Activities for Learning, Inc.

RightStart™ Mathematics

by Joan A. Cotter, Ph.D.

Level E Lessons

For Home Educators

Special thanks to Sharalyn Colvin, who converted *RightStart™ Mathematics: Grade 4 Lessons* into *RightStart™ Mathematics: Level E For Home Educators*.

Note: Rather than use the designation, K-4, to indicate a grade, levels are used. Level A is kindergarten, Level B is first grade, and so forth.

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Home Educators

RIGHTSTART™ MATHEMATICS

by Joan A. Cotter, Ph.D.

The following are items needed to complete the RightStart™ Mathematics Level E Lessons:

STATUS	ITEM	CODE
REQUIRED	***Level E Lessons***	T-E
REQUIRED	***Level E Worksheets***	W-E
REQUIRED	***Math Card Games*** book	M4
RECOMMENDED; choice of abacus	Classic AL Abacus - 8-1/2" x 9-1/2" hardwood frame & beads	A-CL
	Standard AL Abacus - 7-1/2" x 9-1/2" plastic frame & beads	A-ST
	Junior AL Abacus - 5-1/4" x 6" plastic frame & beads	A-JR
RECOMMENDED	Place Value Cards	P
RECOMMENDED	Abacus Tiles	AT
REQUIRED	Cards, Six Special Decks needed for Games	C
RECOMMENDED	Fraction Charts	F
REQUIRED	Basic Drawing Board Geometry Set	DS
RECOMMENDED	Wooden Cubes, 20-1" cubes in set	RH13
REQUIRED	Colored Tiles, apx 200 in set	RH2
REQUIRED	Casio Calculator SL-450	R4
REQUIRED	Centimeter Cubes, 100 in set	R8
RECOMMENDED	Wood Geometry Solids, 12 in set	R14
RECOMMENDED	Math Balance (Invicta)	R7
REQUIRED	4-in-1 Ruler	R10
REQUIRED	Folding Meter Stick	R15
REQUIRED	Goniometer (Angle Measure)	R11

Note: If a child has not previously worked with the AL abacus and is just starting RightStart™ Mathematics , *RightStart Mathematics Transition Lessons* are required before starting the *RightStart Mathematics Level E Lessons* .

TO ORDER OR FOR GENERAL INFORMATION:
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05-08

RightStart™ MATHEMATICS: OBJECTIVES FOR LEVEL E

Name _____

Teacher _____ **Year** _____

	1ST QTR	2ND QTR	3RD QTR	4TH QTR
Numeration				
Understands decimals to two places	N/A			
Can read and write numbers to 99 million				
Understands prime numbers	N/A	N/A		
Can factor numbers into primes	N/A	N/A		
Understands and can use simple percents	N/A			
Multiplication				
Can multiply 4-digit numbers by 2-digit numbers	N/A			
Knows multiplication facts	N/A			
Division				
Can solve division story problems with remainders	N/A	N/A		
Can divide 4-digit numbers by 1-digit using short division	N/A	N/A		
Understands and can find averages	N/A	N/A		
Knows division facts	N/A	N/A		
Fractions				
Can add and subtract simple fractions				
Can convert between improper fractions and mixed fractions	N/A			
Calculator				
Can find squares and square roots	N/A			
Can divide and make sense of the remainder	N/A	N/A		
Money				
Can solve consumer problems involving money				
Problem Solving				
Works well in group to solve problems				
Clearly justifies his/her reasoning				
Geometry				
Can construct and measure angles	N/A	N/A	N/A	
Can sketch 3-dimensional shapes	N/A	N/A	N/A	
Understands rotational symmetry	N/A	N/A	N/A	
Knows terms prism, pyramid, cylinder, and sphere	N/A	N/A	N/A	
Knows terms acute, right, and obtuse angles	N/A	N/A	N/A	
Can find the area of a triangle	N/A	N/A		
Can locate points on a coordinate system	N/A	N/A		
Measurement				
Can measure to fourths & tenths of an inch	N/A			
Can measure to tenths of a centimeter	N/A			
Can find area to tenths of square inches or square cm	N/A			
Can construct and read pie graphs	N/A	N/A	N/A	
Patterns				
Can recognize and continue a pattern				
Can use algebraic thinking to write a pattern symbolically	N/A	N/A	N/A	
Can solve simple equations				
Data and Probability				
Can collect and display data	N/A	N/A	N/A	
Can determine the probability of an event	N/A	N/A		
Knows mean, median, and mode	N/A	N/A		

How This Program Was Developed

We have been hearing for years that Japanese students do better than U.S. students in math in Japan. The Asian students are ahead by the middle of first grade. And the gap widens every year thereafter.

Many explanations have been given, including less diversity and a longer school year. Japanese students attend school 240 days a year.

A third explanation given is that the Asian public values and supports education more than we do. A first grade teacher has the same status as a university professor. If a student falls behind, the family, not the school, helps the child or hires a tutor. Students often attend after-school classes.

A fourth explanation involves the philosophy of learning. Asians and Europeans believe anyone can learn mathematics or even play the violin. It is not a matter of talent, but of good teaching and hard work.

Although these explanations are valid, I decided to take a careful look at how mathematics is taught in Japanese first grades. Japan has a national curriculum, so there is little variation among teachers.

I found some important differences. One of these is the way the Asians name their numbers. In English we count ten, eleven, twelve, thirteen, and so on, which doesn't give the child a clue about tens and ones. But in Asian languages, one counts by saying ten-1, ten-2, ten-3 for the teens, and 2-ten 1, 2-ten 2, and 2-ten 3 for the twenties.

Still another difference is their criteria for manipulatives. Americans think the more the better. Asians prefer very few, but insist that they be imaginable, that is, visualizable. That is one reason they do not use colored rods. You can imagine the one and the three, but try imagining a brown eight–the quantity eight, not the color. It can't be done without grouping.

Another important difference is the emphasis on non-counting strategies for computation. Japanese children are discouraged from counting; rather they are taught to see quantities in groups of fives and tens.

For example, when an American child wants to know 9 + 4, most likely the child will start with 9 and count up 4. In contrast, the Asian child will think that if he takes 1 from the 4 and puts it with the 9, then he will have 10 and 3, or 13.

Unfortunately, very few American first-graders at the end of the year even know that 10 + 3 is 13.

I decided to conduct research using some of these ideas in two similar first grade classrooms. The control group studied math in the traditional workbook-based manner. The other class used the lesson plans I developed. The children used that special number naming for three months.

They also used a special abacus I designed, based on fives and tens. I asked 5-year-old Stan how much is 11 + 6. Then I asked him how he knew. He replied, "I have the abacus in my mind."

The children were working with thousands by the sixth week. They figured out how to add four-place numbers on paper after learning how to do it on the abacus.

Every child in the experimental class, including those enrolled in special education classes, could add numbers like 9 + 4, by changing it to 10 + 3.

