Activities for Learning, Inc.

RightStart™ Mathematics

by Joan A. Cotter, Ph.D.

Level C Lessons

For Home Educators

Special thanks to Sharalyn Colvin, who converted *RightStart™ Mathematics: Grade 2 Lessons* into *RightStart™ Mathematics: Level C For Home Educators*.

Note: Rather than use the designation, K-4, to indicate a grade, levels are used. Level A is kindergarten, Level B is first grade, and so forth.

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Home Educators

RIGHTSTART™ MATHEMATICS

by Joan A. Cotter, Ph.D.

The following are items needed to complete the RightStart™ Mathematics Level C Lessons:

STATUS	ITEM	CODE
REQUIRED	***Level C Lessons***	T-C
REQUIRED	***Level C Worksheets***	W-C
REQUIRED	***Math Card Games*** book	M4
REQUIRED; choice of abacus	Classic AL Abacus - 8-1/2" x 9-1/2" hardwood frame & beads	A-CL
	Standard AL Abacus - 7-1/2" x 9-1/2" plastic frame & beads	A-ST
	Junior AL Abacus - 5-1/4" x 6" plastic frame & beads	A-JR
REQUIRED	Place Value Cards	P
REQUIRED	Abacus Tiles	AT
REQUIRED	Cards, Six Special Decks needed for Games	C
REQUIRED	Fraction Charts	F
REQUIRED	Basic Drawing Board Geometry Set	DS
REQUIRED	Colored Tiles, apx 200 in set	RH2
REQUIRED	Casio Calculator SL-450	R4
REQUIRED	4" Geared Clock	R12
RECOMMENDED	Math Balance (Invicta)	R7
REQUIRED	Tangrams, 2 per set	R9
REQUIRED	Centimeter Cubes, 100 in set	R8
RECOMMENDED	4-in-1 Ruler	R10
RECOMMENDED	Plastic Coins	R5

Note: If a child has not previously worked with the AL abacus and is just starting RightStart™ Mathematics , *RightStart Mathematics Transition Lessons* are required before starting the *RightStart Mathematics Level C Lessons* .

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RightStart™ MATHEMATICS: OBJECTIVES FOR LEVEL C

Name _____ **Year** _____

Teacher _____

	1ST QTR	2ND QTR	3RD QTR	4TH QTR
Numeration				
Can skip count the first ten multiples for the 2s through the 10s	N/A	N/A		
Can read and construct Roman numerals to 1000				
Addition				
Can add 2-digit numbers mentally				
Can add 4-digit numbers				
Knows addition facts				
Subtraction				
Understands subtraction				
Can subtract 2-digit numbers mentally				
Can subtract 4-digit numbers	N/A			
Knows subtraction facts	N/A			
Multiplication				
Understands multiplication				
Knows multiplication facts	N/A	N/A		
Can multiply 4-digit by 1-digit numbers	N/A	N/A	N/A	
Calculator				
Can add and subtract whole numbers	N/A	N/A	N/A	
Can multiply whole numbers				
Money				
Can find the value of up to five coins	N/A			
Problem Solving				
Can solve problems in more than one way				
Geometry				
Can construct an equilateral triangle with drawing tools				
Understands line symmetry	N/A			
Time	N/A	N/A		
Can tell time to the minute				
Measurement				
Can measure in inches				
Can measure in whole centimeters	N/A			
Can find perimeter	N/A			
Can find area in square inches or square cm	N/A			
Fractions	N/A	N/A	N/A	
Understands fractions as a type of division				
Knows unit fractions up to 1/10	N/A			
Understands mixed fractions, for example, 1 1/2	N/A	N/A	N/A	

How This Program Was Developed

We have been hearing for years that Japanese students do better than U.S. students in math in Japan. The Asian students are ahead by the middle of first grade. And the gap widens every year thereafter.

Many explanations have been given, including less diversity and a longer school year. Japanese students attend school 240 days a year.

A third explanation given is that the Asian public values and supports education more than we do. A first grade teacher has the same status as a university professor. If a student falls behind, the family, not the school, helps the child or hires a tutor. Students often attend after-school classes.

A fourth explanation involves the philosophy of learning. Asians and Europeans believe anyone can learn mathematics or even play the violin. It is not a matter of talent, but of good teaching and hard work.

Although these explanations are valid, I decided to take a careful look at how mathematics is taught in Japanese first grades. Japan has a national curriculum, so there is little variation among teachers.

I found some important differences. One of these is the way the Asians name their numbers. In English we count ten, eleven, twelve, thirteen, and so on, which doesn't give the child a clue about tens and ones. But in Asian languages, one counts by saying ten-1, ten-2, ten-3 for the teens, and 2-ten 1, 2-ten 2, and 2-ten 3 for the twenties.

Still another difference is their criteria for manipulatives. Americans think the more the better. Asians prefer very few, but insist that they be imaginable, that is, visualizable. That is one reason they do not use colored rods. You can imagine the one and the three, but try imagining a brown eight–the quantity eight, not the color. It can't be done without grouping.

Another important difference is the emphasis on non-counting strategies for computation. Japanese children are discouraged from counting; rather they are taught to see quantities in groups of fives and tens.

For example, when an American child wants to know 9 + 4, most likely the child will start with 9 and count up 4. In contrast, the Asian child will think that if he takes 1 from the 4 and puts it with the 9, then he will have 10 and 3, or 13.

Unfortunately, very few American first-graders at the end of the year even know that 10 + 3 is 13.

I decided to conduct research using some of these ideas in two similar first grade classrooms. The control group studied math in the traditional workbook-based manner. The other class used the lesson plans I developed. The children used that special number naming for three months.

They also used a special abacus I designed, based on fives and tens. I asked 5-year-old Stan how much is 11 + 6. Then I asked him how he knew. He replied, "I have the abacus in my mind."

The children were working with thousands by the sixth week. They figured out how to add four-place numbers on paper after learning how to do it on the abacus.

Every child in the experimental class, including those enrolled in special education classes, could add numbers like 9 + 4, by changing it to 10 + 3.

I asked the children to explain what the 6 and 2 mean in the number 26. Ninety-three percent of the children in the experimental group explained it correctly while only 50% of third graders did so in another study.

I gave the children some base ten rods (none of them had seen them before) that looked like ones and tens and asked them to make 48. Then I asked them to subtract 14. The children in the control group counted 14 ones, while the experimental class removed 1 ten and 4 ones. This indicated that they saw 14 as 1 ten and 4 ones and not as 14 ones. This view of numbers is vital to understanding algorithms, or procedures, for doing arithmetic.

