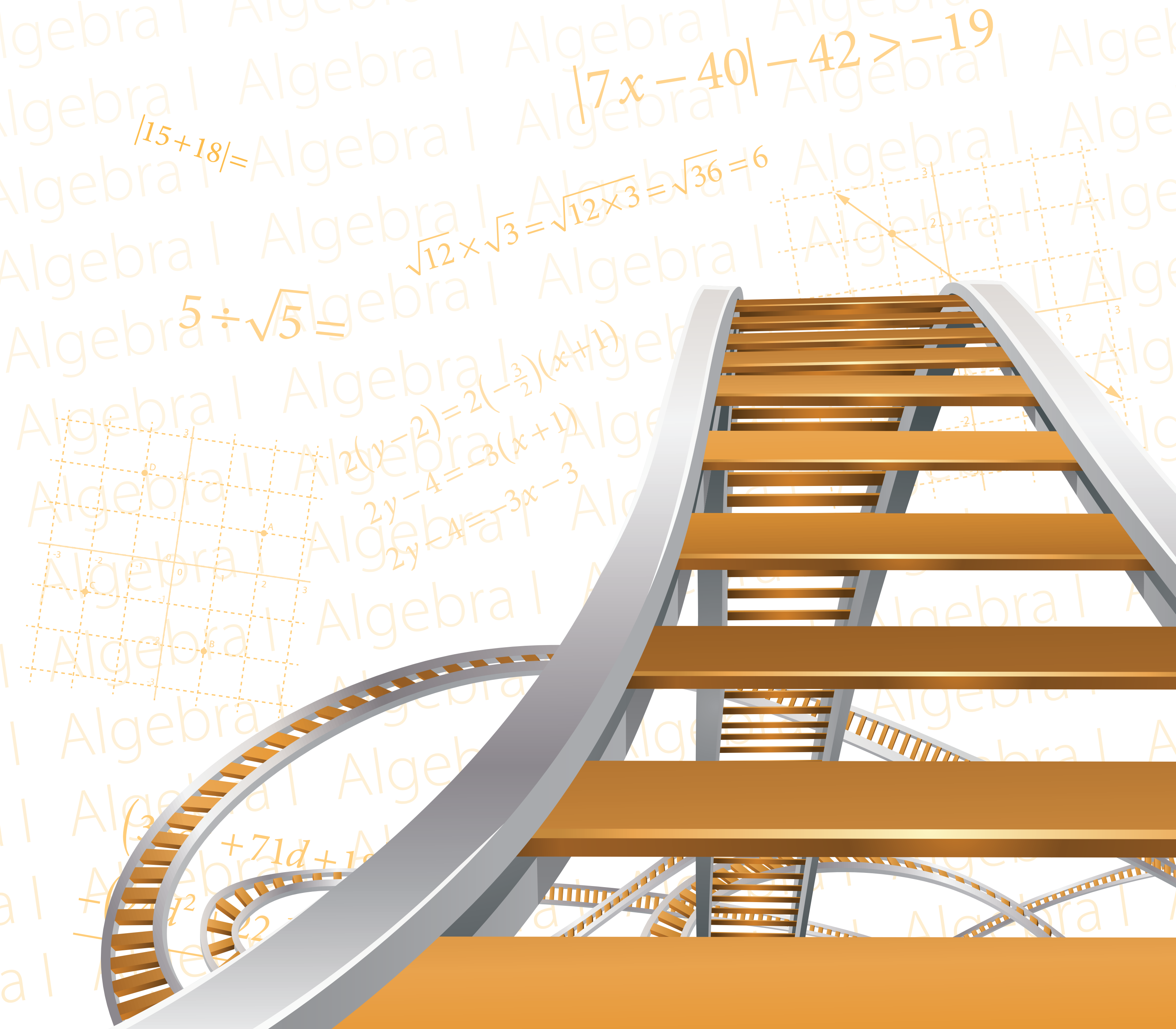
Horizons

Algebra 1

Teacher's Guide



Horizons

Algebra 1

Teacher's Guide

Author:

Shelly Chittam, M.S.

Managing Editor:

Alan Christopherson, M.S.

Editors:

Laura Messner, B.A.

Rachelle Wiersma, M.A.

Graphic Design & Illustration:

Shelly Chittam, M.S.

Alan Christopherson, M.S.

Alpha Omega Publications • Rock Rapids, IA

804 N. 2nd Ave. E.
Rock Rapids, IA 51246-1759

Printed in the United States of America

ISBN 978-0-7403-2554-0

Horizons

Algebra 1

Teacher's Guide

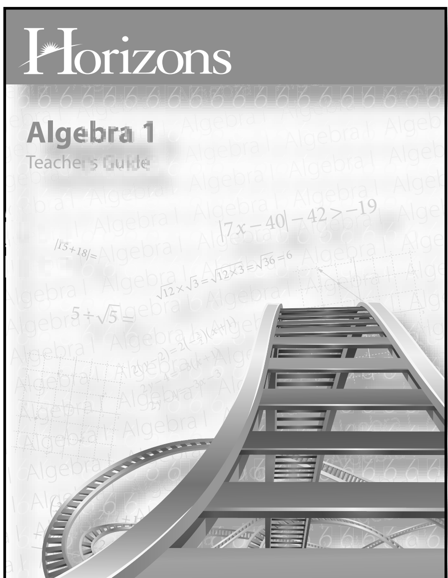
Contents

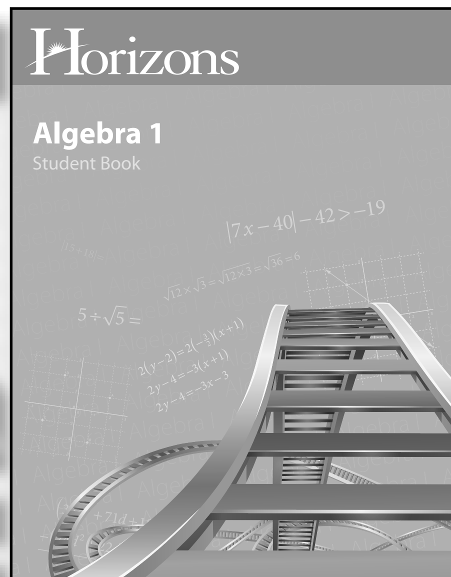
Course Introduction

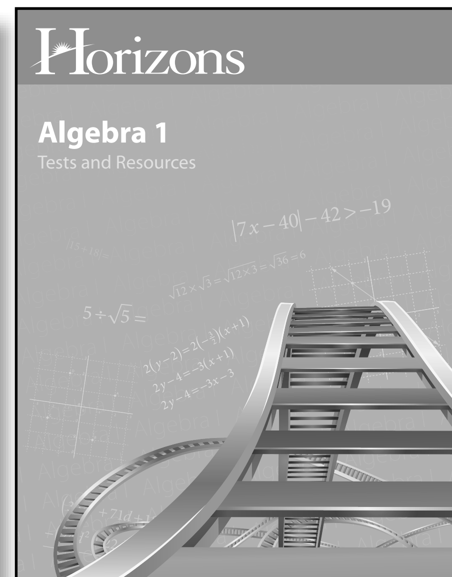
Purpose

This Algebra 1 course has a two-fold purpose. First, students have a thorough review of pre-algebra concepts that are vital for success in upper-level math courses. These concepts include order of operations, signed numbers, roots, exponents, and algebraic properties and notation. Emphasis is placed on practical application of the concepts.

The second purpose of the course is to increase the student's understanding and mastery of algebra, including some advanced algebraic concepts, in preparation for upper-level math courses. After completing this course of study, students should be well prepared for high school level courses in Algebra 2, Geometry, and Trigonometry.







Materials

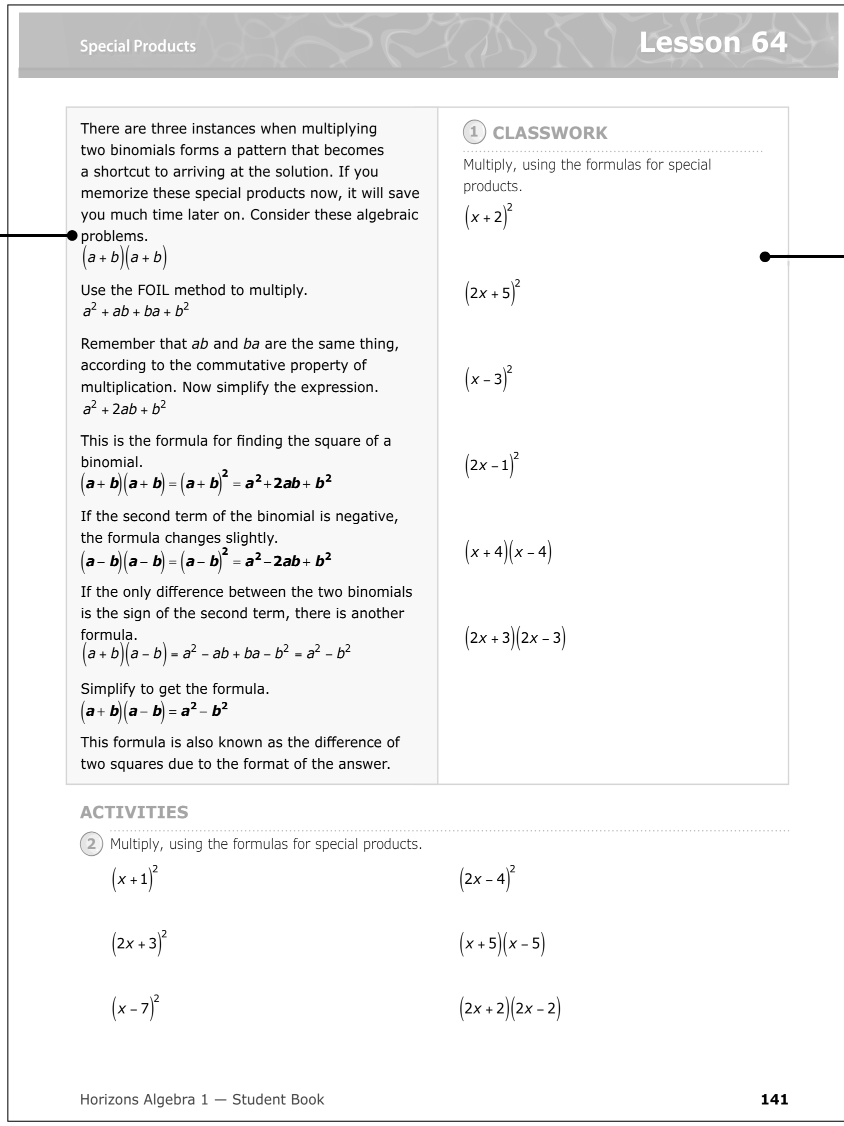
Materials available for this course include the Teacher's Guide, the Student Book, and the Tests and Resources Book. The students will have to supply notebook paper, as well as a scientific calculator, colored pencils, a ruler, and graph paper. Often the Student Book will not have sufficient space for working out all of the steps to the problems. Notebook paper should be used for these situations. Graph paper should have no more than five squares per inch, although quad-rule paper is recommended. The Tests and Resources Book was designed to be a consumable. It has perforated pages for easy tear out. It is recommended that the Student Book remain intact to serve as a resource when students wish to review previously covered concepts.

Layout

Each Lesson in the Student Text has a teaching box in the upper left side of the first page and a Classwork section in the upper right side of the first page. The teaching box is intended for use by both the teacher and the students as an aid to understanding the lesson. New concepts are presented here in detail so students who miss a lesson in class should be able to catch up any missed work with minimal outside help. The Classwork section is intended for the class to do together, with individual students explaining the problems for the class.

Teaching Box

Classwork



Special Products **Lesson 64**

There are three instances when multiplying two binomials forms a pattern that becomes a shortcut to arriving at the solution. If you memorize these special products now, it will save you much time later on. Consider these algebraic problems.

$(a+b)(a+b)$

Use the FOIL method to multiply.

$a^2+ab+ba+b^2$

Remember that *ab* and *ba* are the same thing, according to the commutative property of multiplication. Now simplify the expression.

$a^2+2ab+b^2$

This is the formula for finding the square of a binomial.

$\mathbf{(a+b)(a+b)=(a+b)^2=a^2+2ab+b^2}$

If the second term of the binomial is negative, the formula changes slightly.

$\mathbf{(a-b)(a-b)=(a-b)^2=a^2-2ab+b^2}$

If the only difference between the two binomials is the sign of the second term, there is another formula.

$(a+b)(a-b)=a^2-ab+ba-b^2=a^2-b^2$

Simplify to get the formula.

$\mathbf{(a+b)(a-b)=a^2-b^2}$

This formula is also known as the difference of two squares due to the format of the answer.

1 **CLASSWORK**

Multiply, using the formulas for special products.

$(x+2)^2$

$(2x+5)^2$

$(x-3)^2$

$(2x-1)^2$

$(x+4)(x-4)$

$(2x+3)(2x-3)$

ACTIVITIES

2 Multiply, using the formulas for special products.

$(x+1)^2$ $(2x-4)^2$

$(2x+3)^2$ $(x+5)(x-5)$

$(x-7)^2$ $(2x+2)(2x-2)$

Horizons Algebra 1 — Student Book **141**

Layout continued:

Following the Classwork section is the Activities section. The first problem set in each Activities section is for reinforcement of the concept taught in that lesson. The remaining Activities sections are for review of previously taught concepts. The Activities sections are part of the assignment for each lesson.

3 Multiply. Use the formulas for special products when possible.

$(x+9)^2$	$(2x+3)(x^2+7x-2)$
$(x+1)(x^2+x+3)$	$(4x-3)^2$
$(3x+2)(3x-2)$	$(x+11)(x-11)$
$(x-3)(x^2-4x-3)$	$(2x+9)(3x^2+2x-6)$
$(x+5)(x^2-7x-4)$	$(3x-1)(4x^2+2x-3)$
$(x-1)^2$	$(4x+5)^2$
$(x^2+5x+6)(x^2-2x-3)$	$(5x^2-2x-7)(3x^2+4x+8)$

Activities

4 Solve.

A swimming pool is 3 feet deep on the shallow end. The floor of the pool is level for a distance of 10 feet from the entrance steps, then slopes downward at an even rate until it is 8 feet deep at a distance of 15 feet from the entrance steps. What is the slope of the pool in the first 10 feet?

What is the slope of the floor where the pool is getting deeper?

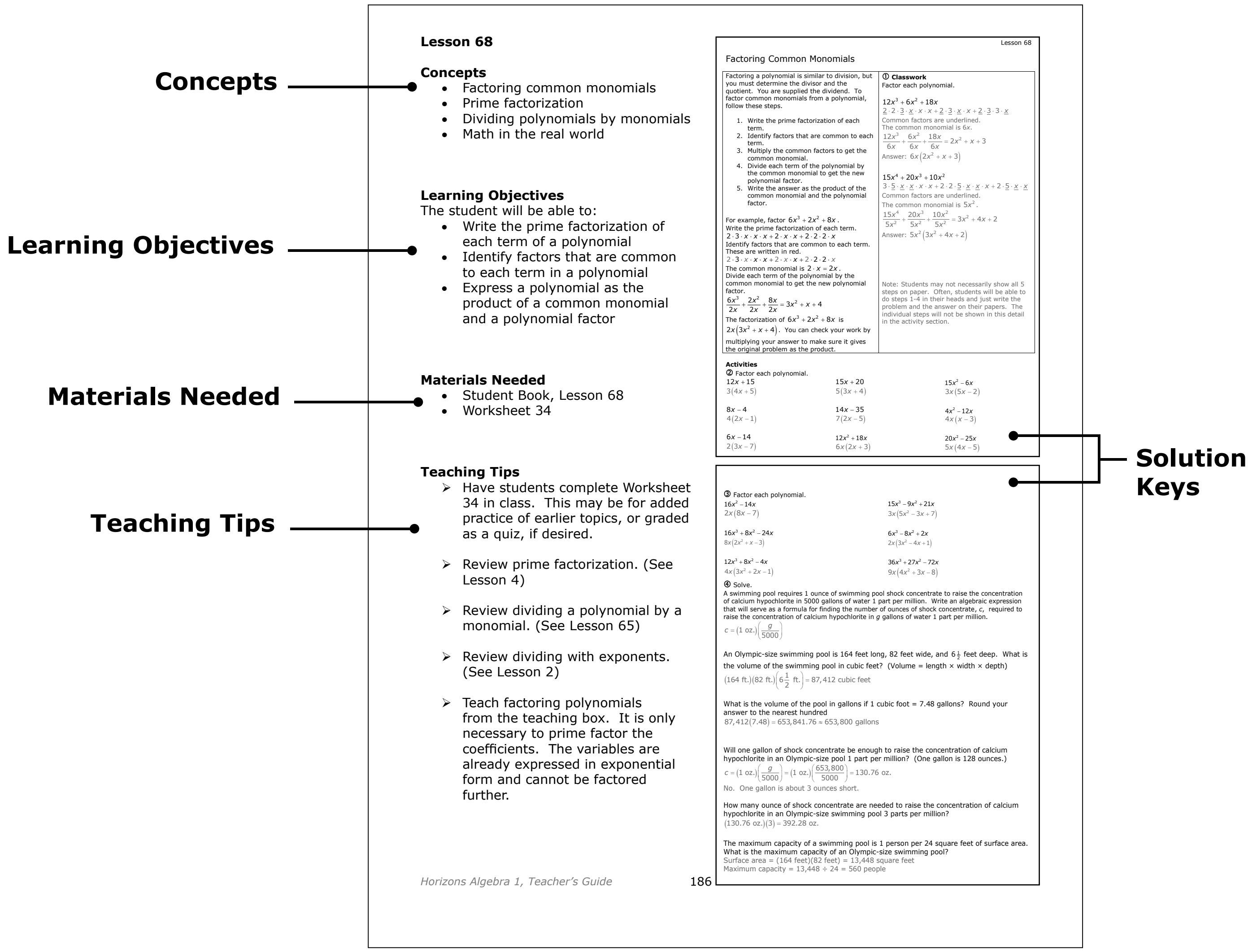
3 FEET DEEP

8 FEET DEEP

142 Horizons Algebra 1 — Student Book

Lesson Plans

Each Lesson Plan lists all concepts taught and reviewed for that individual lesson. The Learning Objectives always relate to the new material taught in that lesson. Each Lesson Plan contains Teaching Tips to aid the teacher in presenting the new material. As often as possible, new material is introduced following a review of related, previously-taught material. The Lesson Plans give detailed helps for the teacher, including sample problems, illustrations, and visual aids. The solution keys for the student activities are also part of each lesson plan.



Lesson Plans continued:

Some Lesson Plans will include a Worksheet. These are found in the Tests and Resources Book. Some Worksheets are for additional practice of a new concept while others are for review or a quiz grade. The Lesson Plan will indicate which case applies for each Worksheet. Those intended for additional practice will appear in the Assignments section at the end of the Lesson Plan.

