UNIT 2

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2.1

Factoring Polynomials

Multiplying polynomials expands them. Factoring polynomials is the reverse. Factoring breaks polynomials down into the basic parts, or factors, that form them.

Factoring GCF

When a polynomial contains a common factor, write the greatest common factor (GCF) in front of parentheses. The factors left after extracting the GCF from each term of the polynomial remain inside the parentheses.

Example 1 **Factor the greatest common factor (GCF) out of the polynomial $6a^3 + 3a^2b + 3a$.**

Inspecting the polynomial reveals that the greatest common factors in each term are 3 and a. Therefore, the common factor is $3a$. Factor $3a$ out of each term of the polynomial.

$6a^3 + 3a^2b + 3a$	Original polynomial.
$3a(\quad\quad\quad)$	Common factor placed in front of parentheses.
$3a(2a^2 + ab + 1)$	$3a$ factored from each term of the polynomial and the remaining factors placed inside the parentheses.

Note that when $3a$ is factored from $3a$, a 1 remains.

Multiplying the factors again will result in the original polynomial.

Factoring a Difference of Squares

To factor a polynomial that is a difference of squares, such as $4x^2 - 36$, remember that this pattern arises from multiplying two binomials where one is a sum and the other a difference.

When a binomial is squared, a perfect square trinomial is formed. In a perfect square trinomial, the first and last terms are perfect squares, and the middle term is equal to the square root of each outer term multiplied then doubled.

To check whether a polynomial is a perfect square trinomial, first see if the first and last terms are perfect squares. If true, then see if the middle term is equal to twice the product of the square roots of the outer terms. If the polynomial passes both checks, it is a perfect square polynomial.

If it is a difference of squares, set up a pair of parentheses, with one set containing a sum and the other a difference. Then, to find the terms of the binomials, take the square roots of both terms of the polynomial.

Example 2 **Factor $4x^2 - 36$.**

$4x^2 - 36$	Original polynomial.
$(\quad + \quad)(\quad - \quad)$	Sum and difference parentheses set up.
$(2x + 6)(2x - 6)$	Square roots of $4x^2$ and 36 filled in for the binomial terms.

$$x^2 + 6x + 9$$
$$\sqrt{x^2} \qquad \sqrt{9}$$
$$(x \bullet 3) \bullet 2 = 6x$$

Factoring a Perfect Square Trinomial

A perfect square trinomial can be factored into two identical binomials. The first term in the binomial is the square root of the first term in the trinomial. The second term in the binomial is the square root of the last term in the trinomial. For the middle sign of the binomial, use the sign of the middle term of the trinomial. Write the two binomial factors as one binomial squared.

Example 3 **Factor $4x^2 + 20x + 25$**

$4x^2 + 20x +25$	Original polynomial.
$2(2x \bullet 5) = 20x$	Perfect square check.
$(\qquad)^2$	Squared parentheses written.
$(2x \qquad)^2$	$\sqrt{4x^2}$ used for the first binomial term.
$(2x \qquad)^2$	Sign of middle term of the trinomial used.
$(2x + 5)^2$	$\sqrt{25}$ used for the second binomial term.

Factoring a Polynomial in the Form $x^2 + bx + c$

Recall that multiplying two binomials usually results in a trinomial:

$$(x - 3)(x + 2) = x^2 - x - 6$$

This can also be reversed by factoring the trinomial into two binomials which are factors of the trinomial. To do so, remember the FOIL acronym and how the different steps produce the trinomial:
The product of the First terms of the binomials is the first term of the trinomial.
The products of the Outer and Inner terms added together form the middle term.
The product of the Last terms of the binomials is the last term of the trinomial.

Factoring a Trinomial in Form $x^2 + bx + c$

1. Write parentheses for two binomials.
2. For the first terms of each parenthesis, write the square root of the first term of the trinomial.
3. To find the second terms of the binomials, identify the factor pair for *c* whose sum is also *b*.

Example 4 **Factor $x^2 - 4x - 12$.**

$x^2 - 4x - 12$ Original trinomial.

$(x \quad)(x \quad)$ The square root of the first term placed as first terms of the binomials.

Factor pairs for –12:	(–1, 12)	(1, –12)	(–2, 6)	**(2, –6)**	(–3, 4)	(3, –4)
Sums:	11	–11	4	**–4**	1	–1

$(x + 2)(x - 6)$ Second terms filled in with the factor pair of –12 whose sum is also –4.

Today's Lesson

Factor by taking out the GCF.

1. $12ab^2 + 3a^2b - 6b$ **2.** $2x^3 - 6xy + 14x^2y^2$

3. $2x^2yz^3 + 4xz^2 - 6x^2yz^2$

Factor the difference of squares.

4. $x^4 - 16$ **5.** $4x^2b^2 - 9$ **6.** $16x^6 - 81$

Factor the perfect square trinomials.

7. $x^2 + 6x + 9$ **8.** $9x^2 + 24x + 16$ **9.** $4x^2 - 24x + 36$

Factor by identifying factor pairs.