I asked the experimental class to mentally add 64 + 20, which only 52% of nine-year-olds on the 1986 National test did correctly; 56% of those in the experimental class could do it.

Since children often confuse columns when taught traditionally, I wrote 2304 + 86 = horizontally and asked them to find the sum any way they liked. Fifty-six percent did so correctly, including one child who did it in his head.

This following year I revised the lesson plans and both first grade classes used these methods. I am delighted to report that on a national standardized test, both classes scored at the 98th percentile.

Some General Thoughts on Teaching Mathematics

1. Only five percent of mathematics should be learned by rote; 95 percent should be understood.
2. Teaching with understanding depends upon building on what the child already knows. Teaching by rote does not care.
3. The role of the teacher is to encourage thinking by asking questions, not giving answers. Once you give an answer, thinking usually stops.
4. It is easier to understand a new model after you have made one yourself. For example, a child needs to construct graphs before attempting to read ready-made graphs.
5. Good manipulatives cause confusion at first. If the new manipulative makes perfect sense at first sight, it wasn't needed. Trying to understand and relating it to previous knowledge is what leads to greater learning, according to Richard Behr and others.
6. Lauren Resnick says, "Good mathematics learners expect to be able to make sense out of rules they are taught, and they apply some energy and time to the task of making sense. By contrast, those less adept in mathematics try to memorize and apply the rules that are taught, but do not attempt to relate these rules to what they know about mathematics at a more intuitive level."
7. According to Arthur Baroody, "Teaching mathematics is essentially a process of translating mathematics into a form children can comprehend, providing experiences that enable children to discover relationships and construct meanings, and creating opportunities to develop and exercise mathematical reasoning."
8. Mindy Holte puts learning the facts in proper perspective when she says, "In our concern about the memorization of math facts or solving problems, we must not forget that the root of mathematical study is the creation of mental pictures in the imagination and manipulating those images and relationships using the power of reason and logic."
9. The only students who like flash cards are those who don't need them.
10. Mathematics is not a solitary pursuit. According to Richard Skemp, solitary math on paper is like reading music, rather than listening to it; "Mathematics, like music, needs to be expressed in physical actions and human interactions before its symbols can evoke the silent patterns of mathematical ideas (like musical notes), simultaneous relationships (like harmonies) and expositions or proofs (like melodies)."
11. "More than most other school subjects, mathematics offers special opportunities for children to learn the power of thought as distinct from the power of authority. This is a very important lesson to learn, an essential step in the emergence of independent thinking." (A quote from *Everybody Counts)*

12. Putting thoughts into words helps the learning process.
13. The difference between a novice and an expert is that an expert catches errors much more quickly. A violinist adjusts pitch so quickly that the audience does not hear it.
14. Europeans and Asians believe learning occurs not because of ability, but primarily because of effort. In the ability model of learning, errors are a sign of failure. In the effort model, errors are natural. In Japanese classrooms, the teachers discuss errors with the whole class.
15. For teaching vocabulary, be sure either the word or the concept is known. For example, if a child is familiar with six-sided figures, we can give him the word, hexagon. Or, if he has heard the word, multiply, we can tell him what it means. It is difficult to learn a new concept and the term simultaneously.
16. Introduce new concepts globally before details. This lets the children know where they are headed.
17. Informal mathematics should precede paper and pencil work. Long before a child learns how to add fractions with unlike denominators, she should be able to add one half and one fourth mentally.
18. Some pairs of concepts are easier to remember if one of them is thought of as dominant. Then the non-dominant concept is simply the other one. For example, if even is dominant over odd; an odd number is one that is not even.
19. Worksheets should also make the child think. Therefore, they should not be a large collection of similar exercises, but should present a variety.
20. In Japan students spend more time on fewer problems. Teachers do not concern themselves with attention spans as is done in the U.S.
21. In Japan the goal of the math lesson is that the student has understood a concept, not necessarily has done something (a worksheet).
22. The calendar should show the entire month, so the children can plan ahead. The days passed can be crossed out or the current day circled.
23. A real mathematical problem is one in which the procedures to find the answer or answers are not obvious. It is like a puzzle, needing trial and error. Emphasize the satisfaction of solving problems and like puzzles, of not giving away the solution to others.
24. Keep math time enjoyable. A person who dislikes math will avoid it. We store our emotional state along with what we've learned. A child under stress stops learning. If a lesson is too hard, end it and play a game. Try again another day.

RightStart™ Mathematics

There are 13 major characteristics that make this research-based program effective.

1. Refers to quantities of up to 5 as a group; discourages counting individually.
2. Uses fingers and tally sticks to show quantities up to 10; teaches quantities 6 to 10 as 5 plus a quantity, for example 6 = 5 + 1.
3. Avoids counting procedures for finding sums and remainders. Teaches five- and ten-based strategies for the facts that are both visual and visualizable.
4. Employs games, not flash cards, for practice.
5. Once quantities 1 to 10 are known, proceeds to 10 as a unit. Uses the "math way" of naming numbers for several months; for example, "1 ten-1" (or "ten-1") for eleven, "1-ten 2" for twelve, "2-ten" for twenty, and "2-ten 5" for twenty-five.
6. Uses expanded notation (overlapping) place-value cards for recording tens and ones; the ones card is placed on the zero of the tens card. Encourages a child to read numbers starting at the left and not backward by starting at the ones column.
7. Proceeds rapidly to hundreds and thousands using manipulatives and place-value cards. Provides opportunities for trading between ones and tens, tens and hundreds, and hundreds and thousands with manipulatives.
8. Only after the above work, about the fourth month of first grade, introduces the traditional English names for quantities 20 to 99 and then 11 to 19.
9. Teaches mental computation. Investigates informal solutions, often through story problems, before learning procedures.
10. Teaches four-digit addition on the abacus, letting the child discover the paper and pencil algorithm. This occurs in Level B. Four-digit subtraction is mastered in Level C.
11. Introduces fractions with a linear visual model.
12. Approaches geometry through drawing boards and tools.
13. Teaches short division (where only the answer is written down) for single-digit divisors, before long division. Both are taught in Level E.

Some Pointers

Kindergarten. Most of the kindergarten lesson plans have two distinct topics, which can be taught on alternate days.