Worksheet
(In *Tests & Resources*)

Multiplying Radicals, Dividing Radicals — **Worksheet 14**

1 Multiply.

$\sqrt{7}\left(8+\sqrt{2}\right)$

$\sqrt{3}\left(8+\sqrt{13}\right)$

$\sqrt{6}\left(2+\sqrt{5}\right)$

$\sqrt{5}\left(9+\sqrt{7}\right)$

$\sqrt{10}\left(4+\sqrt{3}\right)$

$\sqrt{3}\left(4+\sqrt{15}\right)$

$\sqrt{2}\left(5+\sqrt{18}\right)$

$\sqrt{5}\left(4+\sqrt{15}\right)$

$\sqrt{11}\left(8+\sqrt{7}\right)$

$\sqrt{7}\left(9+\sqrt{42}\right)$

2 Divide.

$\left(4\sqrt{15}+7\sqrt{35}\right)\div\sqrt{5}$

$\left(7\sqrt{21}+9\sqrt{14}\right)\div\sqrt{7}$

$\left(2\sqrt{18}+5\sqrt{12}\right)\div\sqrt{2}$

$\left(10\sqrt{3}+2\sqrt{12}\right)\div\sqrt{15}$

$\left(4\sqrt{10}+3\sqrt{15}\right)\div\sqrt{6}$

$\left(4\sqrt{7}+11\sqrt{3}\right)\div\sqrt{5}$

Worksheet Solution
(In *Teacher's Guide*)

Multiplying Radicals, Dividing Radicals — Worksheet 14

① Multiply.

$\sqrt{7}\left(8+\sqrt{2}\right)$
$8\sqrt{7}+\sqrt{7\cdot 2}=8\sqrt{7}+\sqrt{14}$

$\sqrt{3}\left(8+\sqrt{13}\right)$
$8\sqrt{3}+\sqrt{3\cdot 13}=8\sqrt{3}+\sqrt{39}$

$\sqrt{6}\left(2+\sqrt{5}\right)$
$2\sqrt{6}+\sqrt{6\cdot 5}=2\sqrt{6}+\sqrt{30}$

$\sqrt{5}\left(9+\sqrt{7}\right)$
$9\sqrt{5}+\sqrt{5\cdot 7}=9\sqrt{5}+\sqrt{35}$

$\sqrt{10}\left(4+\sqrt{3}\right)$
$4\sqrt{10}+\sqrt{10\cdot 3}=4\sqrt{10}+\sqrt{30}$

$\sqrt{3}\left(4+\sqrt{15}\right)$
$4\sqrt{3}+\sqrt{3\cdot 15}=4\sqrt{3}+\sqrt{3\cdot 3\cdot 5}=$
$4\sqrt{3}+3\sqrt{5}$

$\sqrt{2}\left(5+\sqrt{18}\right)$
$5\sqrt{2}+\sqrt{2\cdot 18}=5\sqrt{2}+\sqrt{36}=$
$5\sqrt{2}+6$ or $6+5\sqrt{2}$

$\sqrt{5}\left(4+\sqrt{15}\right)$
$4\sqrt{5}+\sqrt{5\cdot 15}=4\sqrt{5}+\sqrt{5\cdot 3\cdot 5}=$
$4\sqrt{5}+5\sqrt{3}$

$\sqrt{11}\left(8+\sqrt{7}\right)$
$8\sqrt{11}+\sqrt{11\cdot 7}=8\sqrt{11}+\sqrt{77}$

$\sqrt{7}\left(9+\sqrt{42}\right)$
$9\sqrt{7}+\sqrt{7\cdot 42}=9\sqrt{7}+\sqrt{7\cdot 6\cdot 7}=$
$9\sqrt{7}+7\sqrt{6}$

② Divide.

$\left(4\sqrt{15}+7\sqrt{35}\right)\div\sqrt{5}$
$\frac{4\sqrt{\cancel{15}^{3}}}{\cancel{\sqrt{5}}}+\frac{7\sqrt{\cancel{35}^{7}}}{\cancel{\sqrt{5}}}=4\sqrt{3}+7\sqrt{7}$

$\left(7\sqrt{21}+9\sqrt{14}\right)\div\sqrt{7}$
$\frac{7\sqrt{\cancel{21}^{3}}}{\cancel{\sqrt{7}}}+\frac{9\sqrt{\cancel{14}^{2}}}{\cancel{\sqrt{7}}}=7\sqrt{3}+9\sqrt{2}$

$\left(2\sqrt{18}+5\sqrt{12}\right)\div\sqrt{2}$
$\frac{2\sqrt{\cancel{18}^{9}}}{\sqrt{\cancel{2}}}+\frac{5\sqrt{\cancel{12}^{6}}}{\sqrt{\cancel{2}}}=2\cdot 3+5\sqrt{6}=$
$6+5\sqrt{6}$

$\left(10\sqrt{3}+2\sqrt{12}\right)\div\sqrt{15}$
$\frac{10\cancel{\sqrt{3}}}{\sqrt{\cancel{15}_{5}}}+\frac{2\sqrt{\cancel{12}^{4}}}{\sqrt{\cancel{15}_{5}}}=\frac{10}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}}+\frac{2\sqrt{4}}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}}=$
$\frac{10\sqrt{5}}{\sqrt{5\cdot 5}}+\frac{2\cdot 2\sqrt{5}}{\sqrt{5\cdot 5}}=\frac{10\sqrt{5}+4\sqrt{5}}{5}=\frac{14\sqrt{5}}{5}$

$\left(4\sqrt{10}+3\sqrt{15}\right)\div\sqrt{6}$
$\frac{4\sqrt{10}}{\sqrt{6}}+\frac{3\sqrt{15}}{\sqrt{6}}=\frac{4\sqrt{\cancel{10}^{5}}}{\sqrt{\cancel{6}^{3}}}\cdot\frac{\sqrt{3}}{\sqrt{3}}+\frac{3\sqrt{\cancel{15}^{5}}}{\sqrt{\cancel{6}^{2}}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=$
$\frac{4\sqrt{5\cdot 3}}{\sqrt{3\cdot 3}}+\frac{3\sqrt{5\cdot 2}}{\sqrt{2\cdot 2}}=$
$\frac{4\sqrt{15}}{3}+\frac{3\sqrt{10}}{2}=$ or $\frac{8\sqrt{15}+9\sqrt{10}}{6}$

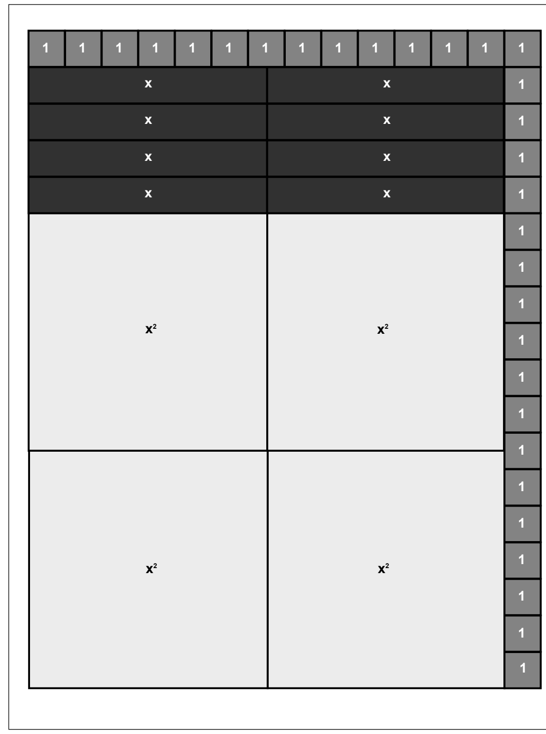
$\left(4\sqrt{7}+11\sqrt{3}\right)\div\sqrt{5}$
$\frac{4\sqrt{7}}{\sqrt{5}}+\frac{11\sqrt{3}}{\sqrt{5}}=\frac{4\sqrt{7}}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}}+\frac{11\sqrt{3}}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}}=$
$\frac{4\sqrt{7\cdot 5}}{\sqrt{5\cdot 5}}+\frac{11\sqrt{3\cdot 5}}{\sqrt{5\cdot 5}}=$
$\frac{4\sqrt{35}}{5}+\frac{11\sqrt{15}}{5}=$ or $\frac{4\sqrt{35}+11\sqrt{15}}{5}$

Learning Styles

Students learn in different ways. Some students can master a concept by listening to instructions or watching someone else do it while others are very "hand-on" and must physically do something to learn a new concept. This book addresses the various learning styles by using a lecture-demonstration method to teach new concepts and review old concepts, and manipulatives are used where appropriate to aid in the understanding of new concepts.

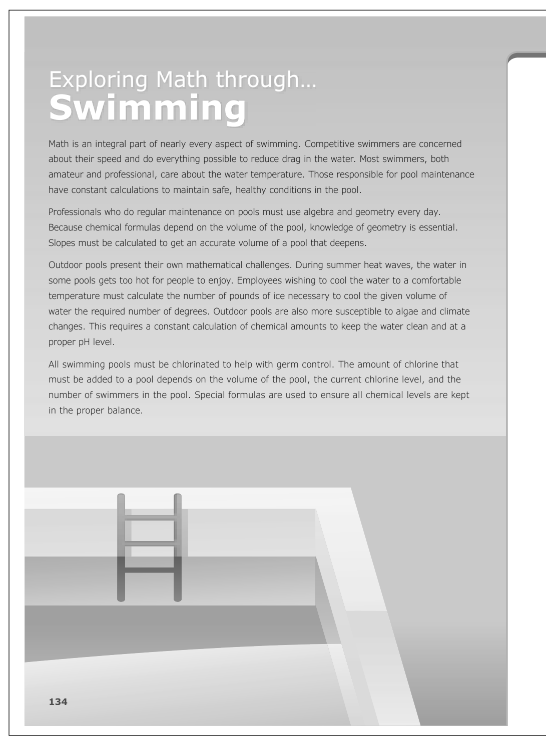
Algebra Tiles

Algebra tiles are located in the Tests and Resources Book. Students should cut these out the first time the Lesson Plan calls for them and store them in a zip-top bag for future use. These manipulatives will assist both visual and kinesthetic learners in mastering algebraic concepts. Details on their use are given in the Lesson Plans where needed.



Exploring Math through . . .

At the beginning of each set of 10 lessons the students will read about a sport or hobby that uses math. The word problems that appear in the section will be based on the featured sport or hobby. Each of the 16 sections of material in this course utilizes a different sport or hobby. None of these activities require a high school education to participate in but all involve extensive mathematics in one way or another.



Exploring Math through...
Swimming

Math is an integral part of nearly every aspect of swimming. Competitive swimmers are concerned about their speed and do everything possible to reduce drag in the water. Most swimmers, both amateur and professional, care about the water temperature. Those responsible for pool maintenance have constant calculations to maintain safe, healthy conditions in the pool.

Professionals who do regular maintenance on pools must use algebra and geometry every day. Because chemical formulas depend on the volume of the pool, knowledge of geometry is essential. Slopes must be calculated to get an accurate volume of a pool that deepens.

Outdoor pools present their own mathematical challenges. During summer heat waves, the water in some pools gets too hot for people to enjoy. Employees wishing to cool the water to a comfortable temperature must calculate the number of pounds of ice necessary to cool the given volume of water the required number of degrees. Outdoor pools are also more susceptible to algae and climate changes. This requires a constant calculation of chemical amounts to keep the water clean and at a proper pH level.

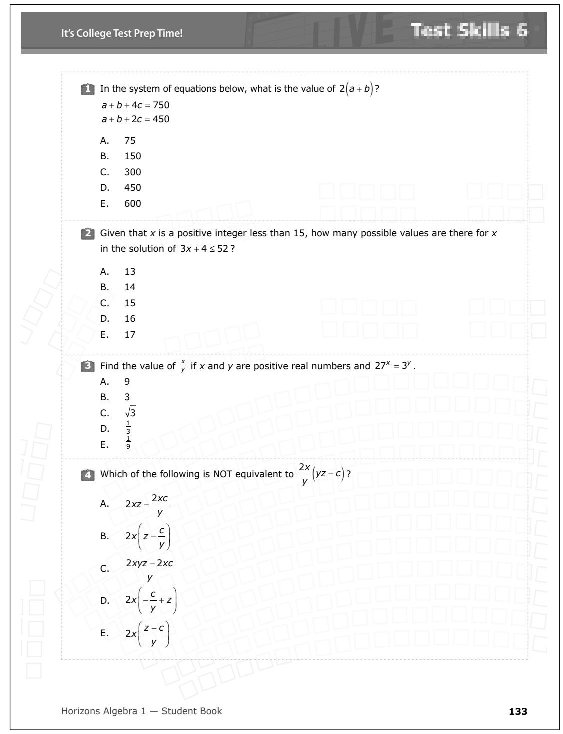
All swimming pools must be chlorinated to help with germ control. The amount of chlorine that must be added to a pool depends on the volume of the pool, the current chlorine level, and the number of swimmers in the pool. Special formulas are used to ensure all chemical levels are kept in the proper balance.

134

College Test Prep

As your students progress through their high school years, they will take a number of standardized tests that measure their skills in math, grammar, writing, vocabulary, and reading comprehension. Most colleges use the scores on these tests to determine whether or not to grant students admission to their colleges. Many scholarships are also based on the test scores, so it is important that students do as well as they can.

At the close of each set of 10 lessons, the students will be given a section of multiple choice questions. These questions are the same style and format as questions that are likely to appear on the math sections of standardized tests. They are also the same difficulty level as the Algebra 1 questions that appear on the tests.


It's College Test Prep Time! Test Skills 6

1 In the system of equations below, what is the value of $2(a+b)$?

$a+b+4c=750$
$a+b+2c=450$

A. 75
B. 150
C. 300
D. 450
E. 600

2 Given that x is a positive integer less than 15, how many possible values are there for x in the solution of $3x+4\le 52$?

A. 13
B. 14
C. 15
D. 16
E. 17

3 Find the value of $\frac{x}{y}$ if x and y are positive real numbers and $27^x=3^y$.

A. 9
B. 3
C. $\sqrt{3}$
D. $\frac{1}{3}$
E. $\frac{1}{9}$

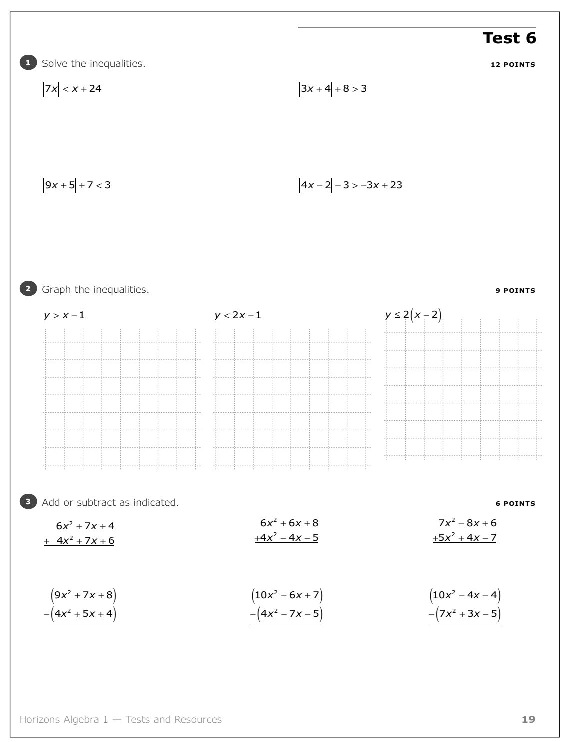
4 Which of the following is NOT equivalent to $\frac{2x}{y}(yz-c)$?

A. $2xz-\frac{2xc}{y}$
B. $2x\left(z-\frac{c}{y}\right)$
C. $\frac{2xyz-2xc}{y}$
D. $2x\left(-\frac{c}{y}+z\right)$
E. $2x\left(\frac{z-c}{y}\right)$

Horizons Algebra 1 — Student Book 133

Evaluation

This course has 16 tests, 4 exams, and 80 worksheets. One test follows each set of 10 lessons, and one exam follows every 40 lessons. Exam 4 is also a final exam. You have the option of administering the first two pages as a fourth quarter exam, or all six pages as a cumulative final exam. Many of the worksheets are used as quizzes at the teacher's discretion. Worksheets that are appropriate for quizzes are identified in the corresponding Lesson Plans.


Test 6

1 Solve the inequalities. 12 POINTS

$|7x|<x+24$ $|3x+4|+8>3$

$|9x+5|+7<3$ $|4x-2|-3>-3x+23$

2 Graph the inequalities. 9 POINTS

$y>x-1$ $y<2x-1$ $y\le 2(x-2)$

3 Add or subtract as indicated. 6 POINTS

$6x^2+7x+4$ + $4x^2+7x+6$ $6x^2+6x+8$ + $4x^2-4x-5$ $7x^2-8x+6$ + $5x^2+4x-7$

$(9x^2+7x+8)$ − $(4x^2+5x+4)$ $(10x^2-6x+7)$ − $(4x^2-7x-5)$ $(10x^2-4x-4)$ − $(7x^2+3x-5)$

Horizons Algebra 1 — Tests and Resources 19

Readiness Evaluation

Why Evaluate Readiness?

Teaching could be defined as the process of starting with what a student knows and guiding him to added knowledge with new material. While this may not be a dictionary definition of teaching, it is descriptive of the processes involved. Determining a student's readiness for Algebra 1 is the first step to successful teaching.

Types of Readiness

True readiness has little to do with chronological age. Emotional maturity and mental preparation are the main components of academic readiness. The teacher who is dealing directly with the student is best able to determine a child's emotional maturity. All emotionally immature students may need special student training in their problem areas. A child's mental *preparation* can be more easily discerned with a simple diagnostic evaluation. Observing the child's attitude of confidence or insecurity while taking the evaluation may help determine emotional readiness.

Determining Readiness

The Algebra 1 *Readiness Evaluation* on the following pages helps the teacher to determine if student(s) are ready to begin studying math at the Algebra 1 level. Complete this evaluation the first or second day of school.

The evaluation should take 45-60 minutes. It would be helpful to evaluate all of the students to determine what each student knows. However, you may want to evaluate only those student(s) whom you sense have not had a thorough preparation for this course. It is especially important to evaluate any student who is using this curriculum for the first time. The student(s) should be able to complete the test on his own with the teacher making sure he understands the directions for each individual activity.

The answer key follows the test. Count each individual answer as a separate point. The total for the test is 60 points. The student(s) should achieve a score of 42 or more points to be ready to begin Algebra 1. Be sure to note the areas of weakness of each student, even those who have scored over 42 points. Students who score under 42 points may need to repeat a previous math level or do some refresher work in their areas of weakness. For possible review of the identified areas of weakness, refer to the chart *Appearance of Concepts* in the *Horizons Pre-Algebra Teacher's Guide*. It will locate lessons where the concepts were taught.

Preparing a Lesson

GENERAL INFORMATION

There is some room on the teacher lessons for you to write your own notes. The more you personalize your teacher's guide in this way, the more useful it will be to you. You will notice that there are 160 student lessons in the curriculum. This allows for the inevitable interruptions to the school year like holidays, test days, inclement weather days, and those unexpected interruptions. It also allows the teacher the opportunity to spend more time teaching any concept that gives the student(s) difficulty. Or, you might wish to spend a day doing some of the fun activities mentioned in the Teaching Tips. If you find that the student(s) need extra drill, use the worksheets.

