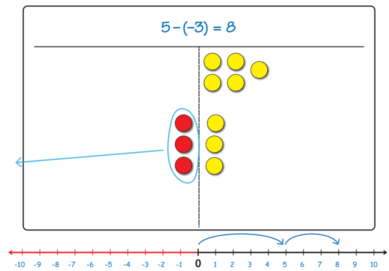
The second number is +3. Students will note that it is not possible to take away 3 yellow counters because there are no yellow counters on the mat.

The fix is to add 3 red counters to the left side of the mat and 3 yellow counters to the right side to form 3 zero pairs. Now we can take away 3 yellow counters. This leaves 8 red counters on the left side of the mat. The solution is –5 – 3 = –8.

We show the subtraction on the number line by starting at –5 and moving 3 spaces to the left.

© Didax

5 – (–3) = ?



Subtraction gets trickier here. The first number is 5, so take 5 yellow (positive) counters and place them on the right side of the mat. The second number is –3, but we're subtracting, so we have to take away 3 red (negative) counters from the 5 yellow counters.

We can't simply add 3 red counters to the mat because that would be an addition equation: 5 + (–3) = ? Since there are no red counters on the mat, we need to add 3 red counters and 3 yellow counters to form 3 zero pairs. Now we can take away –3. This leaves 8 yellow counters on the right side of the mat. The solution is 5 –(–3) = 8.

Students may notice that subtracting a negative number is the same as adding a positive number: 5 + 3 = 8. This models the rule that subtracting a negative number is the same as adding a positive number.

Subtraction Practice on the Integer Operations Mat

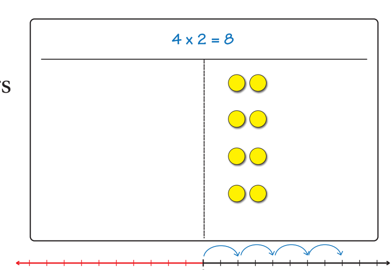
Give students the following subtraction problems to solve on their mats:

7 – 2 = ?	**–5 – 4 = ?**	**–1 – (– 6) = ?**
–4 – 3 = ?	**8 – (– 1) = ?**	**2 + (– 5) = ?**

Multiplication on the Integer Operations Mat

When multiplying integers, the first number tells how many groups and the second number tells how many in a group. For example, 2 × 5 means "2 groups of 5." If the number of groups is positive, counters are added to the mat. If the number of groups is negative, counters are removed from the mat. Let's look at the following examples to understand this.

4 × 2 = ?

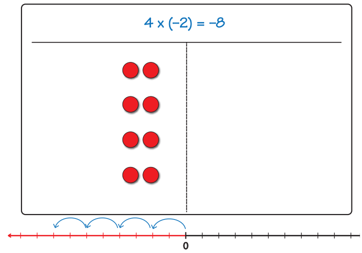


Both numbers are positive, so we add 4 groups of 2 yellow counters to the right side of the mat. How many yellow counters altogether? (8) So the solution is 4 × 2 = 8.

We show this on the number line by starting at 0 and counting by 2 four times to the right.

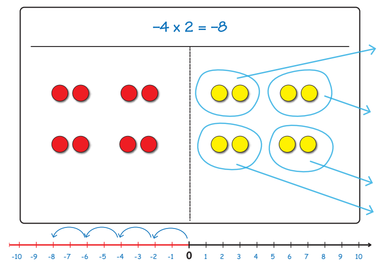
© Didax

$4 \times (-2) = ?$



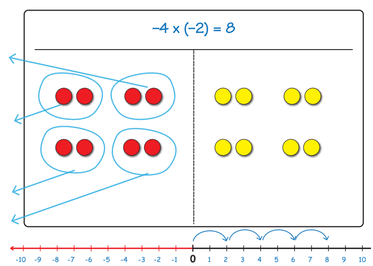
The number of groups is positive, but the number in each group is negative, so $4 \times (-2)$ means "**add** 4 groups of 2 red counters." How many altogether? (–8) The solution is $4 \times (-2) = -8$.