10. $x^2 - 2x - 15$ **11.** $x^2 + 10x + 16$ **12.** $x^2 - 24x - 25$

Use the point-slope formula to write a linear equation from the given information. 1.14

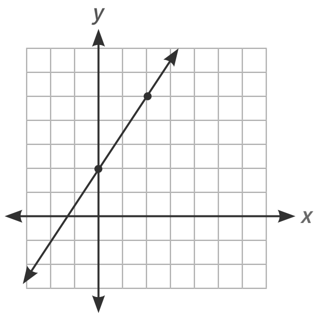
13. $m = \frac{10}{3}$, point = (3, –1) **14.** $m = -\frac{7}{2}$, point = $(2, -\frac{4}{3})$ **15.** $m = \frac{2}{9}$, point = (0, 5)

Write a linear equation for the line passing through the points. 1.14

16. (4, 4) (0, –3) **17.** (–3, –5) (2, 6) **18.** (0, 7) (9, 0)

Write an equation for each graphed line. 1.14

19.



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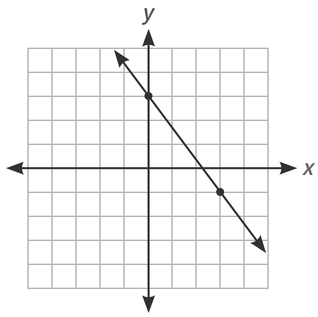

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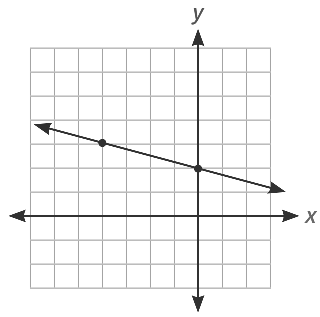

Solve each problem. 1.14

22. An estimator for a roofing company needs to determine the slope of a roof. There is a danger that the workers could lose their footing and slide off the roof if the slope is greater than 8:12 (eight inches of rise for every foot of run). He calculates that the one side of the roof has a run of 17.6 feet and a rise of 6.8 feet.

Hint: View the roof as a line on a graph where the rise represents *y* values and the run represents *x* values. This problem is calculating the slope, or value of *m*. Set the bottom edge of the roof to (0, 0).

a. What is the slope of a roof with an 8:12 pitch? Express the answer as a decimal.

b. What is the slope of a roof that has a run of 17.6 feet and a rise of 6.8 feet? Express the answer as a decimal.

c. Is the roof steep enough that the workers might be in danger of sliding off?

Multiply each polynomial. 1.12

23. $7m^2n \bullet 3n^3o \bullet 2m$ **24.** $(1 + y)(1 - y)$ **25.** $(2a - 3b)(3a^2 + 4)$

Divide each polynomial. 1.12

26. $(6y^3 + 4x^2y - 8x^3) \div 2xy^2$ **27.** $(4ab^2c - 5bc^2 + 6a^3) \div 8abc$

Complete the truth table for the values of *x* given. Circle the correct answer to each question below. 1.11

28. Let P be $x < 5$ and Q be $x > -6$.

	P (x < 5)	Q (x > –6)	P ∧ Q
–8			
0			
11			

29. This truth table illustrates a __?__.

conjunction **disjunction**

30. For a __?__ to be true, both simple statements must be true.

conjunction **disjunction**

Solve each literal equation for the value specified. 1.7

31. Solve $V = \frac{1}{2}lwh$ for h.

32. Solve $w = x - y$ for x.

Divide. Factor the polynomials before dividing. 1.13, 2.1

33. $\frac{x^2 - x - 2}{x^2 + 2x + 1}$

34. $\frac{x^2 - x - 6}{x^2 - 9}$

35. $\frac{x^2 + 4x + 4}{x + 2}$

Solve each equation. 1.6

36. $11x - 8 = 5x - 2$

37. $-2x = 4(9 - x)$

38. $\frac{x}{4} - \frac{1}{2} = \frac{4}{8}$

Use the sets in the text box to answer each question. 1.1

39. Using the formula, figure the total number of subsets for set Y and list them using the roster method.

40. Identify the universal set.

41. What is another name for set X?

42. $5 \in$ __?__ and __?__.

43. Set Y is a proper subset of __?__.

44. State the cardinal number for set U.

45. Set U ∩ Set Y = __?__.

Write the real-number categories each constant belongs to. 1.1

46. 0.125354…

47. –1000

48. 3

49. $0.\overline{39}$

Extra Practice

Factor by taking out the GCF.

50. $15x^2 + 10xy - 15x$

51. $12a^3b + 6ab^2 + 18ab$

52. $12m^2n^3o + 15mn^3o + 9mn^2o^2$

Factor the difference of squares.

53. $4x^6 - 9$

54. $a^{10} - 25$

55. $25b^4 - 16$

Factor the perfect square trinomials.

56. $4b^2 - 12b + 9$

57. $4y^2 - 28y + 49$

58. $9m^2 + 24mn + 16n^2$

Factor by identifying factor pairs.

59. $a^2 + a - 6$

60. $b^2 - 11b + 18$

61. $c^2 + 3c - 18$