STUDENT'S LESSONS
Organization

The lessons are designed to be completed in forty-five to sixty minutes a day. If extra manipulatives or worksheets are utilized, you will need to allow more time for teaching. Each lesson consists of a major concept and practice of previously taught concepts. If the student(s) finds the presence of four or five different activities in one lesson a little overwhelming at the beginning, start guiding the student(s) through each activity. By the end of two weeks, the student(s) should be able to work more independently as she adjusts to the format. Mastery of a new concept is not necessary the first time it is presented. Complete understanding of a new concept will come as the concept is approached from different views using different methods at different intervals.

Tests

Tests are in the *Tests and Resources* book. The test structure is such that the student(s) will have had sufficient practice with a concept to have learned it before being tested. Therefore, no concept is tested until the initial presentation has been completed. For example, Test 2 covers concepts completed in Lessons 8-17. Lessons 18-20 may include the introduction of some new material which will not be covered in Test 2. The Lesson Plans state which Lessons are covered on each Test in the Assignment section of every tenth Lesson. Tests may be administered after every tenth lesson as a separate class day or as part of the following lesson. For example, Test 1 may be administered at the beginning of the class period for Lesson 11 or as a separate day if you wish to give students the entire class period to complete the test. Lessons 149-160 are review for Exam 4 with no new material introduced, so you have the option of combining review lessons to allow enough days in the school year to complete the full curriculum and still allow a full class period for tests. There are a total of 180 Lessons, Tests, and Exams.

TEACHER'S LESSONS
Organization

Each lesson is organized into the following sections: **Concepts**, **Learning Objectives**, **Materials Needed**, and **Teaching Tips**. To be a master teacher you will need to prepare each lesson well in advance.

Concepts

Concepts are listed at the beginning of each lesson. New concepts are listed first followed by concepts that are practiced from previous lessons. The concepts are developed in a progression that is designed to give the student(s) a solid foundation in the math skills while providing enough variety to hold the student's interest.

Learning Objectives

The Learning Objectives list criteria for the student's performance. They state what the student should be able to do at the completion of the lesson. You will find objectives helpful in determining the student's progress, the need for remedial work, and readiness for more advanced information. Objectives are stated in terms of measurable student performance. The teacher then has a fixed level of performance to be attained before the student(s) is ready to progress to the next level.

Materials Needed

Materials Needed lists the things you'll need to find before you teach each lesson. Sometimes you will also find instructions on how to make your own materials. This section also lists the worksheets. There is approximately one worksheet for every two lessons. If worksheets are suggested in a particular lesson you will find them listed. Each worksheet has a worksheet number. The *Teacher's Guide* identifies where these resource worksheets are essential to the lessons. The worksheets will be handy for many purposes. You might use them for extra work for student(s) who demonstrate extra aptitude or ability or as remedial work for the student(s) who demonstrate a lack of aptitude or ability. You may also make your own worksheets and note where you would use them in the materials section on the teacher's lesson.

Teaching Tips

The teaching tips are related to the Activities in the lesson. Some Teaching Tips require the teacher to make a manipulative needed to complete the activity. Teaching Tips are activities that the teacher can do to enhance the teaching process. You will find them useful for helping the student who needs additional practice to master the concepts or for the student who needs to be challenged by extra work.

In the Teaching Tips the teacher will find directions for teaching each lesson. All activities are designed to be teacher directed both in the student lesson and in the teacher's guide. You will need to use your own judgment concerning how much time is necessary to carry out the activities. Each activity is important to the overall scope of the lesson and must be completed.

Please do not put off looking at the activities in the lesson until you are actually teaching. Taking time to preview what you will be teaching is essential. Choose the manipulatives that fit your program best.

Each lesson in the Student Book starts with a **Teaching Box** that discusses the new material being introduced in the lesson. Sample problems are often included in this section. Some students will be able to read and comprehend the information on their own. Other students need to be guided through this section for complete understanding. Next to the Teaching Box is the **Classwork** section. The Classwork section gives the student(s) an opportunity to perform guided practice on the new concept. Following the Teaching Box and Classwork of each lesson are the numbered **Activities** problems for the lesson. Number 2 of the **Activities** section always applies the skills learned in the Teaching Box. The remaining activities review previously taught concepts.

ANSWER KEYS

The reduced page answer keys in the *Teacher's Guide* provide solutions to the activities. It is suggested that you give the student(s) a grade for tests and quizzes only. Daily work is to be a learning experience for the student, so do not put unnecessary pressure on her. You should correct every paper. At the beginning of each class period, the teacher should quickly check for completion of each student paper, without checking each problem for accuracy. The teacher may then either give the answers to the Activities, or have individual students work the problems on the board. Students should check their own papers and make corrections as needed. It is important to allow students the opportunity to ask questions about the previous day's assignment. This will save much time over the teacher grading all of the homework, and allow the students to have immediate follow-up and reinforcement of concepts missed.

WORKSHEETS

Worksheets are in the *Tests and Resources* book. These worksheets have been developed for reinforcement and drill. There is a complete listing of worksheets and where they might best be used on pages of the introduction. Answer keys to the worksheets are provided in the same manner as for the student lessons.

Horizons

Algebra 1 Scope and Sequence

1. **Integers and Real Numbers**
   Kinds of numbers
   Number line
   Absolute value
   Adding real numbers
   Subtracting real numbers
   Multiplying real numbers
   Dividing real numbers
   Exponents and powers
   Order of operations
   Factoring and prime numbers
   Greatest common factor and least common multiple
   Roots and radicals
   Distributive property

2. **Algebra**
   Variables in algebra
   Equations and inequalities
   Translating words into mathematical symbols
   Evaluating algebraic expressions
   Combining like terms
   Removing parentheses
   Using formulas
   Solving word problems
   Functions

3. **Solving Linear Equations**
   Properties of equality
   Solving equations using addition and subtraction
   *Solving equations using multiplication and division
   *Solving multi-step equations
   *Solving equations with variables on both sides
   *Solving decimal equations
   *Absolute value equations
   *Clearing equations of fractions
   Coin and interest problems
   Motion problems
   *Mixture problems
   Formulas
   Ratios and rates
   Percents

4. **Graphing Linear Equations and Functions**
Coordinate plane
Graphing linear equations
Slope
Slope-intercept form
*Graphing horizontal and vertical lines
*Graphing lines using intercepts
*Point-slope form
*Finding the equation of a line given two points
*Direct variation
*Functions and relations

5. **Writing Linear Equations**
Slope-intercept form
*Point-slope form
*Writing linear equations given two points
*Standard form
*Perpendicular lines

6. **Solving and Graphing Linear Inequalities**
Solving inequalities using addition or subtraction
Solving inequalities using multiplication or division
*Adding and subtracting inequalities
*Multiplying and dividing inequalities
*Conjunctions
*Disjunctions
*Absolute value inequalities
*Solving multi-step inequalities
*Solving compound inequalities involving "and" or "or"
*Solving absolute value equations
Graphing inequalities in two variables

7. **Systems of Linear Equations and Inequalities**
*Graphing linear systems
Solving linear systems
*Solving linear systems by linear combinations
Linear systems and problem solving
*Special types of linear systems – no solution or infinite solutions
*Systems of linear inequalities

8. **Exponents and Exponential Functions**
Multiplication properties of exponents
Zero and negative exponents
*Graphs of exponential functions
*Division properties of exponents
*Rational exponents
Scientific notation
*Exponential growth functions
*Exponential decay functions

9. **Quadratic Equations and Functions**
   *Zero product property
   *Solving quadratic equations by factoring
   *Solving equations by taking roots
   *Completing the square
   *Completing the square with leading coefficients
   *The quadratic formula
   *Solving quadratic equations
   *Quadratic functions of the form $f(x) = ax^2$
   *Quadratic functions of the form $f(x) = ax^2 + k$
   *Quadratic functions of the form $f(x) = a(x - h)^2 + k$
   *Zeros of a function
   *Applications of quadratic functions
   *Word problems with quadratic equations
   Simplifying radicals
   *Graphing quadratic functions
   *Solving quadratic functions by graphing
   *Solving quadratic functions by the quadratic formula
   *Using the discriminant
   *Graphing quadratic inequalities

10. **Polynomials and Factoring**
   Classifying and evaluating polynomials
   Adding and subtracting polynomials
   Multiplying by a monomial
   Multiplying binomials
   Multiplying polynomials
   *Special products
   Dividing by a monomial
   Dividing polynomials
   *Solving quadratic equations in factored form
   Factoring common monomials
   *Factoring the difference of two squares
   *Factoring perfect square trinomials
   *Factoring trinomials of the form $x^2 + bx + c$
   *Factoring trinomials of the form $ax^2 + bx + c$
   *Factoring trinomials of the form $ax^2 + bxy + cy^2$
   *Factoring completely
   *Factoring special products
   *Factoring cubic polynomials

11. Rational Expressions and Equations
*Simplifying rational expressions
*Multiplying rational expressions
*Dividing rational expressions
*Adding and subtracting rational expressions
*Adding rational expressions with different denominators
*Subtracting rational expressions with different denominators
*Complex rational expressions
Numerical denominators
*Polynomial denominators
Work problems
Investment problems
Motion problems
Literal equations
Proportions
*Direct and inverse variation

12. Radicals
Expressing square roots
Simplifying radicals
Multiplying radicals
*Dividing radicals and rationalizing denominators
Adding and subtracting radicals
*Multiplying and dividing radical expressions
*Radical equations
*Functions involving square roots
*Operations with radical expressions

13. Geometry
The Pythagorean Theorem
*Distance formula
*Midpoint formula

*New concepts

Where To Use *Algebra 1 Worksheets*

In the *Tests and Resources* book you will find eighty worksheets.
This chart shows where worksheets may be used for *Horizons Algebra 1*.

No.	Concept	Lessons Where Worksheets Are Used
1	Identify Numbers, Signed Numbers, Exponential Expressions	2
2	Order of Operations, Simplifying Exponents	3
3	Order of Operations, Simplifying Exponents	5
4	Prime Factorization, Absolute Value	7
5	Translating Words into Mathematical Statements, Roots	8
6	Translating Words into Mathematical Statements, Distributive Property	12
7	Evaluating Algebraic Expressions, Adding and Subtracting Polynomials	13
8	Evaluating Algebraic Expressions, Adding and Subtracting Polynomials	15
9	Multiplying and Dividing Monomials	18
10	Algebraic Equations, Extraneous Solutions	20
11	Algebraic Equations, Properties of Equality	22
12	Dividing Radicals	23
13	Dividing Radicals	25
14	Multiplying Radicals, Dividing Radicals	28
15	Graphing Linear Equations	30
16	Graphing Linear Equations	32
17	Point-slope Form, Slope-intercept Form	33
18	Slope, Slope-intercept Form	35
19	Slope and *y*-intercept	38
20	Point-slope Form, Standard Form	40
21	Slope-intercept Form, Point-slope Form, Standard Form	42
22	Perpendicular and Parallel Lines	43
23	Equations in Standard Form	45
24	Inequalities	48
25	Graphing Inequalities	50
26	Inequalities with Absolute Value	52
27	Solving Systems of Equations by Adding	53
28	Adding and Subtracting Polynomials, Systems of Equations	55
29	Systems of Equations	58
30	Multiplying Polynomials by Monomials	60
31	Solving Systems of Equations by Graphing	62
32	The FOIL Method, Multiplying Polynomials	63
33	The FOIL Method, Multiplying Polynomials	65
34	Special Products, Dividing Polynomials by Monomials	68
35	Factoring Polynomials with Special Products	70
36	Factoring Polynomials with Special Products	72
37	Factoring Polynomials	73
38	Factoring Polynomials	75
39	Factoring Special Products	78
40	Rational Expressions	80

Where To Use Algebra 1 Worksheets, continued:

No.	Concept	Lessons Where Worksheets Are Used
41	Rational Expressions	82
42	Adding Rational Expressions with Different Denominators	83
43	Multiplying and Dividing Rational Expressions	85
44	Adding Rational Expressions with Different Denominators	88
45	Complex Rational Expressions	89
46	Complex Rational Expressions, Quadratic Equations	92
47	Solving Quadratic Equations	93
48	Solving Quadratic Equations	95
49	The Quadratic Formula	98
50	Sketching the Graph of Parabolas	100
51	Graphing Parabolas, Completing the Square	102
52	Vertex of a Parabola, Zeros of a Function	103
53	Vertex of a Parabola, Zeros of a Function	105
54	Finding Zeros of Quadratic Functions	108
55	Using the Discriminant, Finding Parts of a Parabola	110
56	Parts of a Parabola, Graphing Inequalities	112
57	Money, Investment, and Motion Problems	113
58	Money, Investment, and Motion Problems	115
59	Mixture Problems	118
60	Consecutive Integers, Direct and Inverse Variation	120
61	Consecutive Integers, Direct and Inverse Variation	122
62	Graphing Inequalities on a Number Line	123
63	Graphing Inequalities on a Number Line	125
64	Conjunctions, Disjunctions	128
65	Systems of Inequalities	131
66	Systems of Inequalities	132
67	Graphs of Exponential Functions	133
68	Exponential Growth and Decay	135
69	Ratios and Proportions, Literal Equations	138
70	Factoring Polynomials, Solving Rational Expressions	140
71	Motion and Investment Problems	142
72	Pythagorean Theorem, Length of a Segment	143
73	Pythagorean Theorem, Length of a Segment	145
74	Distance Formula, Midpoint Formula	148
75	Equations of Parallel, Perpendicular, Horizontal, Vertical Lines	150
76	Systems of Equations	152
77	Solving Quadratic Equations	153
78	Quadratic Equations with Radicals	155
79	Slope, y-intercept, Standard Form Equations, Graphing	158
80	Limits of the Domain, Graphing Exponential Functions	160

Horizons Algebra 1 Appearance of Concepts

Lesson 1
Number terminology
Signed numbers
Word problems

Lesson 2
Exponents
Signed numbers
Addition
Subtraction
Multiplication
Division

Lesson 3
Order of operations
Exponents
Signed numbers
Word problems

Lesson 4
Factoring
Prime numbers
Exponents

Lesson 5
Absolute value
Signed numbers
Factoring
Prime numbers
Order of operations

Lesson 6
Greatest common factor
Least common multiple
Factoring
Exponents
Prime numbers

Lesson 7
Roots
Exponents
Absolute value
Signed numbers

Lesson 8
Algebraic expressions
Roots
Greatest common factor
Least common multiple
Word problems

Lesson 9
Algebraic expressions
Roots
Absolute value
Word problems

Lesson 10
Distributive property
Roots
Prime factorization
Exponents
Order of operations

Lesson 11
Algebraic expressions
Exponents
Absolute value
Word problems

Lesson 12
Adding polynomials
Signed numbers
Word problems

Lesson 13
Subtracting polynomials
Distributive property
Order of operations

Lesson 14
Multiplying monomials
Adding polynomials
Subtracting polynomials
Word problems

Lesson 15
Dividing monomials
Adding polynomials
Subtracting polynomials
Multiplying monomials

Lesson 16
Properties of equality
Algebraic equations
Greatest common factor

Lesson 17
Algebraic equations
Properties of equality
Adding polynomials
Subtracting polynomials
Multiplying monomials
Dividing monomials

Lesson 18
Algebraic equations
Fractions
Properties of equality
Least common multiple
Roots
Word problems

Lesson 19
Algebraic equations
Decimals
Fractions
Word problems

Lesson 20
Algebraic equations
Absolute value
Multiplying monomials
Dividing monomials
Fractions
Decimals

Lesson 21
Algebraic equations
Fractions
Decimals
Absolute value
Word problems

Lesson 22
Radical expressions
Rationalizing the denominator
Absolute value
Word problems

Lesson 23
Dividing radicals
Rationalizing the denominator
Decimals
Fractions
Absolute value

Lesson 24
Multiplying radical expressions
Dividing radicals
Rationalizing the denominator
Fractions
Decimals
Algebraic equations
Word problems

Lesson 25
Dividing radical expressions
Absolute value
Fractions
Properties of equality

Lesson 26
Algebraic equations
Properties of equality
Exponents

Lesson 27
Scientific notation
Powers of 10
Adding polynomials
Subtracting polynomials
Absolute value
Radicals

Lesson 28
Rational exponents
Decimals
Fractions
Dividing radicals
Rationalizing the denominator
Word problems

Lesson 29
Coordinate plane
Graphing points
Rational exponents
Radicals

Lesson 30
Solving linear equations
Graphing linear equations
Coordinate plane
Coordinate points

Lesson 31
Slope
Linear equations
Coordinate points
Graphing linear equations

Lesson 32
y-intercept
Slope-intercept form
Slope
Graphing linear equations
Radicals
Extraneous solutions

Lesson 33
Point-slope form
Slope-intercept form
Graphing linear equations
Word problems

Horizons Algebra 1 Appearance of Concepts, continued:

Lesson 34
Horizontal and vertical lines
Writing linear equations
Graphing linear equations
Word problems

Lesson 35
Intercepts
Linear equations
Graphing linear equations
Word problems

Lesson 36
Perpendicular lines
Slope
Linear equations
Graphing intersecting lines

Lesson 37
Parallel lines
Slope
Slope-intercept form
Perpendicular lines
Graphing linear equations
Writing linear equations

Lesson 38
Standard form
Graphing linear equations
Slope-intercept form
Point-slope form
Slope

Lesson 39
Writing linear equations
Slope-intercept form
Slope
Point-slope form

Lesson 40
Writing linear equations
Point-slope form
Standard form
Absolute value
Radicals
Extraneous solutions

Lesson 41
Writing linear equations in standard form
Slope
Writing linear equations in point-slope form
Adding polynomials
Subtracting polynomials
Multiplying monomials
Dividing monomials
Word problems

Lesson 42
Perpendicular lines
Slope
Writing linear equations in point-slope form
Writing linear equations in standard form
Graphing linear equations

Lesson 43
Parallel lines
Slope
Point-slope form
Standard form
Graphing linear equations
Word problems

Lesson 44
Writing linear equations from graphs
Horizontal lines
Vertical lines
Slope
Parallel lines
Perpendicular lines
Word problems

Lesson 45
Inequalities
Absolute value
Extraneous solutions
Square roots
Word problems

Lesson 46
Inequalities
Algebraic equations with Fractions
Properties of equality

Lesson 47
Inequalities
Fractions
Decimals
Word problems

Lesson 48
Inequalities
Absolute value
Multiplying monomials
Dividing monomials

Lesson 49
Inequalities
Absolute value
Word problems

Lesson 50
Graphing linear inequalities
Graphing linear equations
Parallel lines
Perpendicular lines
Slope
Word problems

Lesson 51
Systems of equations
Coordinate points
Order of operations
Word problems

Lesson 52
Adding polynomials
Subtracting polynomials
Systems of equations
Inequalities
Radicals
Absolute value

Lesson 53
Systems of equations
Adding linear equations
Standard form
Fractions
Word problems

Lesson 54
Systems of equations
Subtracting linear equations
Standard form
Slope-intercept form
Perpendicular lines
Parallel lines

Lesson 55
Systems of equations
Multiplying a polynomial by a constant
Adding linear equations
Subtracting linear equations
Order of operations
Inequalities

Lesson 56
Systems of equations
Dividing a polynomial by a constant
Adding linear equations
Subtracting linear equations
Word problems

Lesson 57
Systems of equations
Adding linear equations
Multiplying a polynomial by a constant
Word problems

Lesson 58
Systems of equations
Adding linear equations
Subtracting linear equations
Linear combinations
Word problems

