

RAY'S ELEMENTARY ARITHMETIC — LEVEL 5

UNIT FOUR: DECIMALS — Lessons 77-93

Overview: This unit expands the decimal place values, reviews addition and subtraction of decimals with more place values, and introduces multiplication and division of decimals.

Suggested Schedule: 4 weeks

Background:

We introduced the concept of decimals in earlier levels when we worked with money. We extended the concept to the thousandths place in the previous level. Here we extend the decimal place values to the hundred thousandths so that students can see how the place values can continue to expand infinitely. The whole numbers expand by the power of ten to the left; each next higher place value is 10 times the previous one. For example, ten thousand times ten equals one hundred thousand ($10,000 \times 10 = 100,000$). Since our number system is consistent, the decimal place values (those to the right of the decimal point) also expand by the power of 10; each next higher place value is 10 times the previous one. For example, one-thousandth times 10 equals one-hundredth ($0.001 \times 10 = 0.01$). We also show that one-tenth times 10 equals 1.

Students shouldn't think of a decimal number as a "two-part" number made up of a whole number part and a decimal part. All the digits form one "expanded" number. The digits after the decimal point show an amount less than 1 while the digits before the decimal point show the amount greater than 1.

We won't formally extend the decimal place values any further in subsequent levels since these numbers are not very common. Once numbers get to this point, they are usually converted to exponential numbers and scientific notation, which will be covered in the next level.

We review the fact that a decimal is a special kind of fraction—one that is based on a power of 10. Or to say it another way, where the denominator of the fraction is a power of 10. This reinforces the fact that both fractions and decimals are rational numbers. Both express a relationship; in both cases, between parts to a whole. We review equivalent decimals, comparing decimals, and adding and subtracting decimals. We then add another skill level of multiplying and dividing fractions.



Where You Are Headed:

We are assuming that your child:

- has a solid understanding of what a fraction is and the relationship between parts and a whole, including what the numerator and denominator represent
- recognizes powers of ten for whole numbers and decimal numbers.
- can recognize that decimals are a special type of fraction, where the denominator is a power of ten.
- can add and subtract money as decimals.

Essential Skills

This unit will:

- extend the decimal place values to the hundred thousandths (0.00001).
- introduce multiplying and dividing decimals.
- introduce the concept of multiplying and dividing by a power of ten as it applies to the decimal place values.
- introduce rounding decimals.
- provide practice in operating with decimals in the context of story problems.

Additional Skills Reviewed

- introduce banking and accounting terms such as debit, credit, balance, deposit, and withdrawal
- show students how to balance a savings account register.

- The exercise includes all three types of problems shown in the lesson. The Challenge Question is a challenge only because the wording is slightly different, even though it is the same type of problem.



Teachable Moment

If your child is unable to solve the Challenge Question, it is probably due to his learning style. Students with a random and global thinking style (as opposed to a routine and sequential thinking style) often have trouble handling any questions that don't follow the steps they've seen in the examples.

This is because they don't naturally think in terms of steps. It is also why it takes longer for them to cement the steps into memory. Once they do, it is difficult for them to deviate from what has "hardened" in their minds.

Use this as an opportunity to show your child that his learning style is never an excuse to not try something. In fact, it is all the more reason to try what doesn't come naturally to him. It will expand his brain's ability to learn no matter how information is given—a skill necessary in any career.

Rather than including the question as part of the exercise (graded or not), go through the thinking process together (after you have looked at the answer key!). But don't tell your child each step. Try to ask a leading question that will trigger his own thinking so that he can see the step for himself. If that doesn't work, say something like, "What if we try picturing it this way," and then start the drawing and let him label it.

This uses the To-With-By method of instruction. Show it TO the child until he can see what to do. Then do with it WITH him, allowing him to do more of the work each time. Finally, let him do it BY himself with you nearby to monitor how he does the problem for immediate feedback.

This way your child's learning style doesn't have to keep him from learning the skill; it only means each stage of the method may take longer.

LESSON 109 – Number Problems

This lesson expands the skills needed to solve more involved number puzzles.

- In previous levels students solved number puzzles where they were given two of the three puzzle pieces. All they had to do was determine the operation needed to find the missing piece. In some cases, students had to use two operations to find the piece.

- In this lesson the numbers are related in different ways, so it isn't immediately clear how the number pieces fit together. Students must use the tracking skills they learned in the last two lessons to solve the math mystery.