I asked the children to explain what the 6 and 2 mean in the number 26. Ninety-three percent of the children in the experimental group explained it correctly while only 50% of third graders did so in another study.

I gave the children some base ten rods (none of them had seen them before) that looked like ones and tens and asked them to make 48. Then I asked them to subtract 14. The children in the control group counted 14 ones, while the experimental class removed 1 ten and 4 ones. This indicated that they saw 14 as 1 ten and 4 ones and not as 14 ones. This view of numbers is vital to understanding algorithms, or procedures, for doing arithmetic.

I asked the experimental class to mentally add 64 + 20, which only 52% of nine-year-olds on the 1986 National test did correctly; 56% of those in the experimental class could do it.

Since children often confuse columns when taught traditionally, I wrote 2304 + 86 = horizontally and asked them to find the sum any way they liked. Fifty-six percent did so correctly, including one child who did it in his head.

This following year I revised the lesson plans and both first grade classes used these methods. I am delighted to report that on a national standardized test, both classes scored at the 98th percentile.

Some General Thoughts on Teaching Mathematics

1. Only five percent of mathematics should be learned by rote; 95 percent should be understood.
2. Teaching with understanding depends upon building on what the child already knows. Teaching by rote does not care.
3. The role of the teacher is to encourage thinking by asking questions, not giving answers. Once you give an answer, thinking usually stops.
4. It is easier to understand a new model after you have made one yourself. For example, a child needs to construct graphs before attempting to read ready-made graphs.
5. Good manipulatives cause confusion at first. If the new manipulative makes perfect sense at first sight, it wasn't needed. Trying to understand and relating it to previous knowledge is what leads to greater learning, according to Richard Behr and others.
6. Lauren Resnick says, "Good mathematics learners expect to be able to make sense out of rules they are taught, and they apply some energy and time to the task of making sense. By contrast, those less adept in mathematics try to memorize and apply the rules that are taught, but do not attempt to relate these rules to what they know about mathematics at a more intuitive level."
7. According to Arthur Baroody, "Teaching mathematics is essentially a process of translating mathematics into a form children can comprehend, providing experiences that enable children to discover relationships and construct meanings, and creating opportunities to develop and exercise mathematical reasoning."
8. Mindy Holte puts learning the facts in proper perspective when she says, "In our concern about the memorization of math facts or solving problems, we must not forget that the root of mathematical study is the creation of mental pictures in the imagination and manipulating those images and relationships using the power of reason and logic."
9. The only students who like flash cards are those who don't need them.
10. Mathematics is not a solitary pursuit. According to Richard Skemp, solitary math on paper is like reading music, rather than listening to it; "Mathematics, like music, needs to be expressed in physical actions and human interactions before its symbols can evoke the silent patterns of mathematical ideas (like musical notes), simultaneous relationships (like harmonies) and expositions or proofs (like melodies)."
11. "More than most other school subjects, mathematics offers special opportunities for children to learn the power of thought as distinct from the power of authority. This is a very important lesson to learn, an essential step in the emergence of independent thinking." (A quote from *Everybody Counts)*

12. Putting thoughts into words helps the learning process.
13. The difference between a novice and an expert is that an expert catches errors much more quickly. A violinist adjusts pitch so quickly that the audience does not hear it.
14. Europeans and Asians believe learning occurs not because of ability, but primarily because of effort. In the ability model of learning, errors are a sign of failure. In the effort model, errors are natural. In Japanese classrooms, the teachers discuss errors with the whole class.
15. For teaching vocabulary, be sure either the word or the concept is known. For example, if a child is familiar with six-sided figures, we can give him the word, hexagon. Or, if he has heard the word, multiply, we can tell him what it means. It is difficult to learn a new concept and the term simultaneously.
16. Introduce new concepts globally before details. This lets the children know where they are headed.
17. Informal mathematics should precede paper and pencil work. Long before a child learns how to add fractions with unlike denominators, she should be able to add one half and one fourth mentally.
18. Some pairs of concepts are easier to remember if one of them is thought of as dominant. Then the non-dominant concept is simply the other one. For example, if even is dominant over odd; an odd number is one that is not even.
19. Worksheets should also make the child think. Therefore, they should not be a large collection of similar exercises, but should present a variety.
20. In Japan students spend more time on fewer problems. Teachers do not concern themselves with attention spans as is done in the U.S.
21. In Japan the goal of the math lesson is that the student has understood a concept, not necessarily has done something (a worksheet).
22. The calendar should show the entire month, so the children can plan ahead. The days passed can be crossed out or the current day circled.
23. A real mathematical problem is one in which the procedures to find the answer or answers are not obvious. It is like a puzzle, needing trial and error. Emphasize the satisfaction of solving problems and like puzzles, of not giving away the solution to others.
24. Keep math time enjoyable. A person who dislikes math will avoid it. We store our emotional state along with what we've learned. A child under stress stops learning. If a lesson is too hard, end it and play a game. Try again another day.

RightStart™ Mathematics

There are 13 major characteristics that make this research-based program effective.

1. Refers to quantities of up to 5 as a group; discourages counting individually.
2. Uses fingers and tally sticks to show quantities up to 10; teaches quantities 6 to 10 as 5 plus a quantity, for example 6 = 5 + 1.
3. Avoids counting procedures for finding sums and remainders. Teaches five- and ten-based strategies for the facts that are both visual and visualizable.
4. Employs games, not flash cards, for practice.
5. Once quantities 1 to 10 are known, proceeds to 10 as a unit. Uses the "math way" of naming numbers for several months; for example, "1 ten-1" (or "ten-1") for eleven, "1-ten 2" for twelve, "2-ten" for twenty, and "2-ten 5" for twenty-five.
6. Uses expanded notation (overlapping) place-value cards for recording tens and ones; the ones card is placed on the zero of the tens card. Encourages a child to read numbers starting at the left and not backward by starting at the ones column.
7. Proceeds rapidly to hundreds and thousands using manipulatives and place-value cards. Provides opportunities for trading between ones and tens, tens and hundreds, and hundreds and thousands with manipulatives.
8. Only after the above work, about the fourth month of first grade, introduces the traditional English names for quantities 20 to 99 and then 11 to 19.
9. Teaches mental computation. Investigates informal solutions, often through story problems, before learning procedures.
10. Teaches four-digit addition on the abacus, letting the child discover the paper and pencil algorithm. This occurs in Level B. Four-digit subtraction is mastered in Level C.
11. Introduces fractions with a linear visual model.
12. Approaches geometry through drawing boards and tools.
13. Teaches short division (where only the answer is written down) for single-digit divisors, before long division. Both are taught in Level E.

Some Pointers

Kindergarten. Most of the kindergarten lesson plans have two distinct topics, which can be taught on alternate days.