Transition Lessons. These lessons are designed for children starting Levels C, D, or E (grades 2, 3, or 4) who have not been doing RightStart™ Mathematics previously. The lessons need to be studied before the regular manual, except where noted. The manual tells which lessons to use for the particular grade.

Counting. Counting needs to be discouraged because it is slow and inaccurate. It also interferes with understanding quantity and learning place-value.

Warm-up. The warm-up time is the time for quick review, memory work, and sometimes an introduction to the day's topics. The drawing board makes an ideal slate for quick responses.

Place value. In order to understand addition algorithms, place-value knowledge is essential. From the very beginning, the children are helped to see quantities grouped in fives and tens. Children can understand place value in first grade and even in kindergarten when it is approached as it is in this program.

Worksheets. The worksheets are designed to give the children a chance to think about and to practice the day's lesson. Some lessons, especially in the early grades, have no worksheet.

Games. Games, not worksheets or flash cards, are used for practice. They can be played as many times as necessary until memorization takes place. Games are as important to math as books are to reading.

Some games are incorporated in this manual. Extra games, found in the book, *Math Card Games,* are suggested in the Review and Practice lessons in Levels C to E. There are games for the child needing extra help, as well as for the advanced child.

Teaching. Establish with the children some indication when you want a quick response and when you want a more thoughtful response. Research shows that the quiet time for thoughtful response should be about three seconds. Avoid talking during this quiet time; resist the temptation to rephrase the question. This quiet time gives the slower child time to think. It also gives a quicker child time to think more deeply.

Encourage the child to develop perseverance. Avoid giving answers too quickly. Children tend to stop thinking once they hear the answer.

Help the children realize that it is their responsibility to ask questions when they do not understand. Do not settle for "I don't get it."

Number of lessons. It is not necessary that each lesson be done in one day. Sometimes two days may be more appropriate. However, do complete each manual before going to the next one.

Visualization. The ability to imagine or visualize is an important skill to develop in mathematics and other subjects as well. Often you are called upon to suggest to the children that they imagine a particular topic.

Questions. I really want to hear how this program is working. Please let me know any improvements and suggestions that you may have.

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Level C

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Lesson 78	App. 8	1, heavy paper
Lesson 79,81	App. 9	1, heavy paper
Lesson 80,81	App. 10	1, heavy paper
Lesson 81	App. 11	1, heavy paper

Lesson 1

Venn Diagram & Go to the Dump

OBJECTIVES

1. To introduce the basic Venn diagram
2. To review the facts that make 10

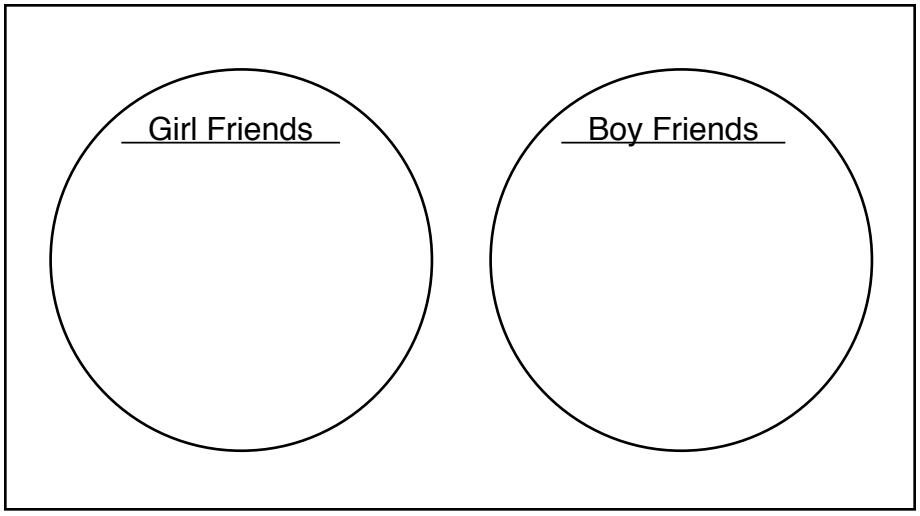
MATERIALS

Abacus
Instructions for playing Go to the Dump (*Math Card Games,* A3)
(This game was played in first grade.)
Cards for playing "Go to the Dump": 6 of each number card from 1 to 9 or 0 to 10

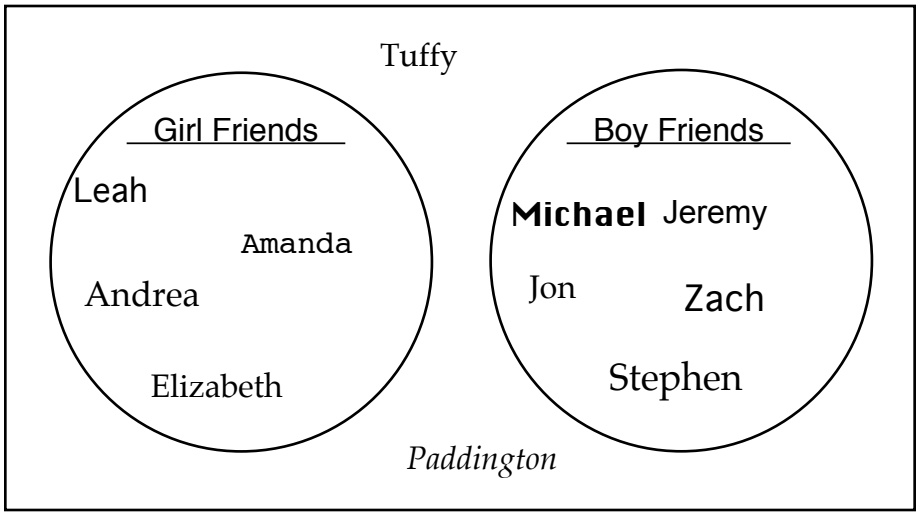
VENN DIAGRAM

Draw a Venn diagram, consisting of a rectangle and two non-overlapping circles, as shown below. In one circle write the phrase, "Girl Friends"; in the other circle write, "Boy Friends." Ask the child to write the names of friends in the correct circle. Those not fitting in either category, for example, pets, have their names inside the rectangle but outside the circles. See the figures below.

Note: Venn diagrams are a way of representing relationships visually. They help organize information in any subject.



A basic Venn diagram. The child is to write the names of friends in the appropriate circles.



A basic Venn diagram with names written in the appropriate circles.