Lesson 59
Systems of equations
Graphing linear equations
Word problems

Lesson 60
Multiplying a polynomial by a monomial
Absolute value
Radicals
Extraneous solutions
Fractions
Properties of equality

Lesson 61
Multiplying binomials
Multiplying a polynomial by a monomial
Systems of equations
Adding linear equations
Subtracting linear equations
Multiplying linear equations
Dividing linear equations

Horizons Algebra 1 Appearance of Concepts, continued:

Lesson 62
The FOIL method
Multiplying binomials
Absolute value
Extraneous solutions
Roots
Word problems

Lesson 63
Multiplying polynomials
Multiplying monomials
Linear equations
Fractions
Word problems

Lesson 64
Special products of binomials
The FOIL method
Multiplying polynomials
Word problems

Lesson 65
Dividing a polynomial by a monomial
Dividing a monomial by a monomial
Exponents

Lesson 66
Dividing a polynomial by a binomial
Order of operations
Exponents
Roots
Inequalities

Lesson 67
Multiplying polynomials
Dividing polynomials
Special products of binomials

Lesson 68
Factoring common monomials
Prime factorization
Dividing a polynomial by a monomial
Word problems

Lesson 69
Factoring the difference of two squares
Systems of equations
Word problems

Lesson 70
Factoring perfect square trinomials
Graphing linear equations
Graphing linear inequalities
Perpendicular lines
Parallel lines

Lesson 71
Factoring trinomials
Factoring common monomials
Factoring the difference of two squares
Factoring perfect square trinomials
Dividing radicals

Lesson 72
Factoring trinomials
Word problems

Lesson 73
Factoring trinomials
Simplifying roots
Word problems

Lesson 74
Factoring the difference of two squares
Factoring perfect square trinomials
Identifying perfect square trinomials

Lesson 75
Factoring completely
Adding fractions with roots
Subtracting fractions with roots
Multiplying fractions with roots
Dividing fractions with roots
Word problems

Lesson 76
Factoring cubic polynomials
Systems of equations
Absolute value
Extraneous solutions
Word problems

Horizons Algebra 1 Appearance of Concepts, continued:

Lesson 77
Factoring by grouping
Factoring completely
Factoring the difference of two squares
Factoring perfect square trinomials

Lesson 78
Rational expressions
Exclusions
Fractions
Word problems

Lesson 79
Adding rational expressions
Subtracting rational expressions
Exclusions
Inequalities
Fractions

Lesson 80
Multiplying rational expressions
Exclusions
Factoring trinomials
Factoring completely
Factoring the difference of two squares
Factoring perfect square trinomials
Factoring by grouping

Lesson 81
Dividing rational expressions
Exclusions
Systems of equations

Lesson 82
Adding rational expressions
Subtracting rational expressions
Inequalities
Absolute value
Word problems

Lesson 83
Adding rational expressions
Common denominators of rational expressions
Exclusions
Factoring polynomials

Lesson 84
Subtracting rational expressions
Lowest common denominator
Exclusions
Word problems

Lesson 85
Multiplying rational expressions
Exclusions
Adding rational expressions
Subtracting rational expressions
Lowest common denominator

Lesson 86
Dividing rational expressions
Exclusions
Word problems

Lesson 87
Complex fractions
Systems of equations
Graphing

Lesson 88
Complex rational expressions
Equations with radicals
Equations with absolute value
Word problems

Lesson 89
Complex rational expressions
Lowest common denominator

Lesson 90
Quadratic equations
Dividing rational expressions
Multiplying rational expressions
Adding rational expressions
Subtracting rational expressions

Lesson 91
Quadratic equations
Solving quadratic equations by factoring
Complex rational expressions

Horizons Algebra 1 Appearance of Concepts, continued:

Lesson 92
Quadratic equations
Solving quadratic equations by taking roots
Solving quadratic equations by factoring
Word problems

Lesson 93
Quadratic equations
Solving quadratic equations by completing the square
Complex rational expressions

Lesson 94
Quadratic equations
Quadratic formula
Solving quadratic equations by factoring
Solving quadratic equations by taking roots

Lesson 95
Discriminant
Double roots
Word problems

Lesson 96
Quadratic equations
Discriminant
Systems of equations
Absolute value
Dividing polynomials

Lesson 97
Functions
Domain
Range
Graphing functions

Lesson 98
Quadratic functions
Parabolas
Conic sections
Word problems

Lesson 99
Parabolas
Vertex
Sketching parabolas

Lesson 100
Completing the square
Quadratic equations

Lesson 101
Quadratic functions
Parabolas
Completing the square

Lesson 102
Quadratic functions
Parabolas
Zeros of a function
Graphing parabolas
Trends in graphs

Lesson 103
Zeros of a function
Completing the square
Word problems

Lesson 104
Quadratic functions
Zeros of a function
Word problems

Lesson 105
Radicals in quadratic equations
Quadratic formula
Systems of equations

Lesson 106
Parabolas
Directrix
Focus
Axis of symmetry
Dividing polynomials

Lesson 107
Parabolas
Vertex
Focus
Directrix
Axis of symmetry
Graphing
Word problems

Lesson 108
Discriminant
Parabolas
Roots of quadratic equations
Factoring polynomials

Lesson 109
Quadratic functions
Parts of a parabola
Discriminant
Roots of equations

Lesson 110
Graphing quadratic inequalities
Order of operations
Radicals
Word problems

Lesson 111
Money
Systems of equations
Graphing quadratic inequalities
Word problems

Lesson 112
Simple interest
Word problems

Lesson 113
Motion
Quadratic formula
Word problems

Lesson 114
Mixtures
Word problems

Lesson 115
Mixtures
Completing the square
Parabolic form
Word problems

Lesson 116
Ratios
Zeros of functions
Radicals
Word problems

Lesson 117
Consecutive integers
Word problems

Lesson 118
Functions
Relations
Word problems

Lesson 119
Direct variation
Quadratic equations
Parabolas
Dividing polynomials

Lesson 120
Inverse variation
Discriminant
Quadratic functions
Completing the square
Parabolas

Lesson 121
Inequalities on a number line
Word problems

Lesson 122
Compound inequalities
Inequalities on a number line

Lesson 123
Compound inequalities
Inequalities on a number line
Functions

Lesson 124
Conjunctions
Compound inequalities
Inequalities on a number line
Word problems

Lesson 125
Disjunctions
Conjunctions
Compound inequalities
Inequalities on a number line

Lesson 126
Inequalities
Absolute value
Inequalities on a number line

Lesson 127
Compound inequalities
Inequalities on a number line
Conjunctions
Disjunctions

Lesson 128
Systems of linear inequalities
Bounded solutions
Unbounded solutions
Inequalities

Horizons Algebra 1 Appearance of Concepts, continued:

Lesson 129
Systems of linear inequalities
Word problems

Lesson 130
Systems of linear inequalities
Direct variation
Inverse variation

Lesson 131
Exponential growth
Compound interest
Word problems

Lesson 132
Exponential decay
Quadratic equations
Factoring
Word problems

Lesson 133
Graphs of exponential functions
Adding polynomials
Subtracting polynomials
Word problems

Lesson 134
Ratios
Proportions
Word problems

Lesson 135
Literal equations
Pythagorean Theorem
Quadratic equations
Completing the square
Word problems

Lesson 136
Work problems
Fractions
Quadratic formula
Solving quadratic equations
Word problems

Lesson 137
Investment problems
Literal equations
Simple interest
Subtracting polynomials
Word problems

Lesson 138
Motion problems
Distance formula
Literal equations
Adding polynomials
Multiplying polynomials by monomials
Word problems

Lesson 139
Square roots without a calculator
Radicals

Lesson 140
Functions
Square roots
Domain
Range
Graphing functions

Lesson 141
Pythagorean Theorem
Hypotenuse
Square roots
Word problems

Lesson 142
Pythagorean Theorem
Literal equations
Systems of equations
Parabolas

Lesson 143
Length of a segment
Pythagorean Theorem
Word problems

Lesson 144
Distance formula
Length of a segment
Adding polynomials

Lesson 145
Middle of a segment
Subtracting polynomials
Multiplying a polynomial by a monomial
Dividing a polynomial by a monomial
Word problems

Horizons Algebra 1 Appearance of Concepts, continued:

Lesson 146
Midpoint formula
Systems of equations
Slope
y-intercept
Graphing linear equations

Lesson 147
Literal equations
Pythagorean Theorem
Distance formula
Midpoint formula
Word problems

Lesson 148
Absolute value
Extraneous solutions
Radicals
Adding polynomials
Subtracting polynomials
Multiplying monomials
Dividing monomials
Multiplying polynomials

Lesson 149
Linear equations
Slope-intercept form
Point-slope form
Intercepts
Parallel lines
Perpendicular lines

Lesson 150
Linear inequalities
Absolute value
Graphing inequalities

Lesson 151
Systems of equations
Multiplying a polynomial by a monomial
Multiplying binomials
Multiplying polynomials
Dividing polynomials

Lesson 152
Factoring polynomials
Rational expressions

Lesson 153
Rational expressions
Complex fractions
Quadratic equations

Lesson 154
Parabolas
Vertex
Focus
Axis of symmetry
Directrix
Graphing parabolas

Lesson 155
Graphing quadratic inequalities
Quadratic equations with radicals
Roots of quadratic equations
Square roots without a calculator

Lesson 156
Investment problems
Motion problems
Mixture problems
Ratios and proportions
Consecutive integers
Word problems

Lesson 157
Exponential growth
Exponential decay
Ratios and proportions
Investment problems
Work problems
Distance problems
Word problems

Lesson 158
Slope
y-intercept
Graphing linear equations
Systems of equations
Direct variation
Inverse variation

Lesson 159
Inequalities on a number line
Conjunctions
Disjunctions
Inequalities with absolute value
Systems of linear inequalities

Lesson 160
Functions with square roots
Graphs of exponential functions
Dividing polynomials by binomials
Pythagorean Theorem
Distance formula
Midpoint formula

Lesson 1

Concepts
- Number terminology
- Signed number rules
- Four operations with signed numbers
- Math in the real world

Learning Objectives
The student will be able to:
- Define terms related to numbers
- Identify numbers as *natural*, *whole*, *integer*, *rational*, *irrational*, and *real*
- Apply the rules of signed numbers
- Add and subtract numbers with different signs
- Multiply and divide numbers with different signs

Materials Needed
- Student Book, Lesson 1
- Exploring Math through Football

Teaching Tips
- Administer the Readiness Test. This test is not to be graded as part of the course grade, but rather as an aid in determining individual student readiness for Algebra 1. Worksheets may be assigned as necessary to assist students who need further help.

- Emphasize that math is necessary for life, not just for those who pursue a career in a math-related field. Introduce the Exploring Math pages. These features will appear throughout the book at the beginning of every 10-lesson segment. Each word problem in the 10 lessons following an Exploring Math page will relate to the featured hobby or sport. Introduce Exploring Math through... Football.

Introduction to…

Exploring Math through…

Often students ask:

Who uses this stuff anyway?

I will NEVER be a math major. Why do I have to learn all this?

Will I ever have to use algebra in the real world?

Math is a school subject that is used daily by people in their work, homes, and play. Many people use math in their jobs, even if those jobs do not require a college degree in mathematics. There is a good chance you will use math on an algebra level when you get a job. Math is also an integral part of recreation. Almost every sport or hobby uses math in some way.

While you may find some of the topics in algebra challenging, they will help you learn more about math and God's carefully designed world. You do not know what plans God has for your life. You may be surprised in the directions God leads you and find that you use math in ways you never expected.

Throughout this book, you will read about several sports and hobbies that require the use of math. Whether or not God's plan for your life includes college, math will play a role in your future.

"For I know the plans I have for you," declares the LORD, "plans to prosper you and not to harm you, plans to give you hope and a future."
Jeremiah 29:11 NIV

Exploring Math through…
Football

Football statistics require a variety of math skills. Signed numbers are used in calculating yardage. Percents calculate player efficiency. This includes finding the percent of passes a quarterback completes, the percent of passes a receiver catches, and the percent of passes a quarterback throws to a particular section of the field. General math calculations are used in determining a player's running speed, keeping score, and deciding if a team should attempt two extra points rather than the standard one extra point after a touchdown.

Order of operations is vital in some football calculations. For example, in each football game, quarterbacks receive a grade known as the Passer Rating. This grade is based on the number of yards gained, touchdowns, interceptions, completions, and pass attempts. The Passer Rating of a college football quarterback is calculated using the formula NCAA QB Passer Rating = $[(8.4y) + (330t) - (200i) + (100c)] \div a$, where y is the number of passing yards, t is the number of touchdowns thrown, i is the number of interceptions thrown, c is the number of completed passes, and a is the number of pass attempts.

Geometry is also a part of football plays. Receivers may run routes that require them to turn a 45-degree angle. The defense must be able to calculate angles while they are running to intercept the receiver, or they will miss a tackle opportunity.

Lesson 1

Kinds of Numbers

Natural numbers are counting numbers. (1, 2, 3, . . .)

Whole numbers are the natural numbers and zero. (0, 1, 2, . . .)

Integers are the positive and negative whole numbers. (. . . -1, 0, 1, . . .)

Rational numbers are numbers that can be written as a fraction. $\left(\frac{1}{2}, \frac{4}{3}, \frac{7}{1}, 10.5\right)$

Irrational numbers are numbers that CANNOT be written as a fraction. $\left(\sqrt{2}, \pi\right)$

Real numbers are numbers in any of the above categories.

Signed Number Rules:
When adding two numbers with the same sign, add the numbers like normal, and keep the same sign in the answer.
(+2) + (+5) = (+7) and (-2) + (-5) = (-7)

When adding two numbers with opposite signs, ignore the signs (use the absolute values) and subtract the smaller number from the larger number. Keep the sign of the larger number as the sign in the answer.
(+5) + (-2) = (5 – 2) = 3. 5 is larger than 2 and 5 is positive in the problem, so the answer is positive.
(+5) + (-2) = (+3).

(-5) + (+2) = -(5 – 2) = -3. 5 is larger than 2 and 5 is negative in the problem, so the answer is negative.
(-5) + (+2) = (-3)

When subtracting signed numbers, change the sign of the second number and add.
$(+5)-(-2)=(+5)+(+2)=5+2=7$

When multiplying two numbers with the same sign, the answer is ALWAYS positive.
$(+5)\times(+4)=20 \quad (-5)\times(-4)=20$

When multiplying two numbers with different signs, the answer is ALWAYS negative.
$(+5)\times(-4)=-20 \quad (-5)\times(+4)=-20$

When multiplying more than two numbers, count the number of negatives. If there is an even number of negative terms, the answer is positive. If there is an odd number of negative terms, the answer is negative.

When dividing signed numbers, follow the rules for multiplying signed numbers.

① **Classwork**

Identify each number as *natural*, *whole*, *integer*, *rational*, *irrational*, or *real*. Some numbers may have more than one answer.

	7	-4	$\sqrt{2}$	0	$1\frac{1}{4}$	$\frac{1}{6}$	π	5.3
Natural	x							
Whole	x			x				
Integer	x	x		x				
Rational	x	x		x	x	x		x
Irrational			x				x	
Real	x	x	x	x	x	x	x	x

Solve, using the rules for signed numbers.

(+42) + (+61) = 42 + 61 = 103

(+42) + (-61) = -(61 – 42) = -19

(+42) – (-61) = 42 + 61 = 103

(-42) – (-61) = (-42) + (+61) = 61 – 42 = 19

(-3)(-4) = 12

(-3)(4) = -12

(-3)(4)(2) = (-12)(2) = -24

(-3)(-4)(2) = (12)(2) = 24

$(+12)\div(-3)=-4$

$(-12)\div(-3)=4$

Activities

② Identify each number as *natural*, *whole*, *integer*, *rational*, *irrational*, or *real*. Some numbers may have more than one answer.

	11	$-\sqrt{3}$	$6\frac{3}{4}$	-7	0	21.62	π	$\frac{5}{6}$	-0.09
Natural	x								
Whole	x				x				
Integer	x			x	x				
Rational	x		x	x	x	x		x	x
Irrational		x					x		
Real	x	x	x	x	x	x	x	x	x

③ Solve, using the rules for signed numbers. Write the problem vertically, if necessary.

(-6) + (+19) = 19 – 6 = 13

(-6) + (-19) = -(6 + 19) = -25

(+6) – (-19) = 6 + 19 = 25

(-6) – (-19) = 19 – 6 = 13

(+23) + (-74) = -(74 – 23) = -51

(-23) + (-74) = -(23 + 74) = -97

(-23) – (-74) = (-23) + (+74) = 74 – 23 = 51

(-23) – (+74) = -(23 + 74) = -97

(-5)(8) = -40

(-5)(-8) = 40

(-36) ÷ (-9) = 4

(36) ÷ (-9) = -4

(10)(-8) = -80

(-8)(-5)(2) = 80

(-81) ÷ (-9) = 9

(81) ÷ (-9) = -9

④ Solve.
In one drive of a football game, the quarterback passed the ball for a 38-yard gain, was sacked for a 7-yard loss, and rushed for a 3-yard gain. How many total yards did the offense move the ball on the drive?

(+38) + (-7) + (+3) = 38 – 7 + 3 = 31 + 3 = 34 yards

If the offense started on the 50-yard line, how many yards away from the goal line are they at the end of the drive?

50 – 34 = 16 yards

Teaching Tips, Cont.

- Define the terms in the teaching box of Lesson 1. Ask students to give other examples of each type of number. They may find it difficult to think of other examples of irrational numbers. Some students may give the square root of other numbers. This is correct UNLESS the student gives the square root of a perfect square.
- Teach the rules for signed numbers from the teaching box. Explain that there are really only two sets of rules to memorize — one set that applies to addition and subtraction, and one set that applies to multiplication and division.
- Complete the Classwork exercises. Have some students work the problems on the board for the class and explain their answers. If you are using the books as consumables, have students mark the correct answers in their books. Otherwise, have the students complete all work on notebook paper. Explain that the value of π is a decimal that never ends and never repeats. In math, it is acceptable to use the value 3.14 or $\frac{22}{7}$ for π when an exact answer is not required.
- The first 100 digits of pi:
3.14159265358979323846264338327950288419716939937510582097494459230781640628620899862803482534211706 7….
(Neither you nor the students are expected to know or memorize this. Often, students will ask, just to see if you know!)

Assignment

- Complete Lesson 1, Activities 2-4.

Lesson 2

Concepts
- Exponents
- Adding and subtracting signed numbers
- Multiplying and dividing signed numbers

Learning Objectives
The student will be able to:
- Define *exponent* and *base*
- Use exponents to express products
- Write exponential notations in expanded form
- Solve exponential expressions

Materials Needed
- Student Book, Lesson 2
- Worksheet 1
- Calculator

Teaching Tips
- Many older calculators will calculate exponential numbers when you repeatedly press the [=] key. Try this on your calculator before class to make sure it works! Have a student press [2] [x] [2] [=] [=] [=] . . . and read the numbers as they appear. Write the numbers on the chalkboard so the class can see them as they are called out. The students should get 4, 8, 16, 32, etc. These numbers will be used later in the lesson. Note: This will not work on the new scientific calculators or those with multiple display lines.

Exponents and Powers

Exponents tell how many times a number is multiplied by itself. The number being multiplied is called the **base**. The exponent is written as a small number on the upper right side of the base. In the expression 4^3, the number 4 is the base and the number 3 is the exponent. $4^3 = 4\times4\times4 = 64$
The answer to an exponential expression is always a multiple of the base.