Transition Lessons. These lessons are designed for children starting Levels C, D, or E (grades 2, 3, or 4) who have not been doing RightStart™ Mathematics previously. The lessons need to be studied before the regular manual, except where noted. The manual tells which lessons to use for the particular grade.

Counting. Counting needs to be discouraged because it is slow and inaccurate. It also interferes with understanding quantity and learning place-value.

Warm-up. The warm-up time is the time for quick review, memory work, and sometimes an introduction to the day's topics. The drawing board makes an ideal slate for quick responses.

Place value. In order to understand addition algorithms, place-value knowledge is essential. From the very beginning, the children are helped to see quantities grouped in fives and tens. Children can understand place value in first grade and even in kindergarten when it is approached as it is in this program.

Worksheets. The worksheets are designed to give the children a chance to think about and to practice the day's lesson. Some lessons, especially in the early grades, have no worksheet.

Games. Games, not worksheets or flash cards, are used for practice. They can be played as many times as necessary until memorization takes place. Games are as important to math as books are to reading.

Some games are incorporated in this manual. Extra games, found in the book, *Math Card Games,* are suggested in the Review and Practice lessons in Levels C to E. There are games for the child needing extra help, as well as for the advanced child.

Teaching. Establish with the children some indication when you want a quick response and when you want a more thoughtful response. Research shows that the quiet time for thoughtful response should be about three seconds. Avoid talking during this quiet time; resist the temptation to rephrase the question. This quiet time gives the slower child time to think. It also gives a quicker child time to think more deeply.

Encourage the child to develop perseverance. Avoid giving answers too quickly. Children tend to stop thinking once they hear the answer.

Help the children realize that it is their responsibility to ask questions when they do not understand. Do not settle for "I don't get it."

Number of lessons. It is not necessary that each lesson be done in one day. Sometimes two days may be more appropriate. However, do complete each manual before going to the next one.

Visualization. The ability to imagine or visualize is an important skill to develop in mathematics and other subjects as well. Often you are called upon to suggest to the children that they imagine a particular topic.

Questions. I really want to hear how this program is working. Please let me know any improvements and suggestions that you may have.

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Table of Contents
Level E

Math Journal

The math journal can be found in the last pages of *Level E Worksheets.* It will be used for solving problems, calculating, and making tables. The lines will help the child align numbers, draw figures, and make tables. He will also become familiar with centimeters and square centimeters because the lines are one centimeter apart.

Practice sheets

Practice sheets are designed to help the child master the facts at his own pace through using strategies. These visual pictures are vitally important for a child's growth in mathematical thinking.

Since these practice sheets will be used more than once, use protector sheets or make copies.

Research shows children require time to learn strategies. They need at least two weeks before new strategies become automatic. When a baby is first learning to walk, progressing from point A to point B is arduous and demands much concentration. However, if that baby is in a hurry to cover that distance, he will resort to all fours. Likewise, a child first learning to think with strategies needs support and encouragement.

Using flash cards to teach the facts by rote is based on outmoded ideas of how the brain works. Such practice often gives the child the erroneous idea that math is a subject in which a person is not expected to think. That would be news to a mathematician or an engineer.

There is another reason to teach by understanding. A child with learning disabilities, about 15% of the population, cannot learn by rote. He must have the structure and mental picture provided by strategies.

Games are a good way to help a child to obtain needed repetition. No two games are the same, in contrast to drill sheets. Games also add a social dimension, thereby adding interest and a chance to practice social skills.

Practice Graph. A good way to keep track of the child's progress is through the Practice Graph. Connect the times. The child learns how to use graphs and is motivated to lower the times, competing only with himself.

When a child's score levels off, or the time is no longer decreasing, move on to another practice sheet.

Lesson 1

Magic Squares

OBJECTIVES

1. To learn about *magic squares* and their *magic numbers*
2. To informally review arithmetic operations, diagonals, and negative numbers

MATERIALS Worksheet 1, "Magic Squares"

WARM-UP Ask the child to name some of the mathematical concepts she has learned. Record them on paper for further discussion. A partial list might include:

1. "Math" way of counting.
2. Venn Diagrams.
3. Addition.
4. Place Value.
5. Geometry terms.
6. Skip counting.
7. Fractions.

ACTIVITIES ***Magic square patterns.*** Write the following magic square. Tell the child it is a *magic square.* Ask her to find what is special about it. Ask her to find as many patterns as possible.

2	7	6
9	5	1
4	3	8

A magic square. All the rows, columns, and diagonals sum to the same number.

After she has worked about 5 to 10 minutes, ask her to share what she found. Ask her to write her observations. Some possibilities include:

Note: If necessary, review the difference between rows (horizontal) and columns (vertical).

1. It includes all the numbers from 1 to 9.
2. Even numbers in the corners; other numbers are odd.
3. Each row adds to 15.
4. Each column adds to 15.
5. Both diagonals add to 15.
6. The overall figure is a square.
7. Opposite corners (and sides) equal 10.

Tell her the sum, 15 in this case, is called the *magic number.*

How many different ways do the numbers in the magic square add up to the magic number? [8, including, 3 horizontal, 3 vertical, and 2 diagonal]

Note: Underlined statements indicate questions you can ask the child.

Making new magic squares. Tell the child she can build her own magic square. Demonstrate as follows. Let's put 5 in the middle and our magic number will still be 15. Now let's put an odd number, say 7, in the corner. See the first figure below.

What other number can we fill in? [3 in the lower right, 7 + 5 = 12, so 3 is needed to make 15] Write the 3 as shown in the next figure.

Note: The magic number is 3 times the number in the middle for a 3 × 3 square. The children will learn this later on.

What other number can we fill in? [none] So we can choose again. Let's write a 1 next to the 7. See the third figure.

What other number can we fill in? [the 7 in the top row and the 9 in the middle column] See the fourth figure. Let her suggest how to complete it. [The lower left can be done with either the diagonal or the bottom row.] Completing it is obvious. Ask her to check all rows, columns, and diagonals to be sure it is a magic square.

7		
	5	

7		
	5	
		3

7	1	
	5	
		3

7	1	7
	5	
	9	3

7	1	7
	5	
3	9	3

7	1	7
5	5	5
3	9	3

Making a magic square.

What is special about the numbers? [odd] Could you make a magic square with all even numbers if the magic number is 15? [no, adding even numbers always gives an even number; 15 is odd]

Worksheet. Ask the child to do the first two parts on the worksheet. Give her about 5 to 10 minutes and then discuss the answers.

1. The first 2 are magic squares because all the rows, columns, and diagonals add to the same number. In the third square, the rows and columns add to the same number, but not the diagonals.

2. The magic number is 3 times the middle number.

Then ask her to complete the worksheet. Ask her to explain her work, especially the last two magic squares. The solutions are shown below.

Note: The answers are boldfaced and underlined.