NOTE The warm-up activities have several purposes. They provide review and practice of concepts previously learned; the child should respond verbally, and/or on a slate or dry erase board. Activities may on occasion introduce a new topic to be studied that day or in the near future; these are marked with asterisks.

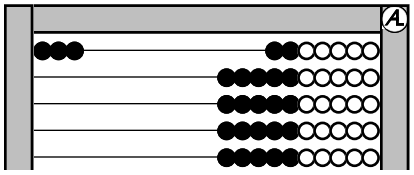
WARM-UP Ask the child to count by 1s to 5-ten, the "math" way of counting: 1, . . . , 10, 1 ten 1, 1 ten 2, . . . ,1 ten-9, 2-ten, 2-ten 1, 2-ten 2, 2-ten 3, . . . , 4-ten 8, 4-ten 9, 5-ten.

Ask the child to count by 1s to 50, using the traditional names.

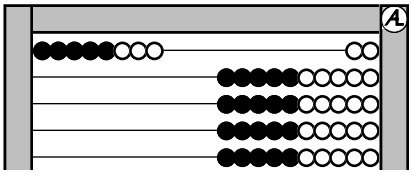
ACTIVITIES ***Reviewing quantities 1 to 10 on the abacus.*** Before giving the abacus to the child, engage her in listing basic rules for its care. The rules should include handling quietly, using fingers (not pencils) for moving the beads, and storing properly.

Note: The child must be encouraged to find strategies other than counting. One who does not progress beyond counting will remain a low achiever.

Hand the abacus to the child, and ask how to clear the abacus. [Move the beads toward the right.] Next ask her to enter quantities on the top wire as you say them; she need not clear between quantities. See the figures below. (She is *not* to count the beads.) Ask for quantities 1 to 10, such as 3, 2, 6, 8, 4, 9, 7, 5, 1, and 10.



The abacus displaying 3.

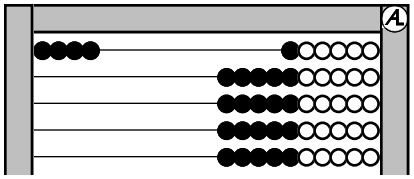


The abacus displaying 8.

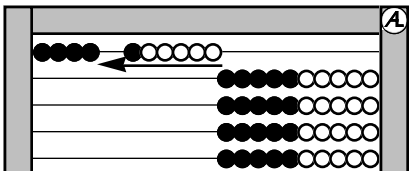
Ask how she can enter 8 without counting. [5 & 3] Repeat for 7 [5 & 2], 6 [5 & 1], and 9 [5 & 4].

Next do the inverse. Enter various quantities up to 10 on the abacus and ask the child to name them quickly.

Facts equaling 10. Enter 4 as is shown below in the left figure and ask what is needed to make 10. [6] Slide the 6 over as shown below on the right and ask if she was correct. Repeat for other numbers: 8 [2], 1 [9], and so on.



4 and what makes 10?



6 is needed with 4 to make 10.

Go to the Dump Game. To practice the facts totaling 10, play Go to the Dump. Encourage the child to use the abacus if she does not remember a fact.

Lesson 2

The Addition Table

OBJECTIVES

1. To continue reviewing the basic addition facts
2. To learn about tables through constructing the addition table

MATERIALS

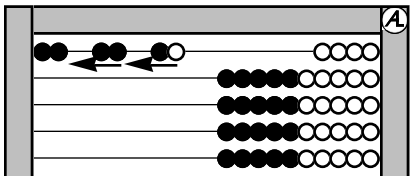
Abacus
Worksheet 1, "Addition Table" (an extra copy is needed during lesson time)

VENN DIAGRAM

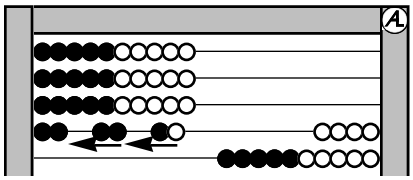
Use the same Venn diagram as in the previous lesson, but change the left heading to "Living Things in the Room" and the right heading to "Non-Living Things in the Room." Let the child read the statements and write the names of items in the room in the correct places. Names should not appear outside the circles.

WARM-UP

Enter 2 beads at a time on the abacus while the child counts by 2s to 50. See the figure below.



Counting by 2s: 2, 4, 6.



Continuing by 2s: 32, 34, 36.

For the following, the child can reply verbally or by writing on a drawing board. Play the Comes After Game; ask the child what number comes after the number you say: 3 [4], 8 [9], 12 [13], 6 [7], 19 [20], and 4 [5].

Then ask what is 3 + 1 [4], 8 + 1 [9], 12 + 1 [13], 6 + 1 [7], 19 + 1 [20], and 4 + 1 [5]. Ask, What is special about adding 1 to a number. [The sum is the next higher number.]

ACTIVITIES

Addition facts equaling 10. Show the child the addition table and explain that it can be used to show all the basic sums. Ask for a way to make 10. For example if he gives 5 + 5, demonstrate locating 5 in the left column with your left index finger and locating 5 in the top row with your right index finger. Slide your fingers together, while staying in the respective column and row, until they meet. Write 10 in the intersecting square. See the left figure following.

Ask the child what other facts total 10. [1 & 9, 2 & 8, 3 & 7, 4 & 6, 5 & 5, 6 & 4, 7 & 3, 8 & 2, and 9 & 1] Ask him to find the corresponding squares and to write the sums on the table as shown below on the right. Also ask him to note the pattern and to explain it. [To keep the same sum, one number goes up while the other number goes down.]

Note: A teacher's job in the past was to help the student get the right answer in the shortest amount of time. Today it is more important to know how a child arrives at the answer. A child should not attempt to memorize the number combinations without efficient strategies.

+	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5					10				
6									
7									
8									
9									

Adding 5 + 5 and recording the sum in the corresponding square in the addition table.

+	1	2	3	4	5	6	7	8	9
1									**10**
2								**10**	
3							**10**		
4						**10**			
5					**10**				
6				**10**					
7			**10**						
8		**10**							
9	**10**								

All sums equaling 10 recorded.

Plus 1 addition facts. Ask the child if the facts making 10 were easy. (These were learned first and reviewed in the previous lesson with the Go to the Dump game.) Next ask what other facts are easy. [adding 1s] Ask how he could find the sum of 4 + 1. [5, next higher number] Ask him to record the 5 on the addition table. See the left table below.

Note: New entries to the table are shown in boldface.