Rules for working with exponents
Any number (except zero) raised to the 0th power equals 1. $3^0 = 1$

Any number raised to the 1st power equals itself. $3^1 = 3$

When multiplying terms with equal bases, add the exponents. $3^2\left(3^3\right) = 3^5$

When dividing terms with equal bases, subtract the exponents. $3^3 \div 3^2 = 3^1$

When the product of two or more factors has an exponent, raise each individual factor to that exponent. $(2\times3)^4 = 2^4\times3^4$ Note that this is the same as 6^4.

When a number has a negative exponent, take the reciprocal of the number (the numerator and denominator switch places) and make the exponent positive. $3^{-2} = \frac{1}{3^2}$ and

$\left(\frac{2}{3}\right)^{-3} = \left(\frac{3}{2}\right)^3 = \frac{3^3}{2^3}$

① **Classwork**
Read and solve the following exponential expressions.

$2^2 =$ 2 squared $= 2\times2 = 4$
$3^2 =$ 3 squared $= 3\times3 = 9$
$2^3 =$ 2 cubed $= 2\times2\times2 = 8$
$3^3 =$ 3 cubed $= 3\times3\times3 = 27$
$4^2 =$ 4 squared $= 4\times4 = 16$
$4^3 =$ 4 cubed $= 4\times4\times4 = 64$

Simplify the expressions. You do not have to solve exponents greater than 3.

$13^0 = 1$

$22^1 = 22$

$6^4\times6^3 = 6^7$

$5^6\div5^4 = 5^2 = 25$

$(4\times5)^2 = 4^2\times5^2 = 16\times25 = 400$

$7^{-2} = \left(\frac{1}{7}\right)^2 = \frac{1^2}{7^2} = \frac{1}{49}$

$\left(\frac{2}{5}\right)^2 = \frac{2^2}{5^2} = \frac{4}{25}$

$\left(\frac{3}{2}\right)^{-3} = \left(\frac{2}{3}\right)^3 = \frac{2^3}{3^3} = \frac{8}{27}$

Activities
② Simplify the expressions. You do not have to solve exponents greater than 3.

$11^0 = 1$

$17^1 = 17$

$8^2\times8^4 = 8^6$

$6^5\div6^3 = 6^2 = 36$

$(3\times4)^2 = 3^2\times4^2 = 9\times16 = 144$

$3^{-3} = \left(\frac{1}{3}\right)^3 = \frac{1^3}{3^3} = \frac{1}{27}$

$\left(\frac{1}{4}\right)^2 = \frac{1^2}{4^2} = \frac{1}{16}$

$\left(\frac{2}{5}\right)^{-2} = \left(\frac{5}{2}\right)^2 = \frac{5^2}{2^2} = \frac{25}{4}$

$27^0 = 1$

$38^1 = 38$

$9^3\times9^2 = 9^5$

$10^5\div10^2 = 10^3 = 1{,}000$

$(2\times4)^3 = 2^3\times4^3 = 8\times64 = 512$

$11^{-2} = \left(\frac{1}{11}\right)^2 = \frac{1^2}{11^2} = \frac{1}{121}$

$\left(\frac{1}{4}\right)^3 = \frac{1^3}{4^3} = \frac{1}{64}$

$\left(\frac{5}{3}\right)^{-3} = \left(\frac{3}{5}\right)^3 = \frac{3^3}{5^3} = \frac{27}{125}$

③ Solve, following the rules of signed numbers.

(+242) + (+397) = 242 + 397 = 639

(-242) + (-397) = -(242 + 397) = -639

(-242) – (+397) = -(242 + 397) = -639

(+242) – (-397) = 242 + 397 = 639

(-29) – (-15) =
(-29) + (+15) = -(29 – 15) = -14

(+29) – (+15) =
(+29) +(-15) = 29 – 15 = 14

(-29) – (+15) = (-29) + (-15) = -44

14 + (-13) + 17 – (-12) =
14 – 13 + 17 + 12 = 30

$(+8)\times(+5) = 40$

$(+8)\times(-5) = -40$

$(-56)\div(+8) = -7$

$(-56)\div(-8) = 7$

$(-4)(5)(-2) = (-20)(-2) = 40$

$(-4)(-5)(-2) = (20)(-2) = -40$

$(72)\div(-9) = -8$

$(72)\div(9) = 8$

Identify Numbers, Signed Numbers, Exponential Expressions Worksheet 1

① Identify each number as *natural*, *whole*, *integer*, *rational*, *irrational*, or *real*. Some numbers may have more than one answer.

	$4\sqrt{11}$	-3	π	0	$1\frac{2}{3}$	$\frac{13}{7}$	65	$-\frac{1}{8}$	41.3
Natural							x		
Whole				x			x		
Integer		x		x			x		
Rational		x		x	x	x	x	x	x
Irrational	x		x						
Real	x	x	x	x	x	x	x	x	x

② Solve, using the rules for signed numbers.

(+48) + (+35) = 48 + 35 = 83

(-48) + (+35) = -(48 – 35) = -13

(-48) + (-35) = -(48 + 35) = -83

(+48) + (-35) = 48 – 35 = 13

(+48) – (-35) = 48 + 35 = 83

(-48) – (-35) = (-48) + (+35) = -(48 – 35) = -13

(11)(12) = 132

(11)(-12) = -132

(-132) ÷ (11) = -12

(-132)÷(-12) = 11

③ Write the following exponential expressions in expanded form and solve.

$3^4 = 3 \times 3 \times 3 \times 3 = 81$

$4^3 = 4 \times 4 \times 4 = 64$

$6^3 = 6 \times 6 \times 6 = 216$

$10^4 = 10 \times 10 \times 10 \times 10 = 10{,}000$

$11^2 = 11 \times 11 = 121$

Note regarding negative exponents in quotients:

Consider the problem $2^2 \div 2^4$. According to the rules of dividing exponents, this equals $2^{2-4} = 2^{-2}$. Written as a fraction, you have

$$\frac{1 \cdot \cancel{2} \cdot \cancel{2}}{1 \cdot \cancel{2} \cdot \cancel{2} \cdot 2 \cdot 2} = \frac{1}{2 \cdot 2} = \frac{1}{2^2} = \frac{1}{4}$$

Teaching Tips, Cont.

- Define *exponent* and *base* from the teaching box. Tell students that the base is the number on the bottom. (This concept will carry over in later years when they are learning logarithms with different bases.) It will also help to remember that the exponent is elevated.

- Demonstrate the proper form for writing numbers with exponents, using the numbers from the calculator as an example.

- Teach the rules for working with exponents from the teaching box.

- Emphasize that any number raised to the zero power is equal to 1. If students are still questioning the validity of this fact, show students that $2^1 \div 2^1$ can be solved by following the rules of exponents: $2^{1-1} = 2^0 = 1$. This problem is obviously equal to 1 because anything divided by itself equals 1. Following the rules of dividing exponents, the resulting term has a zero exponent.

- Complete the Classwork exercises. Have some students work the problems on the board for the class and explain their answers. All students should work the problems in their books. Worksheets that appear in the assignments section may be used at the teacher's discretion. These are designed for additional review of recent topics for students who need more practice prior to being quizzed or tested over the material.

Assignments

- Complete Lesson 2, Activities 2-3.
- Worksheet 1 (Optional).

Lesson 3

Concepts

- Order of operations
- Adding and subtracting signed numbers
- Multiplying and dividing signed numbers
- Math in the real world

Learning Objectives

The student will be able to:

- Memorize the correct sequence for the order of operations
- Apply the order of operations to mathematical expressions
- Calculate correctly the answer to mathematical expressions with multiple terms

Materials Needed

- Student Book, Lesson 3
- Worksheet 2

Teaching Tips

- Ask students what would happen in a football game if there were no rules. How would you know how many points to give a team for a field goal, touchdown, extra point(s), safety, etc? Elicit the idea that rules are necessary for the game to be played properly. Tie this in with the fact that God is a God of order, and the Bible teaches that all things should be done decently and in order. (1 Cor. 14:40)

Order of Operations

There is a specific order you must follow in working more complex math problems to get the correct answer. This is known as the **Order of Operations**. When simplifying mathematical expressions, first look for any **parentheses** and simplify inside each set of parentheses. Second, apply any **exponents** in the problem. Next, do all **multiplication** and **division** together in the order they appear in the expression from left to right. Finally, do all **addition** and **subtraction** together in the order they appear in the expression from left to right. You can remember the proper order of operations by remembering this sentence:
Please **E**xcuse **M**y **D**ear **A**unt **S**ally.
(**P**arentheses, **E**xponents, **M**ultiplication, **D**ivision, **A**ddition, and **S**ubtraction)

To solve the problem $6+2(1+3)^2$, first simplify the parentheses to get $6+2(4)^2$. Next, take care of the exponent: $6+2(16)$ and then do all multiplication. (There is no division in this expression, or that would be done in this step, as well.) You should have $6+32$, which gives you $6+32=38$.

① **Classwork**
Simplify the expressions, following the proper order of operations.

$7-4+3(5-3)^3=$
$7-4+3(2)^3=$
$7-4+3(8)=$
$7-4+24=$
$3+24=27$

$(12-9)^2+20\div4=$
$(3)^2+20\div4=$
$9+20\div4=$
$9+5=14$

$3+2^3-2(24\div6)=$
$3+2^3-2(4)=$
$3+8-2(4)=$
$3+8-8=$
$11-8=3$

Activities
② Simplify each expression, following the proper order of operations.

$21-4\times3=21-12=9$

$18\div9\times3+2-1\times3=2\times3+2-1\times3=6+2-1\times3=6+2-3=8-3=5$

$(8-5)^2-10\div2=(3)^2-10\div2=9-10\div2=9-5=4$

$(3+4)-2^2+3\times4=7-2^2+3\times4=7-4+3\times4=7-4+12=3+12=15$

$(15-12)^3-5^2+2\times7=(3)^3-5^2+2\times7=27-25+2\times7=27-25+14=2+14=16$

$2^3\times3\div(7-1)-4=2^3\times3\div6-4=8\times3\div6-4=24\div6-4=4-4=0$

$(4^2-(13-8)+1)\div6=(4^2-5+1)\div6=(16-5+1)\div6=(11+1)\div6=12\div6=2$

$((6+2\times3)\div3)^2=((6+6)\div3)^2=(12\div3)^2=4^2=16$

③ Solve, following the rules of signed numbers.

(+57) + (+73) = 57 + 73 = 130

(+57) + (-73) = -(73 – 57) = -16

(-57) + (+73) = 73 – 57 = 16

(-57) + (-73) = -(57 + 73) = -130

(+242) – (+397) = (+242) + (-397) = -(397 – 242) = -155

(+242) + (-397) = -(397 – 242) = -155

(-242) + (+397) = 397 – 242 = 155

(-242) – (-397) = (-242) + (+397) = 397 – 242 = 155

(-3)(7)(2) = (-21)(2) = -42

(8)(-7)(1) = (-56)(1) = -56

(-9)(-7)(-1) = (63)(-1) = -63

(-7)(8)(2) = (-56)(2) = -112

(-4)(-9)(3) = (36)(3) = 108

(12)(5)(-2) = (60)(-2) = -120

(-11)(2)(-4) = (-22)(-4) = 88

(-9)(-4)(-3) = (36)(-3) = -108

④ Solve.

The Passer Rating of a college football quarterback is calculated using the formula NCAA QB Passer Rating = [(8.4*y*) + (330*t*) – (200*i*) + (100*c*)] ÷ *a*, where *y* is the number of passing yards, *t* is the number of touchdowns thrown, *i* is the number of interceptions thrown, *c* is the number of completed passes, and *a* is the number of pass attempts.

Calculate the passer rating of a quarterback that had 220 passing yards, 1 touchdown thrown, no interceptions, 13 completed passes, and 17 pass attempts in his last game. Round answers to the nearest hundredth.

Passer Rating = [(8.4)(220) + (330)(1) – (200)(0) + (100)(13)] ÷ 17
= (1848 + 330 – 0 + 1300) ÷ 17 = 3478 ÷ 17 = 204.59

Order of Operations, Simplifying Exponents Worksheet 2

① Simplify each expression, following the proper order of operations.

$8+28\div 4=$
$8+7=15$
$27-3\times 8=$
$27-24=3$
$16-4\times 3+7=$
$16-12+7=4+7=11$
$6+9\times 7\div 3=$
$6+63\div 3=6+21=27$
$30\div 6\times 6+7-3\times 2=$
$5\times 6+7-3\times 2=30+7-3\times 2=30+7-6=37-6=31$
$24\div(3\times 4)+5-4\times 1=$
$24\div 12+5-4\times 1=2+5-4\times 1=2+5-4=7-4=3$
$(11-4)\times 4-11=$
$7\times 4-11=28-11=17$
$(12-7)^2-42\div 7=$
$(5)^2-42\div 7=25-42\div 7=25-6=19$
$(21-4)-3^2+2\times 4=$
$17-3^2+2\times 4=17-9+2\times 4=17-9+8=8+8=16$
$(14-11)^3-4^2-2\times 3=$
$(3)^3-4^2-2\times 3=27-16-2\times 3=27-16-6=11-6=5$
$2^3\times 4\div(9+7)-3^2=$
$2^3\times 4\div 16-9=8\times 4\div 16-9=32\div 16-9=2-9=-7$
$(6^2-(12+13)+7)\div 6=$
$(6^2-25+7)\div 6=(36-25+7)\div 6=(11+7)\div 6=18\div 6=3$
$((8+4\times 3)\div 5)^2=$
$((8+12)\div 5)^2=(20\div 5)^2=4^2=16$

② Simplify the expressions. You do not have to solve exponents greater than 3.

$37^0=1$
$92^1=92$
$14^2\times 14^6=14^8$
$12^9\div 12^7=12^2=144$
$(63\div 9)^2=7^2=49$

$4^{-3}=\left(\frac{1}{4}\right)^3=\frac{1^3}{4^3}=\frac{1}{64}$

$\left(\frac{1}{11}\right)^2=\frac{1^2}{11^2}=\frac{1}{121}$

$\left(\frac{3}{5}\right)^{-2}=\left(\frac{5}{3}\right)^2=\frac{5^2}{3^2}=\frac{25}{9}$

Teaching Tips, Cont.

- Write the following problem on the board: 4 + 10 ÷ 2 =. Ask several students for the answer to the problem. (Students will most likely give 7 as the answer, but the real answer is 9.) For both answers, ask the student supplying the answer to tell how he/she arrived at the answer.
- Explain that without rules in math, we would have the same situation as a football game without rules. There would be no way to tell who was right and who was wrong when two different answers were given.
- Introduce the order of operations in the teaching box. Point out the mnemonic device for remembering the order of operations.
- Complete the Classwork exercises. Have some students work the problems on the board for the class and explain their answers. All students should work the problems in their books.

Assignments

- Complete Lesson 3, Activities 2-4.
- Worksheet 2 (Optional).

Lesson 4

Concepts

- Prime numbers
- Factoring
- Exponents

Learning Objectives

The student will be able to:

- Define *factor*, *prime*, and *composite*
- Find all natural number factors of a given number
- Express the prime factorization of a given number using exponents when appropriate

Materials Needed

- Student Book, Lesson 4
- Algebra tiles (cut from the *Tests and Resources* book)
- Zip-top sandwich bags – 1 per student

Teaching Tips

- Define *factor* from the teaching box. Ask a student to define *natural number*. (Refer to Lesson 1, if necessary.) You may wish to do the following activity repeated from the Horizons Pre-Algebra book.
- Have students take out 12 of the single unit squares from the algebra tiles. Ask them to arrange the squares to form a rectangle. The dimensions of the rectangle are factors. A 3 x 4 rectangle shows that 3 and 4 are factors of 12.
- This activity also works to arrange the squares in equal-sized groups. They should try groups of 1, 2, 3, etc. all the way up to 12. Which group sizes work? Which ones don't? The group sizes that work are the factors of 12.

Lesson 4

Factoring and Prime Numbers

A **factor** is a natural number that divides into another number with no remainder.
4 is a factor of 12 because $12 \div 4 = 3$.
From this example, we can see that 3 is also a factor of 12.
All the factors of 12 are 1, 2, 3, 4, 6, and 12.

Prime numbers are natural numbers whose only factors are 1 and itself. 3 is a prime number because its only factors are 1 and 3.

Composite numbers are all numbers greater than 1 that are not prime.

The numbers 0 and 1 are neither prime nor composite, and 2 is the only even prime number.

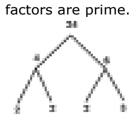
Prime factors of a number are the prime numbers that divide into the number with no remainder.
Prime factorization is the process of finding all the prime numbers that multiply together to get the original number.

There are two ways to find the prime factorization of a number. One is to continually divide by prime numbers until you get a quotient that is prime.

$24 \div 2 = 12$
$12 \div 2 = 6$
$6 \div 2 = 3$

The prime factors of 24 are 2, 2, 2, and 3.

The second way is to make a factor tree. Write the original number as the product of any two factors you think of. Continue factoring these factors until all factors are prime.



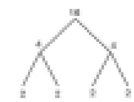
$24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$

① **Classwork**
Find all the factors of each number. Use this information to identify each as either *prime* or *composite*.

7 1, 7 Prime
8 1, 2, 4, 8 Composite
9 1, 3, 9 Composite
10 1, 2, 5, 10 Composite
11 1, 11 Prime

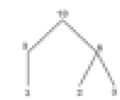
Find the prime factorization of each number.

16



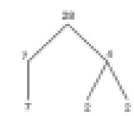
$16 = 2^4$

18



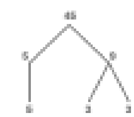
$18 = 2 \times 3^2$

28

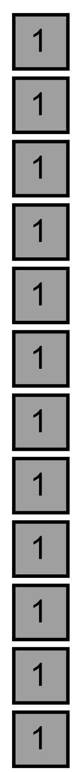


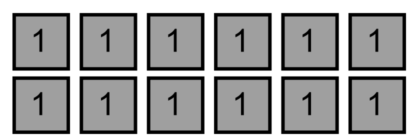
$28 = 2^2 \times 7$

45

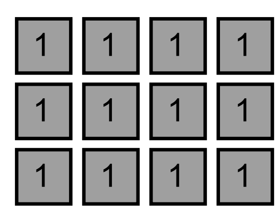


$45 = 3^2 \times 5$





2 and 6 are factors



3 and 4 are factors

1 and 12 are factors

Activities

② Find the prime numbers in the list below by following the directions.

1. Cross out the number 1.
2. Circle the number 2. Cross out every other number after two (the multiples of 2).
3. Circle the number 3. Cross out every third number after three (the multiples of 3).
4. Circle the number 5. Cross out every fifth number after five (the multiples of 5).
5. Circle the number 7. Cross out every seventh number after seven (the multiples of 7).
6. Circle all remaining numbers. The circled numbers are the prime numbers less than 100.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Write the prime numbers less than 100.

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,

43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

③ Find the prime factorization of each number. Use exponents where appropriate.