7	**4**	**4**
2	5	8
6	**6**	**3**

4	1	4
3	**3**	**3**
2	**5**	**2**

5	–2	**9**
8	4	**0**
–1	**10**	3

5	–2	**–3**
–8	0	**8**
3	**2**	**–5**

	1	
	7	
	13	

	2	

The numbers in the blank squares will vary.

Name _____

Date _____

1. Which of these squares is not a magic square? How can you tell? What are the magic numbers of the magic squares?

6	5	10
11	7	3
4	9	8

6	5	1
–1	4	9
7	3	2

6	5	4
2	6	7
7	4	4

2. You can tell the magic number of a 3 × 3 magic square by looking at the number in the middle. What is the relationship?

3. Complete these magic squares. For some of them you may need to make up your own numbers. Be sure to check your magic squares.

7		
	5	8

4	1	4

	–2	
	4	
		3

	–2	
	0	
3		

	1	
	7	

	2	

Lesson 2

Combining Magic Squares

OBJECTIVES

1. To learn about magic *squares* and their *magic numbers*
2. To informally review arithmetic operations, diagonals, and negative numbers

MATERIALS Worksheet 2, "Combining Magic Squares"

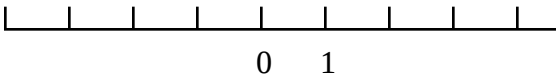
WARM-UP Spend up to 2 minutes discussing one or more of the mathematical concepts named in Lesson 1.

ACTIVITIES

Historical note. The Chinese knew about magic squares thousands of years ago. People in the Middle Ages thought they brought good luck.

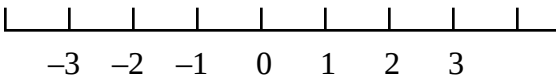
Review. Ask the child what he knows about magic squares. [numbers in a square with all the rows, columns, and diagonals equal to a magic number and that the magic number is 3 times the middle number]

What does negative 1 mean? Draw the following number line with 0 and 1 written.



A number line with two entries.

Where do we write –1 (negative 1), 2, –2, 3, and –3. See the line below. If desired, continue with other numbers.



A number line with more entries.

Note: Do NOT attempt to teach any rules for working with negative numbers.

Discuss ways negative numbers are used. [temperature, sea level, money on hand vs. money owed, and so forth]

Adding two magic squares. What do you think happens when we add two magic squares together? Ask him to complete the two magic squares on the worksheet and to add them together by adding the numbers in the corresponding squares. It is shown below.

6	**4**	**14**
16	8	**0**
2	**12**	10

+

3	**5**	**–2**
–3	**2**	**7**
6	–1	1

=

9	**9**	**12**
13	**10**	**7**
8	**11**	**11**

Adding the corresponding cells of two magic squares results in a new magic square.

Discuss whether it is a magic square. [yes] Ask the child for his strategies for mentally adding the rows, columns, and diagonals. Stress procedures that start with one number, then add the tens of the second number and then the ones of the second number. Discourage the "paper and pencil" algorithm of starting with the ones.

How could you find the new magic NUMBER when you add two magic squares together? [Either add the two magic numbers together, 24 + 6 = 30, or add the middle numbers together and multiply by 3: 8 + 2 = 10; 10 × 3 = 30.]

Adding the same number to a magic square. Ask the child to read Problem 2 on the worksheet. Ask what he thinks will happen. Then ask him to try it. [It is a magic square.] See below.

6	4	14		**14**	**12**	**22**
16	8	0	+ 8 =	**24**	**16**	**8**
2	12	10		**10**	**20**	**18**

Adding the same number to each cell in a magic square gives a new magic square.

Multiplying a magic square by a number. What do you think will happen if you multiply all the numbers in a magic square by a number? Ask him to try it in Problem 3 on the worksheet. The solution is below.

8	3	4		**40**	**15**	**20**
1	5	9	× 5 =	**5**	**25**	**45**
6	7	2		**30**	**35**	**10**

Multiplying each cell in a magic square by the same number gives a new magic square.

Multiplying two magic squares together. What do you think will happen if you multiply two magic squares together? Ask the child to try it in Problem 4. [It does NOT make a new magic square.] In this particular case, the rows and columns add up to the same number, but the diagonals do not. Also, the rows are not 3 times the middle number. The solution is below.

7	8	3		3	8	1		**21**	**64**	**3**
2	6	10	×	2	4	6	=	**4**	**24**	**60**
9	4	5		7	0	5		**63**	**0**	**25**

Multiplying corresponding cells of two magic squares does not result in a new magic square.

Note: The math journal is at the end of the child's Worksheets.

Creative work. Ask the child to make two or more magic squares and then to combine them in some way. He can use his math journal for this work. Ask him to share the results with you.

Name _____

Date _____

1. Complete the two magic squares. Then add the corresponding numbers together. Do the arithmetic mentally. Is the new square a magic square? How can you find the new magic number from the other two magic numbers?

	8	
2		10

+

6	–1	1

=

2. If you add 8 to each number in the above magic square, will you get a new magic square? Try it.

3. What happens if you multiply each number in a magic square by the same number? Try it.

8	3	4
1	5	9
6	7	2

× 5 =

4. Try to make a new magic square by multiplying the corresponding numbers together. Do the arithmetic mentally. Is the new square a magic square?

7	8	3
2	6	10
9	4	5

×

3	8	1
2	4	6
7	0	5

=

Lesson 3 (2 days)

Larger Magic Squares

OBJECTIVES

1. To learn about 4 × 4 magic squares
2. To combine magic squares and writing to make "birthday squares"

MATERIALS Worksheets 3-1 and 3-2, "Birthday Magic Squares"
Calculator

WARM-UP Spend up to two minutes discussing one or more of the mathematical concepts named in Lesson 1.

Ask the child to recite the months of the year. Then ask her to give the month number for various months: March, [3] September, [9] June, [6] November, [11] and August. [8] Then do the inverse; give her the month number and ask for the month: 12, [December] 4, [April] 2, [February] and 10 [October].

ACTIVITIES ***The Durer 4 ×4 magic square.*** Draw the "Durer" 4 × 4 magic square as shown on the left below. Tell the child that it is in a famous painting completed in the year 1514.

16	3	2	13
5	10	11	8
9	6	7	12
4	15	14	1

A magic square, found in a painting by Durer, dated 1514. See the year in the last row.

What is its magic number? [34] What is special about this square?

Ask the child to find as many special things as she can about it. Then ask her to share what she found. Ask her to write her observations. Some possibilities are:

1. It includes all the numbers from 1 to 16.
2. The overall figure is a square.
3. Each row adds to 34.
4. Each column adds to 34.
5. Both diagonals add to 34.
6. The corners add to 34.
7. The inner squares add to 34.
8. The middle two numbers on top and bottom add to 34.
9. The middle two numbers on left and right add to 34.
10. The year 1514 appears in the last row.

