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Unit I — The Structure of Geometry

Part A — What is Geometry?

Lesson 1 — *Origin and Structure*

Objective: To identify the five parts of mathematical speech, and understand their use, as they specifically apply to the study of Geometry.

Important Terms:

Geometry — from the Greek, meaning "earth measure;" the field of mathematics involving properties of, and relationships between, points, lines, planes, and surfaces in space.

Plane Geometry — Often called "Euclidean Geometry," or "Geometry of the Plane," that Geometry which is confined to two-dimensional space only.

Number Symbols — The "things" of mathematics; the objects around which our study revolves. In Algebra, these were the actual numbers. In Geometry, this will be expanded to study "things" like points, lines, planes, angles, polygons, circles, spheres, and more.

Operation Symbols — The "actions" of mathematics. In Algebra, + (addition), – (subtraction), x (multiplication), ÷ (division, and more were used to indicate operations. In Geometry, we will encounter additional operations, including translations, reflections, rotations, and dilations of geometric figures, as well as (union) and (intersection) with sets.

Relation Symbols — Symbols used to show comparisons. In Algebra, we used symbols such as = (is equal to), > (is greater than), and < (is less than). In Geometry, we will extend that list to include relations such as ⊥ (is perpendicular to), || (is parallel to), ≅ (is congruent to), and ~ ("is similar to").

Grouping Symbols — Symbols that associate things, such as ( ) (parentheses), [ ] (brackets), { } (braces), or fraction bars. The meaning and use of these symbols is the same in Geometry as it was in Algebra.

Placeholder Symbols — Often called "variables," these symbols are used to hold the place of something. They may include letters of the alphabet, empty boxes, question marks, and others. They are used in Geometry in the same way they were used in Algebra.

Example: Tell which of the following are relation symbols, and state in words, what each means.

a. $\nless$ b. Δ c. $\perp$ d. $\sqrt{3}$ e. $\cong$

Solution:

a. $\nless$ is a relation symbol, meaning "is not less than."

b. $\triangle$ is not a relation symbol. It is the symbol for the word "triangle," when it is used in Geometry. In Arithmetic, it is sometimes used as a placeholder. For example:

$$5 - \triangle = 2$$

c. $\perp$ is a relation symbol, meaning "is perpendicular to," and is used to state that two lines meet to form a right angle.

d. $\sqrt{3}$ is not a relation symbol. It is a number symbol, read as "the second root of 3", or , more commonly, "the square root of 3." This is an irrational number, approximately equal to 1.732...

e. $\cong$ is a relation symbol. In Geometry, it states that two figures are "congruent." In other words, they have the same shape and are the same size.

Lesson 1 — Exercises:

For each of the following exercises, identify the part of the mathematical speech and state the meaning in words.

1. π
2. m
3. 15.318
4. $\sqrt{11}$
5. $\overline{)\quad}$
6. $\ngtr$
7. $\sqrt[3]{27}$
8. ( )
9. $17\frac{1}{3}$
10. $\cong$
11. e
12. $7.\overline{45}$
13. { }
14. $\frac{10}{7}$
15. 6^3
16. +
17. $\perp$
18. $\sqrt{16}$
19. [ ]
20. •
21. $\neq$

Unit I — The Structure of Geometry

Part A — What is Geometry?

Lesson 2 — *More on Things*

Objective: To further describe and understand the “things” of mathematics as they specifically apply to the study of Geometry.

Important Terms:

Infinite – Becoming increasingly large, or small, beyond any fixed boundary.

Point – A basic element of Geometry, which has no size, and indicates only position. In a drawing, it is represented by a dot, and labeled with a capital letter. For example, • P is read “point P.”

Line – A basic element of Geometry, which has infinite length, but no thickness. In a drawing, it is represented by a line segment with arrowheads on each end, to show that it goes on forever. We name a line by choosing any two points on the line, and labeling each with a capital letter.

For example, 

can be written in symbols as $\overleftrightarrow{AB}$, and is read, “line AB.” Additionally, 

can be written in symbols as $\overleftrightarrow{m}$, and is read, “line m.”

Plane – A basic element of Geometry, which has infinite length and width, but no thickness. We generally represent a plane with a parallelogram, as we might draw a piece of paper using perspective, and name it by using a single capital letter or by using three points on the plane. For example, the diagram in Figure A represents a horizontal plane, named and read as “Plane M.” The diagram in Figure B represents a vertical plane, named and read as “Plane DEF.”

Fig. A

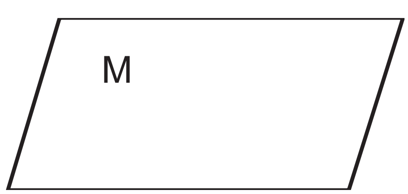
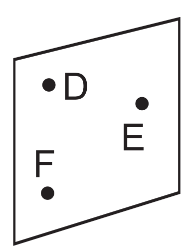


Fig. B



Space – The set of all possible points.

Example 1: List two examples of physical models of a point, a line and a plane.

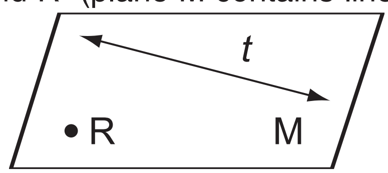
Solution: Point – possible answers include a pin hole, a tiny seed, a grain of sand, a grain of salt, and others.

Line – possible answers include a guy wire used to support a radio tower, a row of trees in a nursery, a string of pixels on a television screen, and others.

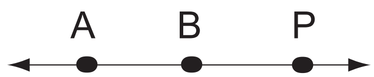
Plane – possible answers include the ceiling of a room, the surface of a tennis court, the side of a box, and others.

Example 2: Draw and label a figure for each of the following descriptions.