+	1	2	3	4	5	6	7	8	9
1				**5**					10
2								10	
3							10		
4	**5**					10			
5					10				
6				10					
7			10						
8		10							
9	10								

Recording the sum of 4 + 1 and 1 + 4 in the corresponding squares.

+	1	2	3	4	5	6	7	8	9
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	10
2	**3**							10	
3	**4**						10		
4	**5**					10			
5	**6**				10				
6	**7**			10					
7	**8**		10						
8	**9**	10							
9	10								

All combinations with a 1 are also recorded.

Then ask how about 1 + 4. [same] Ask him to record that sum on the addition table.

Continue with other number-plus-1 and 1-plus-a-number facts. See the right figure above.

Worksheet. Give the children the worksheet and ask them to record all the facts that make 10 and all the number-plus-1 and 1-plus-a-number facts. These are shown above on the right. Remind him in some way that it is his responsibility to ask questions if he does not understand.

Work on the addition table will continue during the next lessons.

Name _____

Addition Table

+	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

Lesson 3

Reviewing Adding Sums Less Than 10

OBJECTIVES

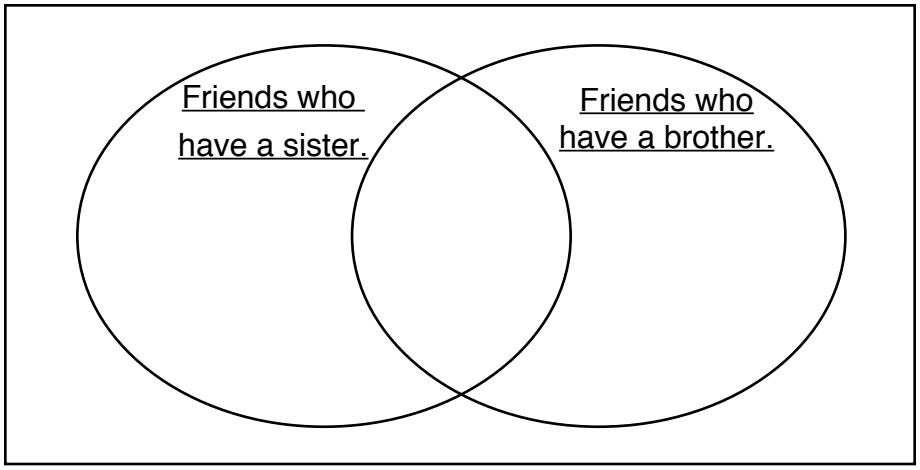
1. To continue reviewing sums of a number plus 2 and a number plus 5
2. To continue constructing the addition table

MATERIALS

Abacus
Worksheet 1, "Addition Table" from the previous lesson

VENN DIAGRAM

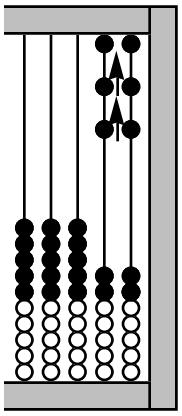
Draw the Venn diagram with overlapping circles as shown below. Let the child read the statements and write names in the correct places. Children who have both a brother and sister need to have their names written in the overlapping portion of the circles.



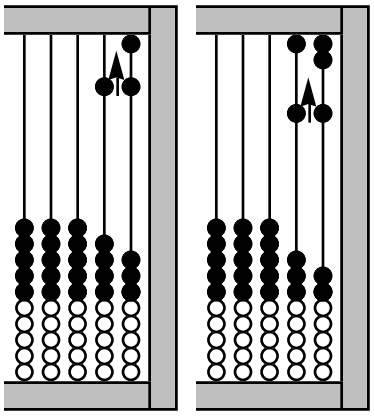
A Venn diagram with overlapping circles.

WARM-UP

Turn the abacus sideways and ask the child to say the even numbers to 20 as you move 2 beads up at a time as shown below on the left.



Counting even numbers: 2, 4, 6.



Counting odd numbers: 1, 3, 5.

Next ask the child to say the odd numbers to 19. Start with 1 on the abacus and move up 2 beads as shown on the right above.

ACTIVITIES

An even number plus 2. Ask the child the following: 2 + 2 [4], 4 + 2 [6], 6 + 2 [8], and 8 + 2 [10] .Ask what pattern she hears. [next even number] Ask her to enter the sums for these facts on the addition table. See the left figure following.

Note: New entries to the table are shown in boldface.

+	1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9	10
2	3	**4**						10	
3	4						10		
4	5	**6**				10			
5	6				10				
6	7	**8**		10					
7	8		10						
8	9	10							
9	10								

Writing the sums for an even number + 2 and 2 + an even number.

+	1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9	10
2	3	**4**	**5**	**6**	**7**	**8**	**9**	10	**11**
3	4	**5**					10		
4	5	**6**				10			
5	6	**7**			10				
6	7	**8**		10					
7	8	**9**	10						
8	9	10							
9	10	**11**							

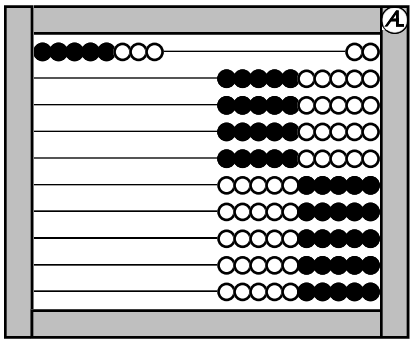
Including the sums for an odd number + 2 and 2 + an odd number.

Then ask how she could think of 2 + 6. [same as 6 + 2] Enter it in the Table. Continue with 2 + 8 and 2 + 4. Ask why some numbers are already written in. [Sums can have more than one strategy.]

An odd number plus 2. Ask the child the following: what is 1 + 2 [3], what is 3 + 2 [5], what is 5 + 2 [7], what is 7 + 2 [9], and what is 9 + 2 [11]. Then ask what is the pattern for adding an odd number plus 2. [next odd number]

Ask the child for the sum of 2 + 7 [9] and how she did it [same as 7 + 2]. Continue for 2 + 3, 2 + 1, 2 + 9, and 2 + 5. Ask her to write in these sums. See the figure on the right above.

Adding 5 to a number. Adding 5 to a number is obvious on fingers or the abacus; it is almost a definition with sums up to 10. See the figure below. Ask the child how much is 5 + 3 [8], and 5 + 1 [6], 5 + 4 [9], 5 + 2 [7], and 5 + 5 [10]. Then ask her for facts with a number 1 to 5 plus 5. Ask her to record on the Table those with 5s that are not already recorded.