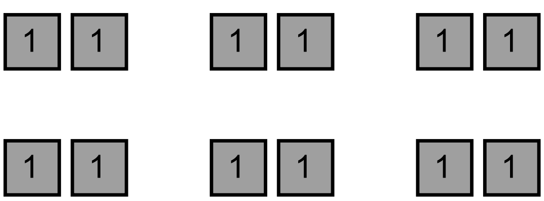
12 12 = 2 x 2 x 3 12 = 2^2 x 3	14 14 = 2 x 7	15 15 = 3 x 5	20 20 = 2 x 2 x 5 20 = 2^2 x 5
21 21 = 3 x 7	22 22 = 2 x 11	24 24 = 2 x 2 x 2 x 3 24 = 2^3 x 3	25 25 = 5 x 5 25 = 5^2

Teaching Tips, Cont.

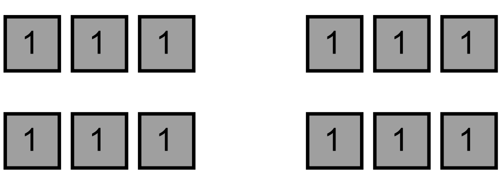
- Define *prime factor* and *prime factorization* from the teaching box.
- Demonstrate the procedure for factorization by division. (See example below.) Emphasize that prime numbers must be used as the divisors when doing repeated division, but any factor may be used in a factor tree.
- Complete the Classwork exercises. Have some students work the problems on the board for the class and explain their answers. All students should work the problems in their books.

Assignment

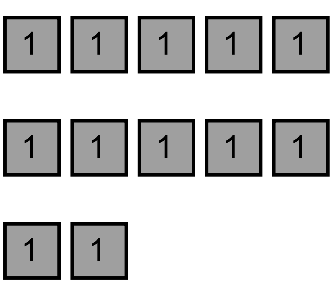
- Complete Lesson 4, Activities 2-3.



2 and 6 are factors



3 and 4 are factors



5 is not a factor

Note: Factorization by division can be done by dividing upside-down:

Step 1:

2 |24
12

Step 2:

2 |24
2 |12
6

Step 3:

2 |24
2 |12
2 | 6
3

Continue dividing the quotient by prime numbers until the quotient is prime. This method makes it easy to identify all of the prime factors.

Lesson 5

Concepts

- Absolute value
- Adding and subtracting signed numbers
- Multiplying and dividing signed numbers
- Prime factorization
- Order of operations

Learning Objectives

The student will be able to:

- Define *absolute value*
- Find the absolute value of positive and negative numbers
- Find the absolute value of mathematical expressions

Materials Needed

- Student Book, Lesson 5
- Worksheet 3

Teaching Tips

- ➢ Have students complete Worksheet 3 in class. This may be for added practice of earlier topics or graded as a quiz.
- ➢ Define *absolute value* from the teaching box. Emphasize that absolute value is a number's distance from zero on the number line. Distance is always positive.
- ➢ Explain that absolute value gives a number's distance from zero. Inverse operations get a number back to its starting point, no matter what that starting point is. Inverse operations are the foundation of math fact families.

Absolute Value

The **absolute value** of a number is the number's distance from zero on a number line. The absolute value of 5, written as $|5|$, is 5, because the number 5 is 5 units away from zero. The absolute value of –5, written as $|-5|$, is also 5, because –5 is 5 units away from zero.

① **Classwork**
Solve the following absolute value problems.

$|37| = 37$ $\quad -|8| = -8$

$|-19| = 19$ $\quad -|-47| = -47$

Activities

② Solve, using the rules of absolute values.

$|3| = 3$

$|49| = 49$

$|-25| = 25$

$|-79| = 79$

$-|11| = -11$

$-|-82| = -82$

$-|16| = -16$

$-|-43| = -43$

$|19+3| = 22$

$-|22+11| = -33$

$|15+18| = 33$

$|5-4| = 1$

$-|17-20| = -3$

$-|2+17| = -19$

$-|33-35| = -2$

$|16|+|4| = 16 + 4 = 20$

$|9|+|-15| = 9 + 15 = 24$

$|-27|+|-3| = 27 + 3 = 30$

$|-29|+|6| = 29 + 6 = 35$

$|5|-|-24| = 5 - 24 = -19$

$-|9|+|-13| = -9 + 13 = 4$

$-|1|-|-28| = -1 - 28 = -29$

$|26|-|35| = 26 - 35 = -9$

$-|-12|-|-15| = -12 - 15 = -27$

$|19+18|+|23-17| = 37 + 6 = 43$

$-|24-16|+|15-17| = -8 + 2 = -6$

$-|26+14|-|42-18| = -40 - 24 = -64$

③ Solve, using the rules for adding signed numbers. Write the problem vertically, if necessary.

(+7) + (+15) = 7 + 15 = 22

(-13) + (+4) = -(13 – 4) = -9

(-18) – (-6) =
(-18) + (+6) = -(18 – 6) = -12

(+17) + (-65) = 17 – 65 = -48

(+29) + (+19) = 29 + 19 = 48

(+34) – (-16) =
(+34) + (+16) = 34 + 16 = 50

(-27) + (+8) = -(27 – 8) = = -19

(-43) + (-12) = -(43 + 12) = -55

(+41) – (+14) = 41 – 14 = 27

(+19) + (-6) = 19 – 6 = 13

(-16) + (+27) = 27 – 16 = 11

(-7) – (-28) =
(-7) + (+28) = 28 – 7 = 21

④ Find the prime factorization of each number. Use exponents where appropriate.

27
$27 = 3 \times 3 \times 3$
$27 = 3^3$

28
$28 = 2 \times 2 \times 7$
$28 = 2^2 \times 7$

30
$30 = 2 \times 3 \times 5$

32
$32 = 2 \times 2 \times 2 \times 2 \times 2$
$32 = 2^5$

33
$33 = 3 \times 11$

35
$35 = 5 \times 7$

⑤ Solve, following proper order of operations.

$5 + 12 \div 3 = 5 + 4 = 9$

$27 - 3 \times 5 = 27 - 15 = 12$

$13 - 2 \times 4 + 6 = 13 - 8 + 6 = 5 + 6 = 11$

$4 + 3^2 + 5 = 4 + 9 + 5 = 13 + 5 = 18$

$12 \div 6 \times 5 + 3 - 1 \times 7 = 2 \times 5 + 3 - 1 \times 7 = 10 + 3 - 1 \times 7 = 10 + 3 - 7 = 13 - 7 = 6$

$16 \div 2^2 + 5 - 3 \times 2 = 16 \div 4 + 5 - 3 \times 2 = 4 + 5 - 3 \times 2 = 4 + 5 - 6 = 9 - 6 = 3$

$(11-2)4 \div 6 - 5 = 9 \times 4 \div 6 - 5 = 36 \div 6 - 5 = 6 - 5 = 1$

$(7-3)^2 - 20 \div 4 = (4)^2 - 20 \div 4 = 16 - 20 \div 4 = 16 - 5 = 11$

$(4+3) - 2^2 + 6 \times 2 = 7 - 2^2 + 6 \times 2 = 7 - 4 + 6 \times 2 = 7 - 4 + 12 = 3 + 12 = 15$

$(11-8)^3 - 5^2 + 7 \times 2 = (3)^3 - 5^2 + 7 \times 2 = 27 - 25 + 7 \times 2 = 27 - 25 + 14 = 2 + 14 = 16$

$3^3 \div 9 + (5+1) - 4 = 3^3 \div 9 + 6 - 4 = 27 \div 9 + 6 - 4 = 3 + 6 - 4 = 9 - 4 = 5$

$(2 \times 8 - (21-16) + 1) \div 6 = (16 - 5 + 1) \div 6 = (11 + 1) \div 6 = 12 \div 6 = 2$

$((2 + 4 \times 3) \div 7)^2 = ((2 + 12) \div 7)^2 = (14 \div 7)^2 = 2^2 = 4$

Order of Operations, Simplifying Exponents Worksheet 3

① Simplify each expression, following the proper order of operations.

$6+9\times8\div12=$
$6+72\div12=6+6=12$

$33\div3\times4+4-2\times7=$
$11\times4+4-2\times7=44+4-2\times7=44+4-14=48-14=34$

$36\div(3\times6)+9-2\times4=$
$36\div18+9-2\times4=2+9-2\times4=2+9-8=11-8=3$

$(25-20)\times4-19=$
$5\times4-19=20-19=1$

$(12-3)^2-20\times3=$
$(9)^2-20\times3=81-20\times3=81-60=21$

$(13+8)-4^2+3\times2=$
$21-4^2+3\times2=21-16+3\times2=21-16+6=5+6=11$

$(9-7)^3+3^2-4\times2=$
$(2)^3+3^2-4\times2=8+9-4\times2=8+9-8=17-8=9$

$3^3\div9\times(11-7)-6=$
$3^3\div9\times4-6=27\div9\times4-6=3\times4-6=12-6=6$

$(8^2-(15+19)+9)\div13=$
$(8^2-34+9)\div13=(64-34+9)\div13=(30+9)\div13=39\div13=3$

$((4+3\times7)\div5)^2=$
$((4+21)\div5)^2=(25\div5)^2=5^2=25$

$(9+18\div3-5)^2\div2+13=$
$(9+6-5)^2\div2+13=10^2\div2+13=100\div2+13=50+13=63$

$1^3+2^3-3^2\times4^0+9\div3=$
$1+8-9\times1+9\div3=1+8-9+3=9-9+3=0+3=3$

② Simplify the expressions. You do not have to solve exponents greater than 3.

$41^0=1$

$53^1=53$

$23^2\times23^6=23^8$

$9^{13}\div9^{11}=9^2=81$

$(54\div6)^2=9^2=81$

$7^{-4}=\left(\frac{1}{7}\right)^4=\frac{1^4}{7^4}=\frac{1}{7^4}$

$\left(\frac{1}{12}\right)^2=\frac{1^2}{12^2}=\frac{1}{144}$

$\left(\frac{2}{3}\right)^{-3}=\left(\frac{3}{2}\right)^3=\frac{3^3}{2^3}=\frac{27}{8}$

Teaching Tips, Cont.

- To illustrate absolute value and inverse operations, ask the students the following questions: If John jogs 1 mile east, turns around, and jogs 1 mile west, how many miles has John jogged? (2) Changing direction does not affect the sign of the answer. Traveling east is like moving in the positive direction on the number line. Traveling west is like moving in the negative direction on the number line. If John starts at mile marker 8 and bicycles for 6 miles, at what mile marker will he end? (14) How many miles must he bicycle to return to mile marker 8? (6) This is using inverse operations.
  8 + 6 = 14 and 14 – 6 = 8

- When working absolute value problems, always solve inside the absolute value sign first (the answer inside the absolute value sign is always positive), then apply any signs and operations outside the absolute value sign.

- Complete the Classwork exercises. Have some students work the problems on the board for the class. All students should work the problems in their books.

Assignment

- Complete Lesson 5, Activities 2-5.

Lesson 86

Concepts

- Dividing rational expressions
- Exclusions
- Math in the real world

Learning Objectives

The student will be able to:

- Factor polynomials in rational expressions
- Simplify rational expressions
- Divide rational expressions

Materials Needed

- Student Book, Lesson 86

Teaching Tips

- Review dividing rational expressions. (See Lesson 81)
- Review factoring polynomials as needed. (See Lessons 68-77)
- Review simplifying rational expressions as needed. (See Lesson 78)
- Remind students to state all exclusions any time they are working a problem with rational expressions.
- Ask the students what special rule applies to exclusions for division with rational expressions. (The denominators in the original problem as well as the denominator of the reciprocal must be considered when stating the exclusions.)

Review: Dividing Rational Expressions

Recall from Lesson 81 that dividing rational expressions follows the same rules as dividing fractions. Factor each numerator and denominator and cancel like factors before you divide. Remember that entire factors must be cancelled. You cannot cancel individual terms. When writing the answer, remember to state any exclusions found in the original problem as well as the denominator after you have taken the reciprocal of the divisor when stating the exclusions.

① Classwork

Solve. Remember to state any exclusions.

$$\frac{3x-15}{4x^2+12x} \div \frac{x^2-x-20}{4x^2+16x} =$$

$$\frac{3x-15}{4x^2+12x} \cdot \frac{4x^2+16x}{x^2-x-20} =$$

$$\frac{3(\cancel{x-5})}{\cancel{4x}(x+3)} \cdot \frac{\cancel{4x}(\cancel{x+4})}{(\cancel{x+4})(\cancel{x-5})} =$$

$$\frac{3}{x+3}; x \neq -4, -3, 0, 5$$

Activities

② Solve. Remember to state any exclusions.

$$\frac{x^2-4}{2x^2+11x+12} \div \frac{2x^2+x-6}{4x^2-9} =$$

$$\frac{x^2-4}{2x^2+11x+12} \cdot \frac{4x^2-9}{2x^2+x-6} =$$

$$\frac{(x-2)(\cancel{x+2})}{(\cancel{2x+3})(x+4)} \cdot \frac{(\cancel{2x+3})(\cancel{2x-3})}{(\cancel{x+2})(\cancel{2x-3})} =$$

$$\frac{x-2}{x+4}; x \neq -4, -2, -\tfrac{3}{2}, \tfrac{3}{2}$$

$$\frac{3x^2+11x-20}{5x^2-7x-6} \div \frac{4x^2+21x+5}{4x^2-7x-2} =$$

$$\frac{3x^2+11x-20}{5x^2-7x-6} \cdot \frac{4x^2-7x-2}{4x^2+21x+5} =$$

$$\frac{(3x-4)(\cancel{x+5})}{(5x+3)(\cancel{x-2})} \cdot \frac{(\cancel{4x+1})(\cancel{x-2})}{(\cancel{4x+1})(\cancel{x+5})} =$$

$$\frac{3x-4}{5x+3}; x \neq -5, -\tfrac{3}{5}, -\tfrac{1}{4}, 2$$

$$\frac{12x^3+44x^2-16x}{6x^4+6x^3-36x^2} \div \frac{6x^3+22x^2-8x}{9x^3-36x} =$$

$$\frac{12x^3+44x^2-16x}{6x^4+6x^3-36x^2} \cdot \frac{9x^3-36x}{6x^3+22x^2-8x} =$$

$$\frac{\cancel{2}\,\cancel{4x}(\cancel{3x-1})(\cancel{x+4})}{\cancel{3x}\,\cancel{6x^2}(x+3)(\cancel{x-2})} \cdot \frac{3\,\cancel{9x}(x+2)(\cancel{x-2})}{\cancel{2x}(\cancel{3x-1})(\cancel{x+4})} =$$

$$\frac{3(x+2)}{x(x+3)} = \frac{3x+6}{x^2+3x}; x \neq -4, -3, -2, 0, \tfrac{1}{3}, 2$$

$$\frac{2x^2+11x+12}{4x^2+3x-1} \div \frac{4x^2+12x+9}{8x^2+18x-5} =$$

$$\frac{2x^2+11x+12}{4x^2+3x-1} \cdot \frac{8x^2+18x-5}{4x^2+12x+9} =$$

$$\frac{(x+4)(\cancel{2x+3})}{(\cancel{4x-1})(x+1)} \cdot \frac{(2x+5)(\cancel{4x-1})}{(\cancel{2x+3})(2x+3)} =$$

$$\frac{(x+4)}{(x+1)} \cdot \frac{(2x+5)}{(2x+3)} =$$

$$\frac{2x^2+13x+20}{2x^2+5x+3}; x \neq -\tfrac{3}{2}, -1, \tfrac{1}{4}, -\tfrac{5}{2}$$

$$\frac{12x^2+x-6}{12x^2-19x-21} \div \frac{9x^2-12x+4}{9x^2-4} =$$

$$\frac{12x^2+x-6}{12x^2-19x-21} \cdot \frac{9x^2-4}{9x^2-12x+4} =$$

$$\frac{(\cancel{3x-2})(\cancel{4x+3})}{(3x-7)(\cancel{4x+3})} \cdot \frac{(3x+2)(\cancel{3x-2})}{(\cancel{3x-2})(\cancel{3x-2})} =$$

$$\frac{3x+2}{3x-7}; x \neq -\tfrac{3}{4}, \tfrac{2}{3}, \tfrac{7}{3}, -\tfrac{2}{3}$$

$$\frac{8x^2-5x-3}{10x^2-33x-54} \div \frac{x^2-3x+2}{5x^2-4x-12} =$$

$$\frac{8x^2-5x-3}{10x^2-33x-54} \cdot \frac{5x^2-4x-12}{x^2-3x+2} =$$

$$\frac{(8x+3)(\cancel{x-1})}{(2x-9)(\cancel{5x+6})} \cdot \frac{(\cancel{5x+6})(\cancel{x-2})}{(\cancel{x-2})(\cancel{x-1})} =$$

$$\frac{8x+3}{2x-9}; x \neq 1, -\tfrac{6}{5}, 2, \tfrac{9}{2}$$

③ Solve. Round answers to the nearest hundredth. You will want to use your scientific calculator.

The trade value, v, of a player is found using the formula $v = \frac{(e-y)^2(y+1)e}{190} + \frac{ey^2}{13}$, where e is the player's estimated value and y is the player's estimated number of years left to play. The value of y is found using the formula $y = 27 - \frac{3a}{4}$, where a is the player's current age. The value of e is found using the formula

$$e = \frac{(p + 0.85r + 0.35d + 0.79a + 1.2s + 0.85b - 1.2t - 0.85g - 1.45m - 0.41f)^{\frac{3}{4}}}{21}$$

where the variables represent the following information for the season:

p is the number of points scored
r is the number of offensive rebounds
d is the number of defensive rebounds
a is the number of assists
s is the number of steals
b is the number blocks
t is the number of turnovers
g is the number of field goals missed
m is the number of missed free throws
f is the number of personal fouls

How many additional years can a team expect a 32-year-old player to play?

$y = 27 - \frac{3(32)}{4} = 27 - 24 = 3$ additional years

What is the estimated value, e, of a player with the following season stats?

2075 points scored
82 offensive rebounds
336 defensive rebounds
385 assists
98 steals
16 blocks
246 turnovers
1066 field goals missed
82 missed free throws
172 personal fouls

$$e = \frac{(2075 + 0.85(82) + 0.35(336) + 0.79(385) + 1.2(98) + 0.85(16) - 1.2(246) - 0.85(1066) - 1.45(82) - 0.41(172))^{\frac{3}{4}}}{21}$$

$$e = \frac{(2075 + 69.7 + 117.6 + 304.15 + 117.6 + 13.6 - 295.2 - 906.1 - 118.9 - 70.52)^{\frac{3}{4}}}{21} = \frac{(1306.93)^{\frac{3}{4}}}{21} = 10.35$$

What is the trade value of a 32-year-old with the above stats?

$$v = \frac{(10.35-3)^2(3+1)(10.35)}{190} + \frac{(10.35)(3^2)}{13} = \frac{(7.35)^2(4)(10.35)}{190} + \frac{(10.35)(9)}{13} = \frac{(54.0225)(4)(10.35)}{190} + \frac{93.15}{13} =$$

$$\frac{13(2236.5315)}{13(190)} + \frac{190(93.15)}{190(13)} = \frac{29074.9095}{2470} + \frac{17698.5}{2470} = \frac{46773.4095}{2470} = 18.94$$

The trade value is used to compare players being considered in a trade to determine if the trade is fair and which team is getting the better deal. It is not a measure of salary.

Teaching Tips, Cont.

- Complete the Classwork exercise. Have one student work the problem on the board for the class and explain the answers. All students should work the problem in their books.

Assignment

- Complete Lesson 86, Activities 2-3.