Discuss these and then ask the following, How are 4 × 4 magic squares different from 3 × 3 magic squares? Some differences are:

1. 3 × 3 squares have nine squares; 4 × 4 has 16 squares.
2. 4 × 4 squares have no middle square.
3. More ways to find the magic number in 4 × 4 squares.

Worksheet. The first worksheet describes a method to make "birthday squares." The month number goes in the first square in the first row, while the day goes in the second square. The age will be the magic number. The solutions for the two birthday squares are given below.

2	14	0	**–6**
10	0	0	**0**
–8	**10**	0	**8**
6	**–14**	**10**	**8**

A birthday square for a 10-year-old born on February 14.

7	9	0	**–5**
0	0	0	**11**
–1	**11**	0	**1**
5	**–9**	**11**	**4**

A birthday square for an 11-year-old born on July 9.

Note: Learning to use a calculator efficiently is an important skill.

Completing 4 ×4 squares. Ask the child to use a calculator and complete the next row of magic squares on the worksheet. She will first need to find the magic numbers. Remind her not to write down or remember any numbers, but to use calculator memory. Discuss how she used the calculator. The solutions are as follows.

16	23	42	19
54	10	11	**25**
19	**59**	20	**2**
11	**8**	**27**	**54**

Magic number is 100, sum of the top row.

18	24	15	**–7**
–11	19	**11**	**31**
11	14	6	**19**
32	**–7**	**18**	7

Magic number is 50, sum of the diagonal.

–10	39	**24**	7
25	**18**	16	1
–3	**–11**	**37**	**37**
48	**14**	**–17**	**15**

Magic number is 60, sum of left column.

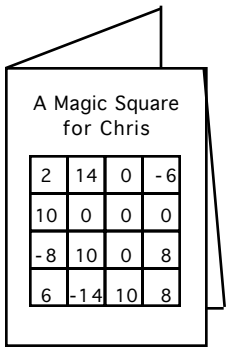
Original birthday squares. Explain to the child that she is to make a birthday square on the first worksheet and then copy it to the second worksheet. Then she can fold the paper into fourths and write a poem inside.

You might want to read some of the following verses to give her some ideas.

To find your age add a column or a row;
Or even a diagonal, as you know.

Add the numbers in a row or in a column,
Or, in a diagonal — but don't be solemn.
Try the inner square or the corners four;
Have a Happy Birthday and many more.

To find your age add a row;
Get the sum any way you know.
Next, add the numbers going down;
Try this without being a clown.



Name _____

Date _____

1. Here is a "birthday square" for Chris, who will be ten years old on February 14. Finish the square so Chris's age will be the magic number. Remember, the four corners must also equal the magic number.

2	14	0	
10	0	0	
		0	

2. Make a "birthday square" for Jamie, who will be eleven years old on July 9. Write the numbers for the month and day in the first two squares. Make the magic number equal to Jamie's age.

____	____	0	
0	0	0	
		0	

3-5. Use a calculator and complete these magic squares. First find the magic numbers.

16	23	42	19
54	10	11	
		20	

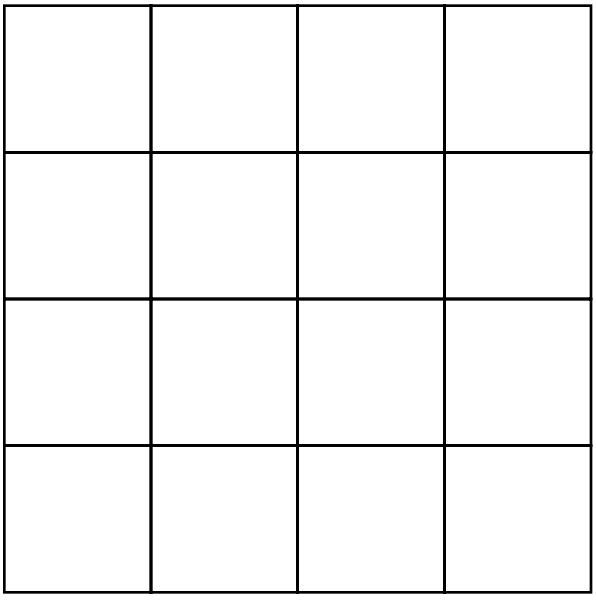
18	24	15	
–11	19		
	14	6	
			7

–10	39		7
25		16	1
–3			
48			

6. Make you own birthday squares. The squares with lines can have any numbers.

____	____	____	
____	____	____	
		____	

____	____	____	
____	____	____	
		____	



Lesson 4

Reviewing Addition

Note: Omit this lesson if the child has done the Transition Lessons.

OBJECTIVES

1. To review adding
2. To learn another way to add columns of numbers
3. To review check numbers, a method of checking addition

MATERIALS

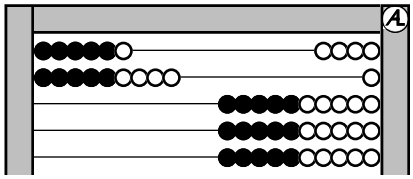
Worksheet 4, "Reviewing Addition"
Worksheet 5, "Addition Facts"

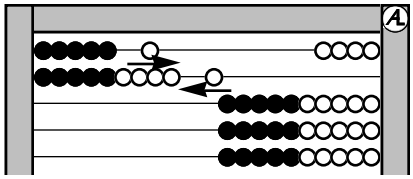
WARM-UP

Write the following for the child to answer:

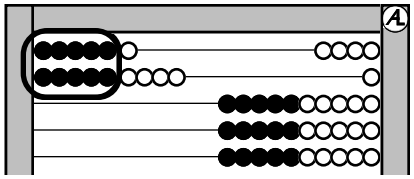
6 + 9 = [15] 7 + 4 = [11] 6 + 7 = [13]

Ask the child for his strategies. Some possible strategies for 6 + 9 are: a) Make Ten: take 1 from 6 and give it 9 to make 5 + 10 (see the left figures below), b) Two Fives: add 10 + the amounts over 5, 1 (6 = 5 + **1**) + 4 (9 = 5 + **4)**. See the figure below on the right.





Adding 6 + 9 by taking 1 from the 6 and giving it to the 9, resulting in 10 + 5 = 15.



Adding 6 + 9 by noticing the two fives make a 10 and the 1 and 4 combine to make 5, making 15.

For 7 + 4 a good strategy is Compensation: since 7 + 3 = 10, 7 + 4 must be 11. For 6 + 7, the Two Fives strategy works; also, if 6 + 6 is known, then 6 + 7 must be 13.

Next write the following and ask him to answer.

25 + 9 = [34] 68 + 4 = [72] 47 + 8 = [55]

Discuss if desired.

ACTIVITIES

Worksheet. Ask the child to figure out Robin's method of adding columns of numbers. [First Robin added the hundreds and wrote it down. Then Robin did the tens and ones and finally added them altogether.]

The solutions to the three problems are shown below.