The sum of 5 + 3 seen visually as 8.

+	1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9	10
2	3	4	5	6	7	8	9	10	11
3	4	5			**8**		10		
4	5	6			**9**	10			
5	6	7	**8**	**9**	10				
6	7	8		10					
7	8	9	10						
8	9	10							
9	10	11							

Writing the sums for 5 + a number and a number + 5.

Worksheet. If needed, ask the child to record on her additional worksheet all the facts for adding 2s and 5s.

Lesson 4

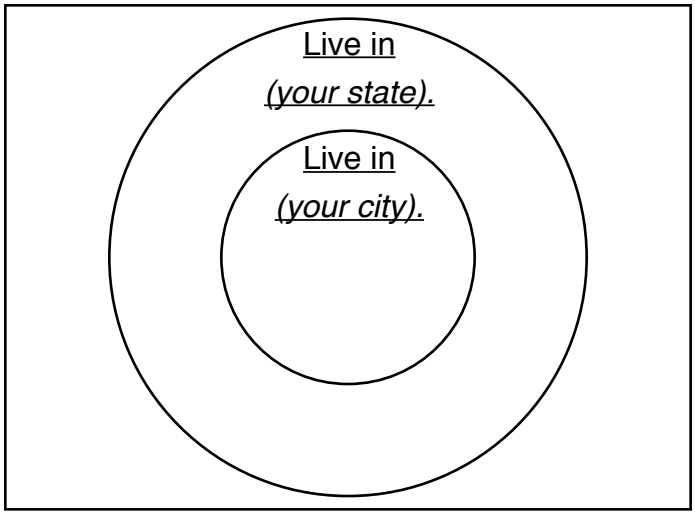
Reviewing Tens

OBJECTIVES
1. To review 10s
2. To review the terms *greater* and *less*

MATERIALS
Abacus
Tiles, about 40 to 50

VENN DIAGRAM Draw the Venn diagram with concentric circles as shown below. Write in the name of your city and state. Ask why the "city" circle is completely inside the "state" circle. [The whole city is in the state.]

Note: Ask the child to write in friends names. Discuss any patterns that can be seen.

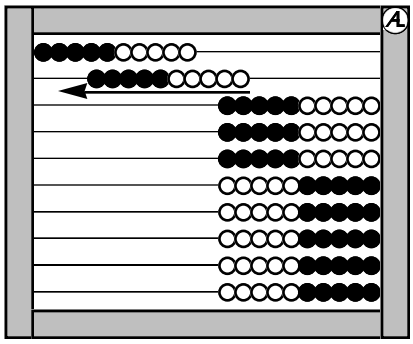


A Venn diagram with one circle inside another.

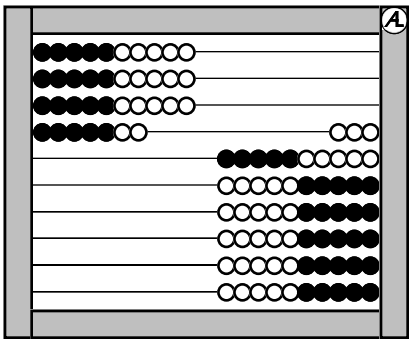
WARM-UP Ask the child to count by 10s to 100 using explicit naming, [1-ten, 2-ten, . . . , 10-ten]

Then ask the child to repeat counting by 10s to 100 using "regular" names. [ten, twenty, . . . one hundred]

ACTIVITIES ***Entering 10s on the abacus.*** Ask the child to show how 10s are counted on the abacus. See the figure below on the left.



Counting by 10s.



37 entered on the abacus.

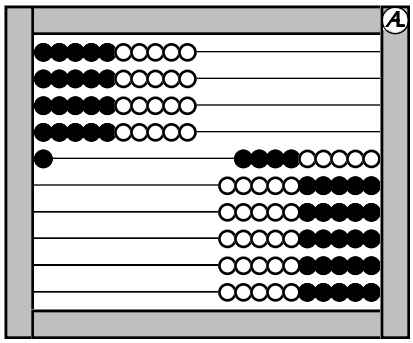
Write

37

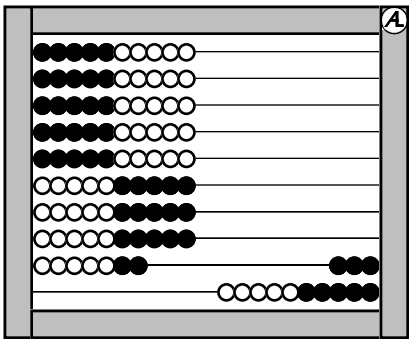
and ask him to explain how to enter 37 on the abacus without counting. [37 is 3 tens and 7 ones; it needs 3 rows and a 5 and a 2. See the figure above on the right.

Repeat for the numbers 64 and 98.

Reading 10s on the abacus. Enter 41 on the abacus as shown below; ask the child to read it without counting and to write the number on the drawing board. [41] Ask how he did it.



41 entered on the abacus.

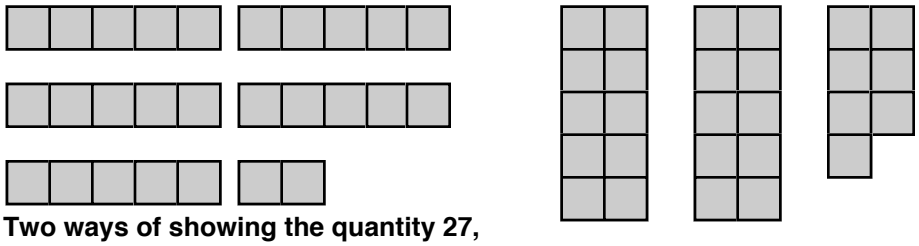


87 entered on the abacus.

Repeat for 87. It may be helpful to point out that the color change at 50. See the figure above on the right. Repeat for 59 and 94.

Counting problem. Give the child between 20 and 30 counters. Do this, perhaps, by asking him to take one or two handfuls from a container. Ask him to count how many he has, but tell him he is to arrange the counters so someone can tell how many there are without counting. Two such arrangements are shown below.

Note: It is virtually impossible to recognize or visualize more than 5 objects without grouping them. That is why the groupings by fives are paramount.



Two ways of showing the quantity 27, so counting is not necessary.