- Some of the number puzzles are buried within a real-life situation. But in most cases, the puzzle just gives the numbers and how they are related. For example, *If 10 is taken from the sum of two numbers, 8 will be left. One of the numbers is 5. What is the other number?*

- Reading through the puzzle, it's easy to see how students could get confused because the numbers aren't connected to any concrete, real-life situation. That's the point of this unit. Just because the steps or answer are not immediately clear, doesn't mean a problem can't be solved.

- The tracking tools guide the student in sorting through the given information. They're our way of training the brain. All three examples in the lesson use the tools.

- The questions in the exercise follow these examples.

- Once again, we highly recommend that the student take the time to make all the drawings and label them with the information given in the puzzle. This way he (and you) can see for sure that he is thinking through the puzzles correctly. This is why we keep the number of questions to a minimum.

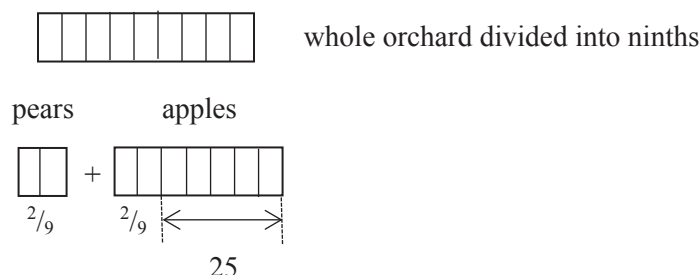
- It may be that your role will primarily be to prod the student through the thinking process, and not let him give up. In other words, this is a prime opportunity to accomplish two goals at once: developing math reasoning skills and emotional/behavioral training, particularly in the life skill of perseverance.

- All students will face challenges in their lives. Here's a chance to show them how to rise to the challenge, and hopefully get a charge out of their accomplishment.

LESSON 110 – Finding Fractional Parts

This lesson uses the same tracking skills to find fractional parts.

- This lesson follows the same procedure as the previous lesson. The only difference is that the parts described in the math mystery are fractions instead of whole numbers.
- While the students may be able to find the fractional amounts just by looking at the picture, they should also be able to multiply or divide a whole number by a fraction, and be able to find an equivalent fraction to solve the problems. If your student is not yet proficient in these skills, you may want to review them before the student does this lesson. Lesson 38 reviews equivalent fractions, Lesson 46 covers multiplication, and Lesson 61 covers division.
- Once again the exercise is kept to a minimum in order to encourage the student to take the time to draw all of the useful pictures that train the brain.
- The Challenge Question is a challenge because the amount “25 more” is added to a portion of the apples that is beyond the amount equal to the number of pear trees (there are 25 more apple trees than pear trees). Since $\frac{2}{9}$ of the orchard is pear trees, the 25 is equal to the number of apple trees greater than $\frac{2}{9}$, which is $\frac{5}{9}$. (Hint: since $\frac{2}{9}$ of the orchard are pear trees, we need to match that amount in apple trees first—another $\frac{2}{9}$ marked off, before adding the 25 more apple trees.



LESSON 111– Varied Problems

This lesson applies the tracking skills to a variety of real-life situations.

- This lesson follows the same procedures as the previous lessons in this unit. The only difference is that the problems are a mixture of situations rather than one type of problem, such as age problems or number puzzles.
- Once again the exercise is kept to a minimum, and all the questions are similar to the examples in the lesson.

LESSON 112 – Parts & Wholes

This lesson introduces a useful problem-solving strategy for solving problems that involve 3 related numbers.

- The strategy is summarized by the sentence, A is B of C. The letters stand for the 3 related numbers, any one of which could be the missing number.
- We can also look at this as “_____ is _____ of _____;” two numbers are given in the problem and the student must “fill in” the third blank.
- This lesson and the next two lessons will each cover a different blank or part of the sentence. This lesson focuses on the A part of the sentence.
- **The keys to using this tool are the two words in between the letters: *is* and *of*.** Students are reminded that we translate *is* as *equals* and *of* as *times*. So another way of looking at this sentence is, “A number = a number X a number,” which means the basic sentence is a multiplication problem.
- The reason we are introducing students to this “tracking sentence” is that in more involved story problems, which operation to use and how the numbers relate to one another is not always clear. The phrasing of the sentence guides students through the reasoning process, sorting out which number goes where.
- For your information, the value of this tracking sentence will be even greater when we get to percent problems.
- For now, we use the fact that the basic sentence is a multiplication problem to hook this new skill to what students already know—a multiplication number bond.