Note: The numbers in parentheses are check numbers. For more information, see either *Math Card Games*, A 63, or *Level D* manual, Lessons 47 and 49.

828 (0)	2942 (8)	2856 (3)
204 (6)	7358 (5)	894 (3)
140 (5)	+8061 (6)	+4305 (3)
+228 (3)	17000	6000
1300	1200	1900
80	150	140
20	11	15
1400 (5) ✓	**18,361 (1) ✓**	**8055 (0) ✓**

Discuss how he liked Robin's method. Is it easier or harder?

Reviewing check numbers. Ask the child to explain check numbers to you, what they are and how to find and use them. [The check number of the sum equals the sum of the check numbers.]

Using the example given, ask him to explain each number in the parentheses following the number. To find a check number, the digits are added together with 9s and multiples of 9s equal to zero. If a sum is more than 9, those digits are added together.

341	(8)		Check number of 341 is 3 + 4 + 1 = 8.
909	(0)		The 9s are 0, so check number of 909 is 0.
696	(3)		For 696, the 9 is 0 and 6 + 6 = 12; 1 + 2 = 3.
378	(0)		3 + 7 + 8 = 18, 1 + 8 = 9, so check number is 0.
+788	(5)		Add 7 + 8 gives 6; 6 + 8 gives 5.
2800			
280			
32			
3112	(7)	✓	Check number is 3 + 1 + 1 + 2 = 7, the same as adding the check numbers, 8 + 0 + 3 + 0 + 5.

Note: Use a check mark to show the sum of the check numbers match the check number of the sum.

The remaining problems can be assigned for homework. The answers are:

1835 **3721** **27,921** **30,486**

Reviewing the addition facts. Explain that the second worksheet reviews most of the basic addition facts. (Some of the simpler facts with a number + 1 and 1 + a number are missing.)

Note: When children don't complete a timed test within the allotted time, they know they have failed. Even the Olympics allow contestants to complete their events.

The child is to start when you say start. Tell him to tell you when he has finished and you will tell him the time, which he is to write on his paper.

Read off the answers, given below, and ask him to mark the wrong answers, perhaps with a colored pen.

91 + 7 = **98**	88 + 7 = **95**	87 + 5 = **92**	52 + 6 = **58**
18 + 5 = **23**	48 + 4 = **52**	84 + 8 = **92**	67 + 6 = **73**
79 + 6 = **85**	85 + 2 = **87**	23 + 7 = **30**	52 + 4 = **56**
75 + 7 = **82**	29 + 3 = **32**	24 + 7 = **31**	99 + 1 = **100**
26 + 8 = **34**	69 + 8 = **77**	59 + 7 = **66**	39 + 9 = **48**
48 + 6 = **54**	53 + 8 = **61**	58 + 1 = **59**	54 + 9 = **63**
57 + 7 = **64**	87 + 3 = **90**	72 + 9 = **81**	56 + 4 = **60**
37 + 8 = **45**	52 + 7 = **59**	16 + 2 = **18**	77 + 4 = **81**
97 + 2 = **99**	14 + 4 = **18**	74 + 3 = **77**	46 + 3 = **49**
23 + 3 = **26**	46 + 9 = **55**	95 + 8 = **103**	46 + 7 = **53**
55 + 5 = **60**	16 + 5 = **21**	51 + 9 = **60**	53 + 9 = **62**
95 + 4 = **99**	24 + 6 = **30**	28 + 9 = **37**	88 + 3 = **91**
44 + 2 = **46**	92 + 5 = **97**	13 + 2 = **15**	23 + 5 = **28**
37 + 9 = **46**	75 + 9 = **84**	92 + 3 = **95**	38 + 8 = **46**
39 + 4 = **43**	69 + 5 = **74**	68 + 2 = **70**	83 + 4 = **87**
69 + 2 = **71**	92 + 2 = **94**	12 + 8 = **20**	91 + 6 = **97**
95 + 3 = **98**	64 + 5 = **69**	51 + 4 = **55**	85 + 6 = **91**
76 + 6 = **82**	43 + 6 = **49**	71 + 8 = **79**	81 + 5 = **86**

Name _____

Date _____

Robin likes to add several numbers together as shown below in the example. Describe Robin's method. _____

_____

_____

Try Robin's method on the next three problems. Check your answers.

```
  341  (8)
  909  (0)
  696  (3)
  378  (0)
+ 788  (5)
-------
 2800
  280
   32
-------
 3112  (7)
```

```
  828
  204
  140
+ 228
-----
```

```
 2942
 7358
+8061
-----
```

```
 2856
  894
+4305
-----
```

Add these.

```
  466
  238
  384
  193
+ 554
-----
```

```
  352
  614
  932
  846
+ 977
-----
```

```
 7493
 3588
 3564
 9807
+3469
-----
```

```
 8576
 5594
 6509
  624
+9183
-----
```

Name _____

Date _____

Time _____ Number right_____/72

91 + 7 = _____	88 + 7 = _____	87 + 5 = _____	52 + 6 = _____
18 + 5 = _____	48 + 4 = _____	84 + 8 = _____	67 + 6 = _____
79 + 6 = _____	85 + 2 = _____	23 + 7 = _____	52 + 4 = _____
75 + 7 = _____	29 + 3 = _____	24 + 7 = _____	99 + 1 = _____
26 + 8 = _____	69 + 8 = _____	59 + 7 = _____	39 + 9 = _____
48 + 6 = _____	53 + 8 = _____	58 + 1 = _____	54 + 9 = _____
57 + 7 = _____	87 + 3 = _____	72 + 9 = _____	56 + 4 = _____
37 + 8 = _____	52 + 7 = _____	16 + 2 = _____	77 + 4 = _____
97 + 2 = _____	14 + 4 = _____	74 + 3 = _____	46 + 3 = _____
23 + 3 = _____	46 + 9 = _____	95 + 8 = _____	46 + 7 = _____
55 + 5 = _____	16 + 5 = _____	51 + 9 = _____	53 + 9 = _____
95 + 4 = _____	24 + 6 = _____	28 + 9 = _____	88 + 3 = _____
44 + 2 = _____	92 + 5 = _____	13 + 2 = _____	23 + 5 = _____
37 + 9 = _____	75 + 9 = _____	92 + 3 = _____	38 + 8 = _____
39 + 4 = _____	69 + 5 = _____	68 + 2 = _____	83 + 4 = _____
69 + 2 = _____	92 + 2 = _____	12 + 8 = _____	91 + 6 = _____
95 + 3 = _____	64 + 5 = _____	51 + 4 = _____	85 + 6 = _____
76 + 6 = _____	43 + 6 = _____	71 + 8 = _____	81 + 5 = _____

Lesson 9

Equivalent Fractions

Note: If the child is doing the Transition Lessons, do Lesson T-33 before this lesson.