Lesson 87

Concepts

- Complex fractions
- Systems of equations
- Graphing

Learning Objectives

The student will be able to:

- Define *complex fraction*
- Simplify complex fractions

Materials Needed

- Student Book, Lesson 87

Teaching Tips

- ➢ Review lowest common multiple as needed. (See Lesson 6)
- ➢ Define *complex fraction* from the teaching box.
- ➢ Teach simplifying complex fractions from the teaching box.
- ➢ Ask the students what mathematical operator is indicated by a fraction bar. (Division)
- ➢ An alternate method of simplifying complex fractions is shown at the right. You may teach the alternate method at your own discretion. Students who have a difficult time simplifying complex fractions with the LCD may find it easier using the alternate method. However, students should be encouraged to use the LCD method as much as possible because it will make complex fractions with rational expressions easier to simplify later.

Complex Fractions

A **complex fraction** is a fraction that has a fraction in the numerator, the denominator, or both. The following are all complex fractions.

$\frac{\frac{1}{2}}{3}$ Fraction in the numerator

$\frac{2}{\frac{3}{4}}$ Fraction in the denominator

$\frac{\frac{2}{3}}{\frac{3}{4}}$ Fractions in the numerator and denominator

To simplify a complex fraction, find the lowest common denominator (LCD) of all fractions in both the numerator and the denominator. Do not use whole numbers that appear in the numerator or denominator of the complex fraction. Multiply the numerator and denominator of the complex fraction by the LCD.

Simplify $\frac{\frac{2}{3}}{\frac{3}{4}}$. The LCD is $3(4)=12$.

Multiply the numerator and denominator by 12.

$$\frac{{}_4\cancel{12}\left(\frac{2}{\cancel{3}}\right)}{{}_3\cancel{12}\left(\frac{3}{\cancel{4}}\right)}=\frac{4(2)}{3(3)}=\frac{8}{9}$$

① Classwork

Simplify the complex fractions.

$\frac{\frac{1}{2}}{3}$

The LCD is 2. (The 3 in the denominator is not a fraction and is not used in finding the LCD.)

$$\frac{\cancel{2}\left(\frac{1}{\cancel{2}}\right)}{2(3)}=\frac{1}{6}$$

$\frac{2}{\frac{3}{4}}$

The LCD is 4. (The 2 in the numerator is not a fraction and is not used in finding the LCD.)

$$\frac{4(2)}{\cancel{4}\left(\frac{3}{\cancel{4}}\right)}=\frac{8}{3}$$

$\frac{\frac{1}{4}}{\frac{3}{5}+\frac{1}{2}}$

The LCD is $5(2^2)=20$.

$$\frac{{}_5\cancel{20}\left(\frac{1}{\cancel{4}}\right)}{20\left(\frac{3}{5}+\frac{1}{2}\right)}=\frac{5}{{}_4\cancel{20}\left(\frac{3}{\cancel{5}}\right)+{}_{10}\cancel{20}\left(\frac{1}{\cancel{2}}\right)}=$$

$$\frac{5}{12+10}=\frac{5}{22}$$

Activities

② Simplify the complex fractions.

$\frac{\frac{1}{3}}{5}$ LCD = 3.

$$\frac{\cancel{3}\left(\frac{1}{\cancel{3}}\right)}{3(5)}=\frac{1}{15}$$

$\frac{\frac{3}{8}}{2}$ LCD = 8.

$$\frac{\cancel{8}\left(\frac{3}{\cancel{8}}\right)}{8(2)}=\frac{3}{16}$$

$\frac{\frac{5}{9}}{7}$ LCD = 9.

$$\frac{\cancel{9}\left(\frac{5}{\cancel{9}}\right)}{9(7)}=\frac{5}{63}$$

$\frac{7}{\frac{5}{6}}$ LCD = 6.

$$\frac{6(7)}{\cancel{6}\left(\frac{5}{\cancel{6}}\right)}=\frac{42}{5}$$

$\frac{6}{\frac{2}{7}}$ LCD = 7.

$$\frac{7(6)}{\cancel{7}\left(\frac{2}{\cancel{7}}\right)}=\frac{{}_{21}\cancel{42}}{\cancel{2}}=21$$

$\frac{4}{\frac{6}{11}}$ LCD = 11.

$$\frac{11(4)}{\cancel{11}\left(\frac{6}{\cancel{11}}\right)}=\frac{44}{6}=\frac{22}{3}$$

$\frac{\frac{1}{4}}{\frac{3}{5}}$ LCD = $5(4)=20$.

$$\frac{{}_5\cancel{20}\left(\frac{1}{\cancel{4}}\right)}{{}_4\cancel{20}\left(\frac{3}{\cancel{5}}\right)}=\frac{5}{12}$$

$\frac{\frac{3}{4}}{\frac{5}{8}}$ LCD = 8.

$$\frac{{}_2\cancel{8}\left(\frac{3}{\cancel{4}}\right)}{\cancel{8}\left(\frac{5}{\cancel{8}}\right)}=\frac{6}{5}$$

$\frac{\frac{2}{5}}{\frac{3}{8}}$ LCD = $5(8)=40$.

$$\frac{{}_8\cancel{40}\left(\frac{2}{\cancel{5}}\right)}{{}_5\cancel{40}\left(\frac{3}{\cancel{8}}\right)}=\frac{16}{15}$$

Alternate Method for Complex Fractions

Rewrite as a division problem.

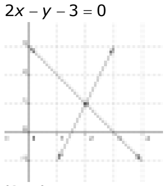
$$\frac{\frac{2}{3}}{\frac{3}{4}}=\frac{2}{3}\div\frac{3}{4}$$

Take the reciprocal of the divisor and multiply.

$$\frac{2}{3}\times\frac{4}{3}=\frac{8}{9}$$

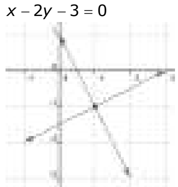
③ Solve the systems of equations by graphing. Express the solution as a coordinate point.

$x + y - 3 = 0$
$2x - y - 3 = 0$



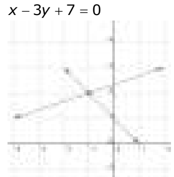
(2, 1)

$2x + y - 1 = 0$
$x - 2y - 3 = 0$



(1, -1)

$x + y - 1 = 0$
$x - 3y + 7 = 0$



(-1, 2)

④ Solve the systems of equations using the method of your choice. Express the solution as a coordinate point.

$3x + y - 5 = 0$
$-2x - y + 6 = 0$

$3x + y = 5$
$-2x - y = -6$
$x = -1$

$3(-1) + y - 5 = 0$
$-3 + y = 5$
$y = 8$

(-1, 8)

$x + 2y + 11 = 0$
$-x - 3y - 20 = 0$

$x + 2y = -11$
$-x - 3y = 20$
$-y = 9$
$y = -9$

$x + 2(-9) + 11 = 0$
$x - 18 = -11$
$x = 7$

(7, -9)

$x + y - 7 = 0$
$2x + y - 5 = 0$

$x + y = 7$
$2x + y = 5$
$-x = 2$
$x = -2$

$-2 + y - 7 = 0$
$y = 9$

(-2, 9)

$3x + 4y + 9 = 0$
$-x - y - 1 = 0$

$3x + 4y = -9$
$-x - y = 1$

$3x + 4y = -9$
$-3x - 3y = 3$
$y = -6$

$-x - (-6) - 1 = 0$
$-x + 6 = 1$
$-x = -5$
$x = 5$

(5, -6)

$10x - 3y - 10 = 0$
$7x - 2y - 8 = 0$

$10x - 3y = 10$
$7x - 2y = 8$

$20x - 6y = 20$
$21x - 6y = 24$
$-x = -4$
$x = 4$

$10(4) - 3y - 10 = 0$
$40 - 3y = 10$
$-3y = -30$
$y = 10$

(4, 10)

$3x - 6y - 15 = 0$
$4x - 8y - 20 = 0$

$3x - 6y = 15$
$4x - 8y = 20$

$x - 2y = 5$
$x - 2y = 5$
$0 = 0$

All real numbers

$10x - 3y - 12 = 0$
$7x - 2y - 9 = 0$

$10x - 3y = 12$
$7x - 2y = 9$

$20x - 6y = 24$
$21x - 6y = 27$
$-x = -3$
$x = 3$

$10(3) - 3y - 12 = 0$
$30 - 3y = 12$
$-3y = -18$
$y = 6$

(3, 6)

$4x - 12y - 8 = 0$
$6x - 18y - 12 = 0$

$4x - 12y = 8$
$6x - 18y = 12$

$x - 3y = 2$
$x - 3y = 2$
$0 = 0$

All real numbers

Note: Solutions show one method of solving. Students may use a different method, but will still get the same answer.

Teaching Tips, Cont.

- If you have taught both methods of simplifying complex fractions, tell the students that they may use either method for this lesson.
- Encourage the students to use the method presented in the teaching box of this lesson. The complex fractions become more involved in the next two lessons and students should be used to the method presented in this lesson.
- Complete the Classwork exercises. Have some students work the problems on the board for the class and explain their answers. All students should work the problems in their books.

Assignment

- Complete Lesson 87, Activities 2-4.

Lesson 88

Concepts
- Complex rational expressions
- Equations with radicals
- Equations with absolute value
- Math in the real world

Learning Objectives
The student will be able to:
- Define *complex rational expression*
- Simplify complex rational expressions

Materials Needed
- Student Book, Lesson 88
- Worksheet 44

Teaching Tips
- Have students complete Worksheet 44 in class. This may be for added practice of earlier topics, or graded as a quiz, if desired.
- Review complex fractions. (See Lesson 87)
- Review rational expressions. (See Lessons 78-86)
- Have the students compare the complex fractions in the teaching boxes of Lessons 87 and 88. Ask the students what is different about the complex fractions in Lesson 88. (There are variables in the complex fractions in Lesson 88.)
- Teach complex rational expressions from the teaching box. Explain that the method used to solve complex fractions with rational expressions is the same as the method used to solve complex fractions in Lesson 87.

Complex Rational Expressions

A **complex rational expression** is a fraction that has a rational expression in the numerator, the denominator, or both. The following are all complex rational expressions.

$\dfrac{\frac{1}{x+2}}{3}$ Rational expression in the numerator

$\dfrac{2}{\frac{3}{3x+4}}$ Rational expression in the denominator

$\dfrac{\frac{2}{x-3}}{\frac{3}{4x+1}}$ Rational expression in both

To simplify a complex rational expression, find the lowest common denominator (LCD) of all fractions in both the numerator and the denominator. Do not use whole numbers that appear in the numerator or denominator of the complex fraction. Leave the LCD as factors. Multiply the numerator and denominator of the complex fraction by the LCD.

Simplify $\dfrac{\frac{2}{x-3}}{\frac{3}{4x+1}}$. The LCD is $(x-3)(4x+1)$.

Multiply the numerator and denominator by $(x-3)(4x+1)$.

$$\frac{\cancel{(x-3)}(4x+1)\left(\frac{2}{\cancel{x-3}}\right)}{(x-3)\cancel{(4x+1)}\left(\frac{3}{\cancel{4x+1}}\right)}=\frac{2(4x+1)}{3(x-3)}=\frac{8x+2}{3x-9}$$

① **Classwork**

Simplify the complex rational expressions.

$\dfrac{\frac{1}{x+2}}{3}$

The LCD is $x + 2$. (The 3 in the denominator is not a fraction and is not used in finding the LCD.)

$$\frac{\cancel{x+2}\left(\frac{1}{\cancel{x+2}}\right)}{(x+2)(3)}=\frac{1}{3x+6}$$

$\dfrac{2}{\frac{3}{3x+4}}$

The LCD is $3x + 4$. (The 2 in the numerator is not a fraction and is not used in finding the LCD.)

$$\frac{(3x+4)(2)}{\cancel{(3x+4)}\left(\frac{3}{\cancel{3x+4}}\right)}=\frac{6x+8}{3}$$

$\dfrac{\frac{1}{x+4}}{\frac{3}{2x+5}}$

The LCD is $(x+4)(2x+5)$.

$$\frac{\cancel{(x+4)}(2x+5)\left(\frac{1}{\cancel{x+4}}\right)}{(x+4)\cancel{(2x+5)}\left(\frac{3}{\cancel{2x+5}}\right)}=$$

$$\frac{2x+5}{3x+12}$$

Activities

② Simplify the complex rational expressions.

$\dfrac{\frac{1}{x-3}}{5}$ LCD = $x - 3$.

$$\frac{\cancel{(x-3)}\left(\frac{1}{\cancel{x-3}}\right)}{(x-3)(5)}=\frac{1}{5x-15}$$

$\dfrac{\frac{1}{x-4}}{\frac{3}{x+5}}$ LCD = $(x-4)(x+5)$.

$$\frac{\cancel{(x-4)}(x+5)\left(\frac{1}{\cancel{x-4}}\right)}{(x-4)\cancel{(x+5)}\left(\frac{3}{\cancel{x+5}}\right)}=\frac{x+5}{3x-12}$$

$\dfrac{6}{\frac{2}{2x+7}}$ LCD = $2x + 7$.

$$\frac{(2x+7)(\cancel{6}_3)}{\cancel{(2x+7)}\left(\frac{\cancel{2}}{\cancel{2x+7}}\right)}=6x+21$$

$\dfrac{7}{\frac{5}{6x+1}}$ LCD = $6x + 1$.

$$\frac{(6x+1)(7)}{\cancel{(6x+1)}\left(\frac{5}{\cancel{6x+1}}\right)}=\frac{42x+7}{5}$$

$\dfrac{\frac{3}{3x-8}}{2}$ LCD = $3x - 8$.

$$\frac{\cancel{(3x-8)}\left(\frac{3}{\cancel{3x-8}}\right)}{(3x-8)(2)}=\frac{3}{6x-16}$$

$\dfrac{\frac{3}{5x-4}}{\frac{5}{8x+3}}$ LCD = $(5x-4)(8x+3)$.

$$\frac{\cancel{(5x-4)}(8x+3)\left(\frac{3}{\cancel{5x-4}}\right)}{(5x-4)\cancel{(8x+3)}\left(\frac{5}{\cancel{8x+3}}\right)}=\frac{24x+9}{25x-20}$$

③ Solve. Identify any extraneous solutions.

$\sqrt{-6x+10}+3=11$

$\sqrt{-6x+10}=8$

$\left(\sqrt{-6x+10}\right)^2=8^2$

$-6x+10=64$

$-6x=54$

$x=-9$

check:

$\sqrt{-6(-9)+10}+3=11$

$\sqrt{54+10}+3=11$

$\sqrt{64}+3=11$

$8+3=11$

$|-4x-9|+7>3$

$|-4x-9|>-4$ ALL REAL NUMBERS

The absolute value of anything can never be negative, so the answer is all real numbers.

$|4x-3|-2=x+10$

$|4x-3|=x+12$

$4x-3=x+12$ or $4x-3=-(x+12)$

$3x=15$ $\quad$ $4x-3=-x-12$

$x=5$ $\quad$ $5x=-9\Rightarrow x=-\frac{9}{5}$

check:

$|4(5)-3|-2=1(5)+10$

$|20-3|-2=5+10$

$17-2=15$

$\left|4\left(-\frac{9}{5}\right)-3\right|-2=1\left(-\frac{9}{5}\right)+10$

$\left|-\frac{36}{5}-\frac{15}{5}\right|-\frac{10}{5}=-\frac{9}{5}+\frac{50}{5}$

$\frac{51}{5}-\frac{10}{5}=\frac{41}{5}$

$|3x+6|+13<7$

$|3x+6|<-6$ NO SOLUTION

The absolute value of anything can never be negative, so the answer is no solution.

$|2x+7|+6x=3x-11$

$|2x+7|=-3x-11$

$2x+7=-3x-11$ or $2x+7=-(-3x-11)$

$5x=-18$ $\quad$ $2x+7=3x+11$

$x=-\frac{18}{5}$ $\quad$ $-x=4\Rightarrow x=-4$

check:

$\left|2\left(-\frac{18}{5}\right)+7\right|+6\left(-\frac{18}{5}\right)=3\left(-\frac{18}{5}\right)-11$

$\left|-\frac{36}{5}+\frac{35}{5}\right|-\frac{108}{5}=-\frac{54}{5}-\frac{55}{5}$

$\left|-\frac{1}{5}\right|-\frac{108}{5}=-\frac{109}{5}$

$\frac{1}{5}-\frac{108}{5}\neq-\frac{109}{5}$ extraneous

$|2(-4)+7|+6(-4)=3(-4)-11$

$|-8+7|-24=-12-11$

$|-1|-24=-23$

$1-24=-23$

$\sqrt{4x-7}+1=-4$

$\sqrt{4x-7}=-5$

$\left(\sqrt{4x-7}\right)^2=(-5)^2$

$4x-7=25$

$4x=32$

$x=8$

check:

$\sqrt{4(8)-7}+1=-4$

$\sqrt{32-7}+1=-4$

$\sqrt{25}+1=-4$

$5+1\neq-4$ extraneous

④ Solve.

According to the formula $y=27-\frac{3a}{4}$, where *a* is a basketball player's current age and y is the number of years left to play professional ball, what is the age of a basketball player when he is likely to end his career? State your answer as an inequality.

$0\le 27-\frac{3a}{4}$ $\quad$ $3a\le 108$

$0\le 108-3a$ $\quad$ $a\le 36$ $\quad$ A basketball player's age is less than or equal to 36 years.

Adding Rational Expressions with Different Denominators Worksheet 44

① Solve. Remember to state any exclusions.

$\frac{2x}{3x+4}+\frac{1}{3}=$

$\frac{(2x)(3)}{(3x+4)(3)}+\frac{(1)(3x+4)}{(3)(3x+4)}=$

$\frac{6x+3x+4}{9x+12}=\frac{9x+4}{9x+12}; x \neq -\frac{4}{3}$

$\frac{5x^2}{2x+5}+2=$

$\frac{(5x^2)(1)}{(2x+5)(1)}+\frac{(2)(2x+5)}{(1)(2x+5)}=$

$\frac{5x^2+4x+10}{2x+5}; x \neq -\frac{5}{2}$

$\frac{3x}{5x+2}-\frac{2x}{5}=$

$\frac{(3x)(5)}{(5x+2)(5)}-\frac{(2x)(5x+2)}{(5)(5x+2)}=$

$\frac{15x-10x^2-4x}{25x+10}=$

$\frac{-10x^2+11x}{25x+10}; x \neq -\frac{2}{5}$

$\frac{5}{4x+5}+\frac{3x}{7}=$

$\frac{(5)(7)}{(4x+5)(7)}+\frac{(3x)(4x+5)}{(7)(4x+5)}=$

$\frac{35+12x^2+15x}{28x+35}=$

$\frac{12x^2+15x+35}{28x+35}; x \neq -\frac{5}{4}$

$\frac{4}{2x+3}+\frac{1}{3x-1}=$

$\frac{4(3x-1)}{(2x+3)(3x-1)}+\frac{1(2x+3)}{(2x+3)(3x-1)}=$

$\frac{12x-4}{6x^2+7x-3}+\frac{2x+3}{6x^2+7x-3}=$

$\frac{14x-1}{6x^2+7x-3}; x \neq -\frac{3}{2}, \frac{1}{3}$

$\frac{3}{x+4}+\frac{2x}{4x+1}=$

$\frac{3(4x+1)}{(x+4)(4x+1)}+\frac{2x(x+4)}{(x+4)(4x+1)}=$

$\frac{12x+3}{4x^2+17x+4}+\frac{2x^2+8x}{4x^2+17x+4}=$

$\frac{2x^2+20x+3}{4x^2+17x+4}; x \neq -4, -\frac{1}{4}$

$\frac{5}{2x+1}+\frac{4}{2x-1}=$

$\frac{5(2x-1)}{(2x+1)(2x-1)}+\frac{4(2x+1)}{(2x+1)(2x-1)}=$

$\frac{10x-5}{4x^2-1}+\frac{8x+4}{4x^2-1}=$

$\frac{18x-1}{4x^2-1}; x \neq -\frac{1}{2}, \frac{1}{2}$

$\frac{8x}{x-2}+\frac{2x^2}{4x+3}=$

$\frac{8x(4x+3)}{(x-2)(4x+3)}+\frac{2x^2(x-2)}{(x-2)(4x+3)}=$

$\frac{32x^2+24x}{4x^2-5x-6}+\frac{2x^3-4x^2}{4x^2-5x-6}=$

$\frac{2x^3+28x^2+24x}{4x^2-5x-6}; x \neq -\frac{3}{4}, 2$

Teaching Tips, Cont.