When he has finished, point to a child's arrangement and ask him to explain how we can tell how many it is without counting. Ask him to take an additional handful of objects and make a different arrangement that can be used to determine the number of objects without counting.

Comparing quantities. Point to 2 of the child's arrays that are adjacent, but with differing quantities. Ask which array has more pieces. Assuming the quantities are 23 and 25, tell the child we can say, 25 is more than 23.

Note: Sometimes children do not know that the term *greater* means more.

Then tell him in mathematical language we say, 25 is *greater* than 23. What does greater mean? [more]

Next ask what is the opposite of more [less] and what is the opposite of greater [less]. Ask, If 25 is greater than 23 what can we say about 23. [23 is less than 25.]

Lesson 9

Doubles and Near Doubles

OBJECTIVE 1. To find the *doubles* and *near doubles* on the addition table

MATERIALS Abacus
Worksheet 1, "Addition Table" from the previous lesson
Math journal (in back of the Worksheets)

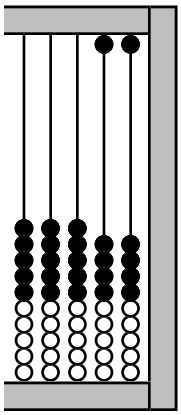
WARM-UP Ask the child to count by 10s to 200 and by 5s to 100.

Also ask her to say the even numbers to 20 and backward to 2 and the odd numbers to 19 and backward to 1.

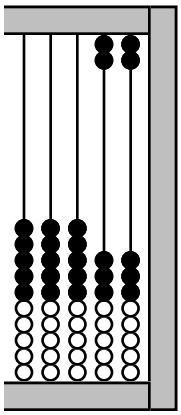
Note: If desired, the child can work on an appropriate Practice Sheets daily.

Play the Comes Before Game and ask the child, What even number comes *before* 10? [8] Before 6? [4] Before 8? [6] Before 2? [0] Before 4? [2] Then ask, What odd number comes *before* 11? [9] Before 7? [5] Before 3? [1] Before 9? [7] Before 5? [3]

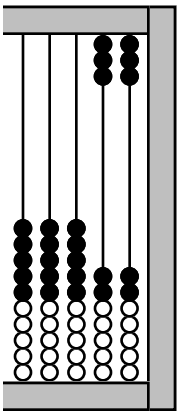
ACTIVITIES ***Doubles.*** Ask the child, What strategy do we call facts like 2 + 2, 3 + 3, and 6 + 6? [doubles] Ask her to turn the abacus sideways and to enter the doubles as shown and to recite the sums. [1 + 1 = 2, 2 + 2 = 4, 3 + 3 = 6, . . . , 9 + 9 = 18]



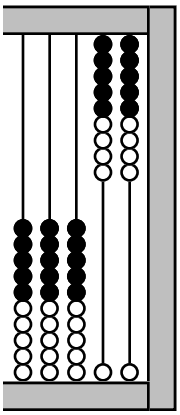
The double, 1 + 1 = 2.



The double, 2 + 2 = 4.



The double, 3 + 3 = 6.



The double, 9 + 9 =18.

Ask the child to write in any missing doubles on the addition table and to circle all the doubles. See the figure below on the left. Ask about the pattern, Why are they all even numbers?

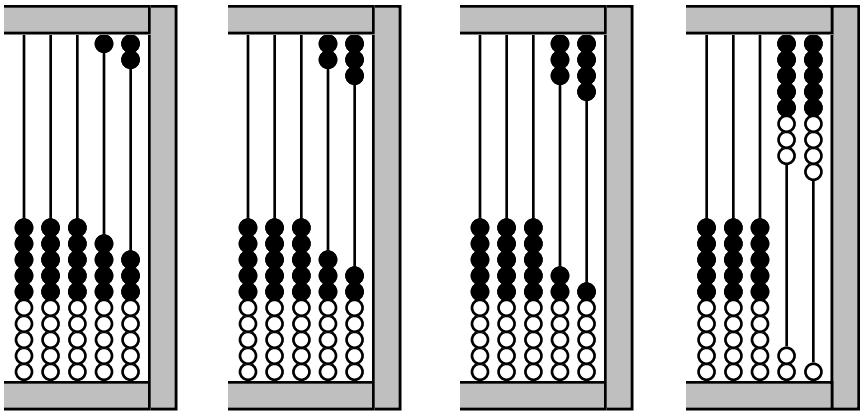
+	1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9	10
2	3	4	5	6	7	8	9	10	11
3	4	5	6		8		10	11	12
4	5	6		8	9	10		12	13
5	6	7	8	9	10	11	12	13	14
6	7	8		10	11	12	13	14	15
7	8	9	10		12	13	14	15	16
8	9	10	11	12	13	14	15	16	17
9	10	11	12	13	14	15	16	17	18

The addition table with the doubles circled.

+	1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9	10
2	3	4	5	6	7	8	9	10	11
3	4	5	6	7	8		10	11	12
4	5	6	7	8	9	10		12	13
5	6	7	8	9	10	11	12	13	14
6	7	8		10	11	12	13	14	15
7	8	9	10		12	13	14	15	16
8	9	10	11	12	13	14	15	16	17
9	10	11	12	13	14	15	16	17	18

The addition table with the doubles and near doubles circled.

Near Doubles. Enter 1 and 1 again and explain to the child that you are going to enter 1 extra bead so you have a *near double.* See the figure below. Ask what the next near double is. [2 + 3] Ask the child to enter the near doubles with you and to say the sums. [1 + 2 = 3, 2 + 3 = 5, 3 + 4 = 7, 4 + 5 = 9, 5 + 6 = 11, 6 + 7 = 13, 7 + 8 = 15, 8 + 9 = 17]



Near double, 1 + 2 = 3. **Near double, 2 + 3 = 5.** **Near double, 3 + 4 = 7.** **Near double, 8 + 9 =17.**

Ask the child to circle the near doubles on the addition table. See the figure above on the right. Ask if the near doubles are even or odd [odd] and why.

Then tell the child to write in the doubles and near doubles on her extra addition tables if the practice is needed.

Missing strategies. Ask the child to name the facts on the addition table that have missing sums. [6 + 3, 3 + 6, 4 + 7, and 7 + 4]

Take a fact, for example, 3 + 6, and ask the child what strategy she could use to remember it. [One possibility is to think of 4 + 6, which is 10, so 3 + 6 must be 9.] Ignore counting strategies. Then ask the child to explain her reasoning.