OBJECTIVES

1. To find equivalent fractions using drawing tools
2. To learn the word *terms* as related to fractions

MATERIALS

Worksheet 11, "Equivalent Fractions"
Drawing board, T-squares, and 30-60 triangle
Tape for fastening the worksheet to the drawing board. (3M's Removable Tape works best.)

WARM-UP

Write the puzzle numbers of 5 1 1 6. Some solutions are:

$$5 = 11 - 6$$
$$5 + 1 = 1 \times 6$$
$$5 + 1 \times 1 = 6$$

Ask the child to write the multiples of 2, 4, 6, and 8 in two rows and ask for various multiplication facts. Also ask for the 9s facts.

ACTIVITIES

Reviewing fraction charts. Take out the worksheet. What does the top half of the sheet show? [2 fraction charts; that is dividing rectangles into halves, thirds, down to tenths] What pattern do you notice about the size of the pieces? Which is largest and which is smallest? [$\frac{1}{2}$ is largest and $\frac{1}{10}$ is smallest] Why is $\frac{1}{10}$ less than $\frac{1}{9}$? [Dividing something into 10 pieces means each piece must be smaller than if it was divided into 9 pieces.]

Note: Recipes and roadway signs use a slanted line for fractions, but in mathematics the horizontal line is used.

Write

$$\frac{2}{5}$$

and ask the child to read it. [two fifths] What does it mean? [two $\frac{1}{5}$s or 2 divided into 5 pieces] Ask for both meanings on the charts. [two of the $\frac{1}{5}$s]

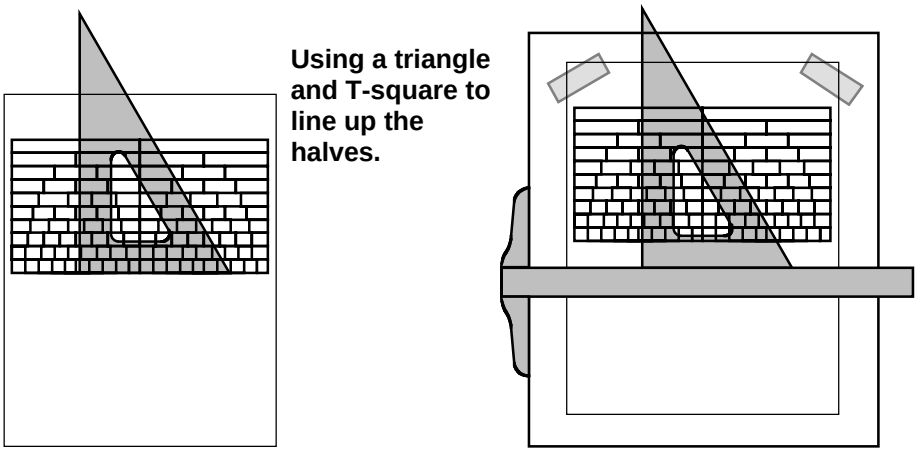
Seeing halves. Can you find other fractions equal to $\frac{1}{2}$? Write them as the child finds them:

$$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10}$$

What pattern do you see? [The upper number is half of the lower number.] Tell her the numbers in a fraction are called *terms.*

Equivalent fractions. How could you use a drawing triangle to find all the halves? See the figure below on the left.

Note: The best tape to use is 3M's removable tape, which can be reused several times.



Using a triangle and T-square to line up the halves.

Tell the child to tape the worksheet to the drawing board, in the upper corners as shown on the previous page.

Ask her to move the triangle to show two halves. What other fractions equal two halves? [$\frac{3}{6}$, $\frac{4}{8}$, . . . $\frac{10}{20}$]

What other fractions are equal to $\frac{3}{9}$? [$\frac{1}{3}$, $\frac{2}{6}$] Write them down.

$$\frac{3}{9} \quad \frac{1}{3} \quad \frac{2}{6}$$

Which of these fractions has the lowest numbers, called the *lowest terms?* [$\frac{1}{3}$]

Repeat for $\frac{6}{8}$. What other fractions are equal to $\frac{6}{8}$? [$\frac{3}{4}$] Write them down.

$$\frac{6}{8} \quad \frac{3}{4}$$

Which fraction has the lowest terms? [$\frac{3}{4}$]

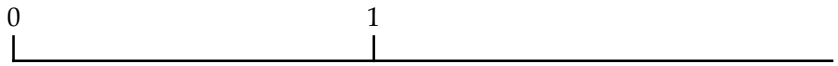
Repeat for $\frac{9}{6}$. What other fractions are equal to $\frac{9}{6}$? [$\frac{3}{2}$, $\frac{6}{4}$, $\frac{12}{8}$, $\frac{15}{10}$] Write them down.

$$\frac{3}{2} \quad \frac{6}{4} \quad \frac{12}{8} \quad \frac{15}{10}$$

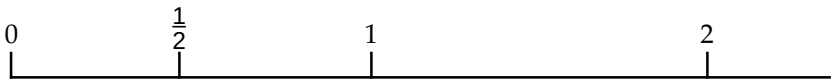
Which fraction has the lowest terms? [$\frac{3}{2}$]

Fraction number line. Draw a horizontal line; label it with a 0 and 1 as shown below.

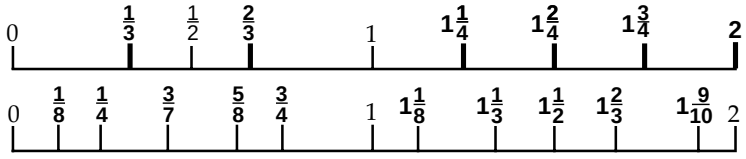
Where would you put a mark for 2? Ask the child to make the mark and write the number. Repeat for $\frac{1}{2}$. See below.



Ask her to mark and write other fractions, such as $\frac{1}{4}$, $\frac{3}{4}$, $1\frac{1}{2}$, and $\frac{5}{4}$.



Worksheet. The first two questions on the worksheet are similar to the above activity, except it is to be done with the precision of the drawing tools. The answers are below.