$-4 \times 2 = ?$



The number of groups is negative, so -4×2 means "**remove** 4 groups of 2 yellow counters." Add 4 groups of 2 yellow counters and 4 groups of 2 red counters to the mat to make zero pairs, and then remove the 4 groups of yellow counters . What's left? (8 red counters) The solution is $-4 \times (2) = -8$.

$-4 \times (-2) = ?$



The number of groups is negative, so $-4 \times (-2)$ means "**remove** 4 groups of 2 red counters." Add 4 groups of 2 red counters and 4 groups of 2 yellow counters to the mat to make zero pairs, and then remove the 4 groups of 2 red counters . What's left? (8 yellow counters) The solution is $-4 \times (-2) = 8$.

Students may notice that multiplying a negative number by a negative number results in a positive product.

> **Multiplication Practice on the Integer Operations Mat**
>
> Give students the following multiplication problems to solve on their mats:
>
> | | | |
> |---|---|---|
> | $3 \times 3 = ?$ | $-3 \times 2 = ?$ | $2 \times (-5) = ?$ |
> | $-3 \times (-3) = ?$ | $-5 \times 3 = ?$ | $3 \times (-5) = ?$ |

Rules for Multiplication and Division with Integers

At this point, based on the work above, it is helpful to introduce the rules for multiplying integers, especially since these rules also apply to division of integers.

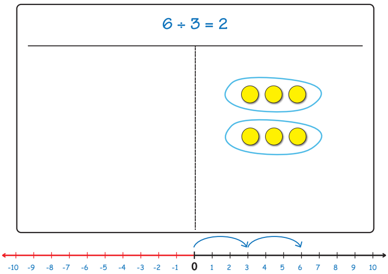
Positive × positive = positive	**Positive ÷ positive = positive**
Positive × negative = negative	**Positive ÷ negative = negative**

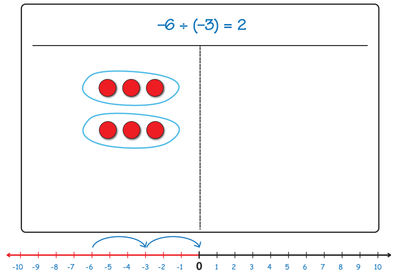
Negative × positive = negative
Negative × negative = positive

Negative ÷ positive = negative
Negative ÷ negative = positive

Division on the Integer Operations Mat

When dividing integers, we determine how many groups of the second number can be made from the first number. For example:





6 ÷ 3 = ?

6 is positive, so we add 6 yellow counters to the right side of the mat. How many groups of 3 can be made from 6? (2) So 6 ÷ 3 = 2.

–6 ÷ (–3) = ?

6 is negative, so we add 6 red counters to the left side of the mat. How many groups of –3 can be made from –6? (2) So –6 ÷ (–3) = 2.

–6 ÷ 3 = ?

6 is negative, so we add 6 red counters to the left side of the mat. How many groups of +3 can be made from –6? (None, because there no yellow counters on the mat.) But the rule tells us that dividing a negative by a positive results in a negative. If we divide –6 into groups of 3, we get –2. So –6 ÷ 3 = –2.

6 ÷ (–3) = ?

Again, the rule tells us that dividing a positive by a negative results in a negative. If we divide 6 into groups of –3, we get –2. So 6 ÷ (–3) = –2.

Division Practice on the Integer Operations Mat

Give students the following division problems to solve on their mats with the help of the rules for division of integers:

8 ÷ 4 = ?	**–8 ÷ 4 = ?**	**8 ÷ (– 2) = ?**
–8 ÷ (– 2) = ?	**–10 ÷ (–5) = ?**	**10 ÷ (– 2) = ?**

© Didax