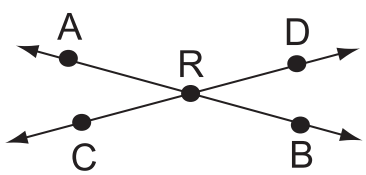
a. Plane M contains $\overleftrightarrow{t}$ and R (plane M contains line t, and point R)
Possible Solution:



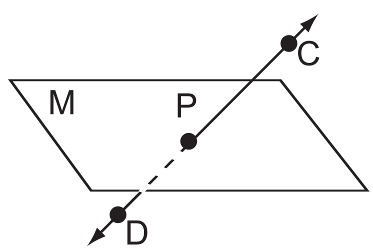
b. P is on $\overleftrightarrow{AB}$ (point P is on line AB)
Possible Solution:



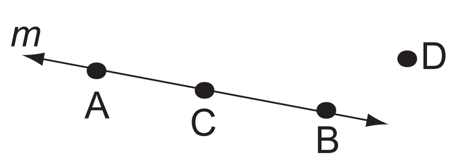
c. $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ intersect at R (line AB and line CD intersect at point R)
Possible Solution:



d. Plane M and $\overleftrightarrow{CD}$ intersect at P (plane M and line CD intersect at point P)
Possible Solution:



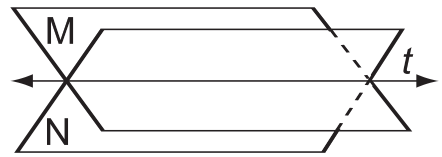
e. A, B, and C lie on $\overleftrightarrow{m}$ but D does not lie on $\overleftrightarrow{m}$ (point A, point B, and point C lie on line m, but point D does not lie on line m)
Possible Solution:



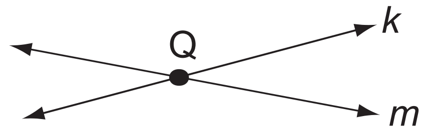
Example 2: (cont'd)

f. Plane M intersects plane N in $\overleftrightarrow{t}$ (plane M intersects plane N in line *t*)

Possible Solution:



Example 3: Write a description of the following diagram and represent that statement with geometric symbols.



Possible Solution: Point Q is the intersection of line *k* and line *m* (Q is the intersection of $\overleftrightarrow{k}$ and $\overleftrightarrow{m}$)

Lesson 2 — Exercises:

In exercises 1 through 3, replace each __?__ with a geometric term that makes the statement true.

1. A __?__ has no thickness, but extends infinitely in two opposite directions.

2. A __?__ has no thickness, but extends infinitely in all directions along a flat surface.

3. A __?__ is a location. It has no thickness or length.

For each item in exercises 4 through 6, list three other examples of physical models which are different from the examples given in this lesson.

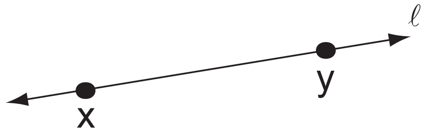
4. Point

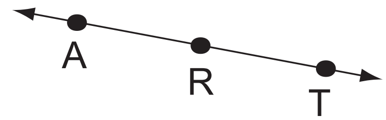
5. Line

6. Plane

Lesson 2 — Exercises: (cont'd)

Name each of the lines in exercises 7 and 8, in two different ways.

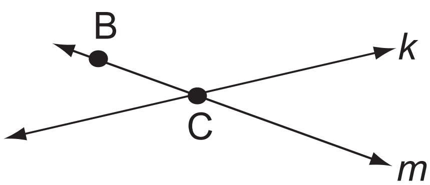
7. 

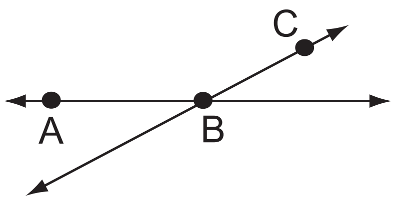
8. 

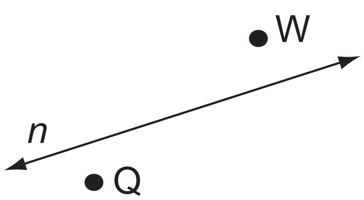
Use a ruler or straight edge to draw each line indicated in exercises 9-11, labeling each appropriately.

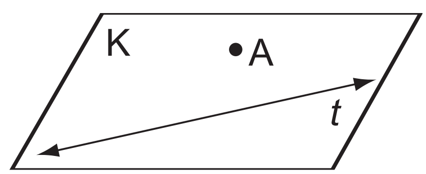
9. $\overleftrightarrow{AB}$ 10. $\overleftrightarrow{KL}$ 11. $\overleftrightarrow{PU}$

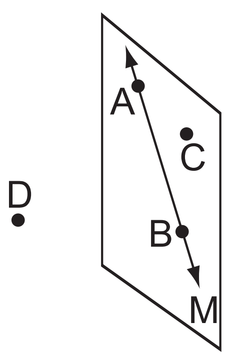
Write a complete description for each diagram in exercises 12-17.

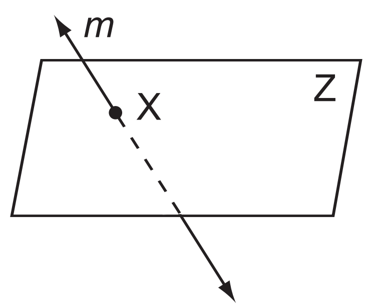
12. 

13. 

14. 

15. 

16. 

17. 

18. Sketch and label a diagram showing two planes that do not intersect.

19. Sketch and label a diagram showing a vertical plane intersected by a horizontal line. Draw hidden parts with dashes.

20. Sketch and label a diagram showing that points P and Q both lie in the intersection of planes A and B.