Note: The addition table will be used again. It could be stored in the child's math folder.

Repeat for the other missing sums. After the discussion of the strategies, ask the child to write the sums in her addition table.

Writing doubles and near doubles. Ask the child to write the doubles and near doubles in her math journal. Also ask her to explain why they are even or odd numbers.

Lesson 26

Problems Using Multiplication

OBJECTIVES

1. To solve problems using multiplication
2. To write multiplication equations

MATERIALS

Math journal
Tiles or other objects in 2 colors, at least 20 (optional)
Worksheet 13, "Writing the Equations"

WARM-UP

Ask the child to say the multiples of 4 to 40 as she moves over groups of 4s.

Ask the child to say the months of the year and to sing "Thirty Days." (Lesson 23). Then ask him which months have 30 days. [September, April, June, and November]

Point to any hour numbers on a clock and ask the child to say the minute number. For example, point to 2 [10], 4 [20], and 6 [30].

ACTIVITIES

Review. Ask the child what addition is. Then ask what multiplication is.

Problem. Give the child the following problem. Sammy wanted to buy 4 balloons. Each balloon cost 15¢. How much will the 4 balloons cost? [60¢]

Note: Asking the child to write out his problem solutions helps him clarify his thinking. It is also an effective way to assess his understanding.

Give the child several minutes to think about the problem. Then ask him to share his thoughts. Listen carefully to be sure he understands.

Ask the child how he could write the equation. [15¢ + 15¢ + 15¢ + 15¢ = 60¢ or 15¢ × 4 = 60¢] Discuss both equations.

Writing the solution. Ask the child to explain in writing one way he could solve the problem. Tell him to draw pictures and use words to explain how to do it. Give him 10 minutes or so.

Multiplying with 1. Write

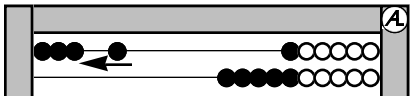
$$3 + 1 = [4]$$

and ask what it means [3 and 1 added together] and how much it is. [4]

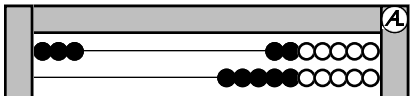
Now write

$$3 \times 1 = [3]$$

and ask the child what it means. [3 taken 1 time] Then ask how much it is. [3] Ask him to demonstrate both on an abacus.



Showing 3 + 1.

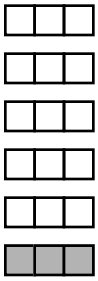


Showing 3 × 1.

Ask the child how much is 5 taken 1 time [5], 8 taken 1 time [8], and 100 taken 1 time [100].

Working with arrays. Lay out three rows each with 7 tiles; change color or space after 5 as shown below on the left. Be sure the rows are distinct.

An array of three 7s.



An array of six 3s.

Note: A child needs to see the connection between addition and multiplication.

Ask the child what he sees. [3 rows of 7 or 7 taken 3 times] (Do not correct him if he says 3 taken 7 times or uses the word *times*.] Ask him to write the equation using adding and to figure out the amount. [7 + 7 + 7 = 21]

Next ask if he could write the equation using multiplication. [7 × 3 = 21] Compare the 2 equations.

Repeat for a 3 × 6 array with tiles as shown above on the right. Again ask the child to write the 2 equations and figure out the answers. [3 + 3 + 3 + 3 + 3 + 3 = 18 and 3 × 6 = 18] Compare the 2 equations. Ask which one is easier to write.

Worksheet. Give the child the worksheet. Part of the worksheet consists of 5 arrays, for which the child writes the corresponding addition and multiplication equations. The preferred equation is given below with the other possibility given in parentheses. The child is also asked to write the pattern he sees with the 5s table.

(6 + 6 + 6 + 6 + 6 = 30)
6 × 5 = 30

(4 + 4 + 4 + 4 + 4 + 4 + 4 = 28)
4 × 7 = 28

(1 + 1 + 1 + 1 = 4)
1 × 4 = 4

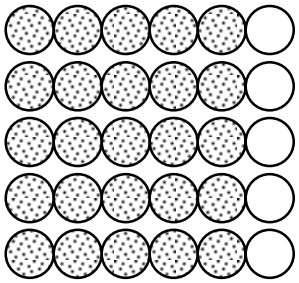
(8 + 8 + 8 + 8 + 8 + 8 = 48)
8 × 6 = 48

(5 + 5 + 5 + 5 = 20)
5 × 4 = 20

5 × 1 = **5**
5 × 2 = **10**
5 × 3 = **15**
5 × 4 = **20**
5 × 5 = **25**
5 × 6 = **30**
5 × 7 = **35**
5 × 8 = **40**
5 × 9 = **45**
5 × 10 = **50**
5 × 11 = **55**

Name _____

Write the equations.



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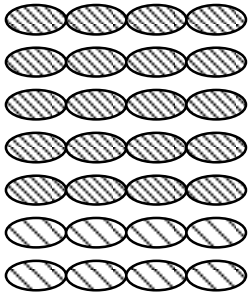

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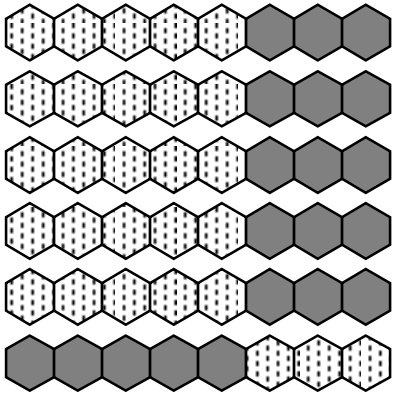

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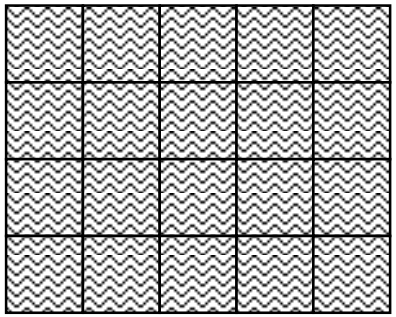

_____

_____

5 × 1 =	_____
5 × 2 =	_____
5 × 3 =	_____
5 × 4 =	_____
5 × 5 =	_____
5 × 6 =	_____
5 × 7 =	_____
5 × 8 =	_____
5 × 9 =	_____
5 × 10 =	_____
5 × 11 =	_____

What pattern do you see?

_____

_____