$\frac{4}{8}$: **$\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{6}$, $\frac{5}{10}$**

$\frac{3}{2}$: **$\frac{6}{4}$, $\frac{9}{6}$, $\frac{12}{8}$, $\frac{15}{10}$**

$\frac{2}{6}$: **$\frac{1}{3}$, $\frac{3}{9}$**

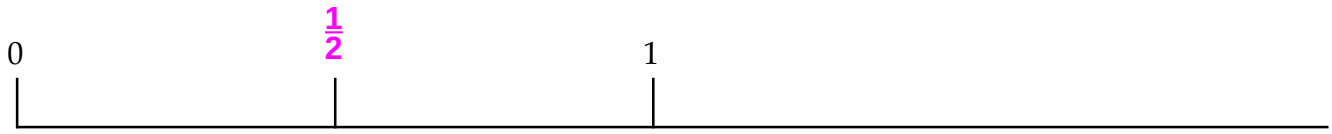
$\frac{2}{3}$: **$\frac{4}{6}$, $\frac{6}{9}$**

Name _____

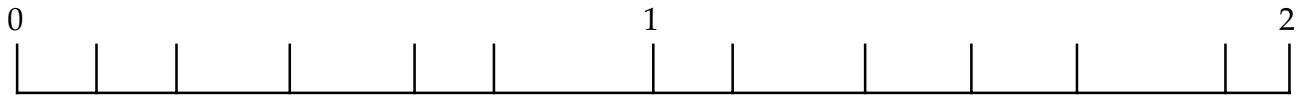
Date _____

1										1									
$\frac{1}{2}$									$\frac{1}{2}$	$\frac{1}{2}$									$\frac{1}{2}$
$\frac{1}{3}$								$\frac{1}{3}$		$\frac{1}{3}$								$\frac{1}{3}$	
$\frac{1}{4}$							$\frac{1}{4}$			$\frac{1}{4}$							$\frac{1}{4}$		
$\frac{1}{5}$						$\frac{1}{5}$				$\frac{1}{5}$						$\frac{1}{5}$			
$\frac{1}{6}$					$\frac{1}{6}$					$\frac{1}{6}$					$\frac{1}{6}$				
$\frac{1}{7}$				$\frac{1}{7}$						$\frac{1}{7}$				$\frac{1}{7}$					
$\frac{1}{8}$			$\frac{1}{8}$							$\frac{1}{8}$			$\frac{1}{8}$						
$\frac{1}{9}$		$\frac{1}{9}$								$\frac{1}{9}$		$\frac{1}{9}$							
$\frac{1}{10}$	$\frac{1}{10}$									$\frac{1}{10}$	$\frac{1}{10}$								

On the number line below mark and label these fractions: $\frac{1}{2}$ $\frac{1}{3}$ $\frac{2}{3}$ $1\frac{1}{4}$ $1\frac{2}{4}$ $1\frac{3}{4}$ 2.
Use the two fraction charts above and your drawing tools.



On the number line below, write the fraction with the lowest terms for each mark. Use your drawing tools.



Write other fractions equal to the one at the left.

$\frac{4}{8}$ _____

$\frac{3}{2}$ _____

$\frac{2}{6}$ _____

$\frac{2}{3}$ _____

Lesson 26

Percent Practice

OBJECTIVES

1. To find the percent of various numbers in the hundred chart
2. To review some general math and map concepts

MATERIALS

Worksheets 23-1 and 23-2, "Percent Practice"
A map of the U.S. and a list of the 50 states

WARM-UP

Write the puzzle numbers of 4 9 7 2. Some solutions are:

$$4\,9 = 7^{2}$$
$$4 - 9 + 7 = 2$$
$$4 = 9 - 7 + 2$$

Write $100\% - 51\% = [49\%]$

to be solved mentally with only the answer written. Some methods: 100 – 50 is 50, so 100 – 51 = 49; 100 – 50 = 50 and 50 – 1 = 49.

Spend a few minutes doing a Practice sheet with the child.

ACTIVITIES

Worksheet 23-1. Take out the worksheet. What chart is on the worksheet? [hundred chart, numbers 1 to 100]

Review by asking: What does percent mean? [a fraction of 100] Ask the child to read the questions and answer them. Then ask for explanations.

1. What percent of the numbers have only 1 digit? [**9%**]
2. What percent of the numbers have 3 digits? [**1%**]
3. What percent of the numbers have 2 digits? [**90%**]
4. What percent of the numbers are even numbers? [**50%**]
5. What percent of the numbers are odd numbers? [**50%**]
6. What percent of the numbers are multiples of 10? [**10%**]
7. What percent of the numbers are not multiples of 10? [**90%**]
8. What percent of the numbers are multiples of 5? [**20%**]
9. What percent of the numbers are not multiples of 5? [**80%**]
10. What percent of the numbers > 80? [**20%**]
11. What percent of the numbers < 15? [**14%**]
12. What percent of the numbers are perfect squares? [**10%**]
13. What percent of the numbers have a "1" in them? [**20%**]
14. What percent of the numbers are multiples of 9? [**11%**]
15. What percent of the numbers are not multiples of 9? [**89%**]

If interest remains, continue to ask him similar questions.

Worksheet 23-2. Assign the second worksheet to be done outside of lesson time. The child will need a map of the U.S. and a list of the 50 states. The answers are as follows:

16%	**16%**	**68%**	**20%**	**80%**
26%	**74%**	**20%**	**10%**	**36%**
26%	**16%**	**2%**	**8%**	**30%**
14%	**4%**	**98%**	**0%**	**6%**
4%				

Name _____

Date _____

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

The questions refer to the numbers in the hundred chart.

1. What percent of the numbers have only 1 digit? _____

2. What percent of the numbers have 3 digits? _____

3. What percent of the numbers have 2 digits? _____

4. What percent of the numbers are even numbers? _____

5. What percent of the numbers are odd numbers? _____

6. What percent of the numbers are multiples of 10? _____

7. What percent of the numbers are not multiples of 10? _____

8. What percent of the numbers are multiples of 5? _____

9. What percent of the numbers are not multiples of 5? _____

10. What percent of the numbers > 80? _____

11. What percent of the numbers < 15? _____

12. What percent of the numbers are perfect squares? _____ (Perfect squares are numbers such as 1, 4, 9, 16, that is, the products of 1×1, 2×2, 3×3 and so on.)

13. What percent of the numbers have a “1” in them? _____

14. What percent of the numbers are multiples of 9? _____

15. What percent of the numbers are not multiples of 9? _____

Name _____

Date _____

The questions refer to the 50 states of the United States.

1. What percent of the states' names start with the letter *M*? _____

2. What percent of the states' names start with the letter *N*? _____

3. What percent of the states' names do not start with the letters *M* or *N*? _____

4. What percent of the states' names have two words? _____

5. What percent of the states' names have only one word? _____

6. What percent of the states were part of the original colonies? _____

7. What percent of the states joined after the original colonies? _____

8. What percent have the Mississippi River as part of their border? _____

9. What percent of the states have a coastline on the Pacific Ocean? _____

10. What percent have a coastline on the Atlantic Ocean or Gulf of Mexico?_____

11. What percent of the states border Canada? _____

12. What percent of the states border the Great Lakes? _____

13. What percent of the states border both a Great Lake and an ocean? _____

14. What percent of the states border Mexico? _____

15. What percent of the states border only other states? _____

16. What percent of the states border Kentucky? _____

17. What percent of the states do not border any other state? _____

18. What percent of the states are in North America? _____

19. What percent of the states are in South America? _____

20. What percent of the states have borders that are only straight lines? _____

21. What percent of the states have borders that are an arc, or curve, of a circle? (Look near Delaware.) _____