- Make sure all students understand that the LCD is found using the denominators of the individual fractions. The numerators of the individual fractions are not used to determine the LCD.
- Note: When simplifying expressions with variables in the denominator, it is customary to list values of the variable that must be excluded as part of the solution. (The variable cannot equal anything that would cause a denominator to equal zero since you cannot divide by zero.) For now, it is important that the students understand the basic concept of simplifying complex rational expressions. Exclusions will be included when the complex expressions are solved.
- Complete the Classwork exercises. Have some students work the problems on the board for the class and explain their answers. All students should work the problems in their books.

Assignment

- Complete Lesson 88, Activities 2-4.

Lesson 89

Concepts

- Complex rational expressions
- Lowest common denominator

Learning Objectives

The student will be able to:

- Identify the lowest common denominator (LCD) of a complex rational expression
- Simplify complex rational expressions

Materials Needed

- Student Book, Lesson 89
- Worksheet 45

Teaching Tips

- Review complex fractions. (See Lesson 87)
- Review rational expressions. (See Lessons 78-86)
- Review complex rational expressions. (See Lesson 88)
- Remind students to use only the denominators when finding the LCD. If there are multiple fractions in the numerator or denominator of the main fraction, all denominators must be used to find the LCD.

Review of Complex Expressions

Recall that complex expressions are fractions that have a fraction, rational expression, or both in the numerator, denominator, or both. No matter what the configuration, find the LCD of all fractions in the complex expression and multiply each term in the numerator and denominator by the LCD.

Simplify the complex expression.

$$\frac{\frac{3}{x}+\frac{4}{x+1}}{\frac{x-1}{x^2-1}}$$

Factor each denominator and cancel like terms before finding the LCD.

$$\frac{\frac{3}{x}+\frac{4}{x+1}}{\frac{\cancel{x-1}}{(x+1)(\cancel{x-1})}}$$

The LCD is $x(x+1)$.

Multiply each term in the numerator and denominator by the LCD. Cancel where appropriate and combine like terms.

$$\frac{\cancel{x}(x+1)\left(\frac{3}{\cancel{x}}\right)+x(\cancel{x+1})\left(\frac{4}{\cancel{x+1}}\right)}{x(\cancel{x+1})\left(\frac{1}{\cancel{x+1}}\right)}=$$

$$\frac{3(x+1)+4x}{x}=\frac{3x+3+4x}{x}=\frac{7x+3}{x}$$

① **Classwork.**

Simplify the complex expression.

$$\frac{\frac{4}{x}+\frac{3}{x+2}}{\frac{x+1}{x^2-1}}$$

$$\frac{\frac{4}{x}+\frac{3}{x+2}}{\frac{\cancel{x+1}}{(\cancel{x+1})(x-1)}}$$

The LCD is $x(x+2)(x-1)$.

$$\frac{\cancel{x}(x+2)(x-1)\left(\frac{4}{\cancel{x}}\right)+x(\cancel{x+2})(x-1)\left(\frac{3}{\cancel{x+2}}\right)}{x(x+2)(\cancel{x-1})\left(\frac{1}{\cancel{x-1}}\right)}=$$

$$\frac{(x+2)(x-1)(4)+x(x-1)(3)}{x(x+2)(1)}=$$

$$\frac{(x^2+x-2)(4)+(x^2-x)(3)}{(x^2+2x)(1)}=$$

$$\frac{4x^2+4x-8+3x^2-3x}{x^2+2x}=$$

$$\frac{7x^2+x-8}{x^2+2x}$$

Activities

② Simplify the complex expressions.

$$\frac{\frac{2}{x}+\frac{3}{x+2}}{\frac{x-2}{x^2-4}}$$

$$\frac{\frac{2}{x}+\frac{3}{x+2}}{\frac{\cancel{x-2}}{(x+2)(\cancel{x-2})}}$$

The LCD is $x(x+2)$.

$$\frac{\cancel{x}(x+2)\left(\frac{2}{\cancel{x}}\right)+x(\cancel{x+2})\left(\frac{3}{\cancel{x+2}}\right)}{x(\cancel{x+2})\left(\frac{1}{\cancel{x+2}}\right)}=$$

$$\frac{(x+2)(2)+x(3)}{x(1)}=\frac{2x+4+3x}{x}=$$

$$\frac{5x+4}{x}$$

$$\frac{\frac{5}{x}+\frac{1}{2x-3}}{\frac{2x+3}{4x^2-9}}$$

$$\frac{\frac{5}{x}+\frac{1}{2x-3}}{\frac{\cancel{2x+3}}{(\cancel{2x+3})(2x-3)}}$$

The LCD is $x(2x-3)$.

$$\frac{\cancel{x}(2x-3)\left(\frac{5}{\cancel{x}}\right)+x(\cancel{2x-3})\left(\frac{1}{\cancel{2x-3}}\right)}{x(\cancel{2x-3})\left(\frac{1}{\cancel{2x-3}}\right)}=$$

$$\frac{(2x-3)(5)+x(1)}{x(1)}=\frac{10x-15+x}{x}=$$

$$\frac{11x-15}{x}$$

③ Simplify the complex expressions.

$$\frac{\frac{7}{3x}+\frac{5}{x+4}}{\frac{x-4}{x^2-16}}$$

$$\frac{\frac{7}{3x}+\frac{5}{x+4}}{\frac{\cancel{x-4}}{(x+4)(\cancel{x-4})}}$$

The LCD is $3x(x+4)$.

$$\frac{\cancel{3x}(x+4)\left(\frac{7}{\cancel{3x}}\right)+3x(\cancel{x+4})\left(\frac{5}{\cancel{x+4}}\right)}{3x(\cancel{x+4})\left(\frac{1}{\cancel{x+4}}\right)}=$$

$$\frac{(x+4)(7)+3x(5)}{3x(1)}=$$

$$\frac{7x+28+15x}{3x}=$$

$$\frac{22x+28}{3x}$$

$$\frac{\frac{2}{2x+1}+\frac{3}{x-2}}{\frac{x+1}{x^2-x-2}-\frac{2x+4}{x^2-4}}$$

$$\frac{\frac{2}{2x+1}+\frac{3}{x-2}}{\frac{\cancel{x+1}}{(\cancel{x+1})(x-2)}-\frac{2(\cancel{x+2})}{(\cancel{x+2})(x-2)}}$$

The LCD is $(2x+1)(x-2)$.

$$\frac{(\cancel{2x+1})(x-2)\left(\frac{2}{\cancel{2x+1}}\right)+(2x+1)(\cancel{x-2})\left(\frac{3}{\cancel{x-2}}\right)}{(2x+1)(\cancel{x-2})\left(\frac{1}{\cancel{x-2}}\right)-(2x+1)(\cancel{x-2})\left(\frac{2}{\cancel{x-2}}\right)}=$$

$$\frac{(x-2)(2)+(2x+1)(3)}{(2x+1)(1)-(2x+1)(2)}=$$

$$\frac{(2x-4)+(6x+3)}{2x+1-4x-2}=$$

$$\frac{8x-1}{-2x-1}$$

$$\frac{\frac{2x+5}{2x}+\frac{3x+1}{x^2+2x+1}}{\frac{x+1}{6x^2+2x}}$$

$$\frac{\frac{2x+5}{2x}+\frac{3x+1}{(x+1)^2}}{\frac{x+1}{2x(3x+1)}}$$

The LCD is $2x(x+1)^2(3x+1)$.

$$\frac{\cancel{2x}(x+1)^2(3x+1)\left(\frac{2x+5}{\cancel{2x}}\right)+2x\cancel{(x+1)^2}(3x+1)\left(\frac{3x+1}{\cancel{(x+1)^2}}\right)}{\cancel{2x}(x+1)^2(\cancel{3x+1})\left(\frac{x+1}{\cancel{2x}(\cancel{3x+1})}\right)}=$$

$$\frac{(x+1)^2(3x+1)(2x+5)+2x(3x+1)(3x+1)}{(x+1)^2(x+1)}=$$

$$\frac{(x^2+2x+1)(6x^2+17x+5)+(6x^2+2x)(3x+1)}{(x^2+2x+1)(x+1)}=$$

$$\frac{(6x^4+29x^3+45x^2+27x+5)+(18x^3+12x^2+2x)}{x^3+x^2+2x^2+2x+x+1}=$$

$$\frac{6x^4+47x^3+57x^2+29x+5}{x^3+3x^2+3x+1}$$

$$\frac{\frac{x^2+7x+6}{x+1}-\frac{x^2+4x+3}{x+3}}{\frac{x+1}{x^2+7x+6}}$$

$$\frac{\frac{(x+6)(\cancel{x+1})}{\cancel{x+1}}-\frac{(x+1)(\cancel{x+3})}{\cancel{x+3}}}{\frac{\cancel{x+1}}{(x+6)(\cancel{x+1})}}$$

The LCD is $(x+6)$.

$$\frac{(x+6)(x+6)-(x+6)(x+1)}{(\cancel{x+6})\left(\frac{1}{\cancel{x+6}}\right)}=$$

$$\frac{(x^2+12x+36)-(x^2+7x+6)}{1}=$$

$$5x+30$$

Complex Rational Expressions Worksheet 45

① Simplify the complex expressions.

$$\frac{\frac{9}{4x}+\frac{3}{x+2}}{\frac{2x-4}{x^2-4}}$$

$$\frac{\frac{9}{4x}+\frac{3}{x+2}}{\frac{2\cancel{(x-2)}}{(x+2)\cancel{(x-2)}}}$$

The LCD is $4x(x+2)$.

$$\frac{\cancel{4x}(x+2)\left(\frac{9}{\cancel{4x}}\right)+4x\cancel{(x+2)}\left(\frac{3}{\cancel{x+2}}\right)}{4x\cancel{(x+2)}\left(\frac{2}{\cancel{x+2}}\right)}=$$

$$\frac{(x+2)(9)+4x(3)}{4x(2)}=$$

$$\frac{9x+18+12x}{8x}=$$

$$\frac{21x+18}{8x}$$

$$\frac{\frac{2x+1}{5x}+\frac{2x+5}{x^2-2x+1}}{\frac{3x+1}{15x^2+10x}}$$

$$\frac{\frac{2x+1}{5x}+\frac{2x+5}{(x-1)^2}}{\frac{3x+1}{5x(3x+2)}}$$

The LCD is $5x(x-1)^2(3x+2)$.

$$\frac{\cancel{5x}(x-1)^2(3x+2)\left(\frac{2x+1}{\cancel{5x}}\right)+5x\cancel{(x-1)^2}(3x+2)\left(\frac{2x+5}{\cancel{(x-1)^2}}\right)}{\cancel{5x}(x-1)^2\cancel{(3x+2)}\left(\frac{3x+1}{\cancel{5x}\cancel{(3x+2)}}\right)}=$$

$$\frac{(x-1)^2(3x+2)(2x+1)+5x(3x+2)(2x+5)}{(x-1)^2(3x+1)}=$$

$$\frac{(x^2-2x+1)(6x^2+7x+2)+(15x^2+10x)(2x+5)}{(x^2-2x+1)(3x+1)}=$$

$$\frac{(6x^4-5x^3-6x^2+3x+2)+(30x^3+95x^2+50x)}{3x^3+x^2-6x^2-2x+3x+1}=$$

$$\frac{6x^4+25x^3+89x^2+53x+2}{3x^3-5x^2+x+1}$$

$$\frac{\frac{3}{x+6}+\frac{4}{7x-2}}{\frac{2x+2}{7x^2+5x-2}-\frac{5x+10}{7x^2+12x-4}}$$

$$\frac{\frac{3}{x+6}+\frac{4}{7x-2}}{\frac{2\cancel{(x+1)}}{\cancel{(x+1)}(7x-2)}-\frac{5\cancel{(x+2)}}{\cancel{(x+2)}(7x-2)}}$$

The LCD is $(x+6)(7x-2)$.

$$\frac{\cancel{(x+6)}(7x-2)\left(\frac{3}{\cancel{x+6}}\right)+(x+6)\cancel{(7x-2)}\left(\frac{4}{\cancel{7x-2}}\right)}{(x+6)\cancel{(7x-2)}\left(\frac{2}{\cancel{7x-2}}\right)-(x+6)\cancel{(7x-2)}\left(\frac{5}{\cancel{7x-2}}\right)}=$$

$$\frac{(7x-2)(3)+(x+6)(4)}{(x+6)(2)-(x+6)(5)}=$$

$$\frac{(21x-6)+(4x+24)}{2x+12-5x-30}=$$

$$\frac{25x+18}{-3x-18}$$

$$\frac{\frac{x^2+7x+6}{x+1}-\frac{x^2+4x+3}{x+3}}{\frac{5x+5}{x^2+7x+6}}$$

$$\frac{\frac{(x+6)\cancel{(x+1)}}{\cancel{x+1}}-\frac{(x+1)\cancel{(x+3)}}{\cancel{x+3}}}{\frac{5\cancel{(x+1)}}{(x+6)\cancel{(x+1)}}}$$

The LCD is $(x+6)$.

$$\frac{(x+6)(x+6)-(x+6)(x+1)}{\cancel{(x+6)}\left(\frac{5}{\cancel{x+6}}\right)}=$$

$$\frac{(x^2+12x+36)-(x^2+7x+6)}{5}=$$

$$\frac{5x+30}{5}=$$

$$\frac{\cancel{5}(x+6)}{\cancel{5}}=$$

$$x+6$$

Teaching Tips, Cont.

- Encourage the students to simplify the fractions by cancelling like terms as much as possible before multiplying.
- Complete the Classwork exercise. Have one student work the problem on the board for the class and explain the answer. All students should work the problem in their books.
- Note: The solution for the second problem in the first row of Worksheet 45 can be simplified by combining the terms in the denominator after the first step.

Assignment

- Complete Lesson 89, Activities 2-3.
- Worksheet 45.

Lesson 90

Concepts
- Quadratic equations
- Dividing rational expressions
- Multiplying rational expressions
- Adding rational expressions
- Subtracting rational expressions

Learning Objectives
The student will be able to:
- Define *quadratic equation*
- Identify whether or not an equation is a quadratic equation
- Explain why a given equation is not a quadratic equation

Materials Needed
- Student Book, Lesson 90

Teaching Tips
- Review multiplying by a binomial. (See Lesson 60)
- Review the FOIL method. (See Lesson 62)
- Define *quadratic equation* from the teaching box.
- Teach the conditions for an equation to be a quadratic equation:
  - The equation must be in the format $ax^2 + bx + c$.
  - The variable a cannot equal 0.
  - No variable may have an exponent greater than 2.

Quadratic Equations

You are familiar with a variety of polynomials, such as monomials, binomials, trinomials, as well as polynomials with more than three terms. This lesson deals with a specific type of polynomial known as a quadratic equation. A **quadratic equation** is a polynomial of the second degree in the form $ax^2 + bx + c = 0$.

The standard form trinomials you have worked with already this year are quadratic equations. While the easiest quadratic equations to recognize are those that follow the rule exactly, the most important thing to remember is that $a \neq 0$ and no variable may have an exponent greater than 2.

Identify whether or not the equation simplifies to a quadratic equation. If not, explain why.

$8(x-1)=0$

Multiply to get $8x-8=0$. This is not a quadratic equation because it is missing the ax^2 term.

① **Classwork**

Identify whether or not the equation simplifies to a quadratic equation. If not, explain why.

$x(x+3)=0$

$x^2+3x=0$

This is a quadratic equation.

$(x^2)(x+5)=0$

$x^3+5x^2=0$

This is not a quadratic equation. The variable has an exponent greater than 2.

Activities

② Identify whether or not the equation simplifies to a quadratic equation. If not, explain why.

$x(x-4)=0$

$x^2-4x=0$

This is a quadratic equation.

$(x-2)(x+6)=0$

$x^2+4x-12=0$

This is a quadratic equation.

$2(x^2+5x)=0$

$2x^2+10x=0$

This is a quadratic equation.

$(3x-1)(x^2+4)=0$

$3x^3-x^2+12x-4=0$

This is not a quadratic equation. The variable has an exponent greater than 2.

$(x^2)(x-2)=x^3$

$x^3-2x^2=x^3$

$-2x^2=0$

This is a quadratic equation.

$(x^2-3)(2x+5)=0$

$2x^3+5x^2-6x-15=0$

This is not a quadratic equation. The variable has an exponent greater than 2.

$(x-3)(x+3)=0$

$x^2-9=0$

This is a quadratic equation.

$(3x+4)(2x+1)=0$

$6x^2+11x+4=0$

This is a quadratic equation.

$(7x-2)(x^2-4)=0$

$7x^3-2x^2-28x+8=0$

This is not a quadratic equation. The variable has an exponent greater than 2.

$(2x+5)(2x-5)=0$

$4x^2-25=0$

This is a quadratic equation.

$(x^2-4)(-x^2+4)=0$

$-x^4+8x^2-16=0$

This is not a quadratic equation. The variable has an exponent greater than 2.

$4(x^2+3x+2)=0$

$4x^2+12x+8=0$

This is a quadratic equation.

③ Solve. Remember to state any exclusions.

$$\frac{2x^2+x-6}{2x^2-x-6} \div \frac{2x^2+3x-2}{4x^2+4x-3} =$$
$$\frac{2x^2+x-6}{2x^2-x-6} \cdot \frac{4x^2+4x-3}{2x^2+3x-2} =$$
$$\frac{(2x-3)\cancel{(x+2)}}{\cancel{(2x+3)}(x-2)} \cdot \frac{\cancel{(2x+3)}\cancel{(2x-1)}}{\cancel{(x+2)}\cancel{(2x-1)}} =$$
$$\frac{2x-3}{x-2}; x \neq -2, -\tfrac{3}{2}, \tfrac{1}{2}, 2$$

$$\frac{8x+20}{8x^2+6x-9} \div \frac{2x^2-3x-20}{12x^2-9x} =$$
$$\frac{8x+20}{8x^2+6x-9} \cdot \frac{12x^2-9x}{2x^2-3x-20} =$$
$$\frac{4\cancel{(2x+5)}}{\cancel{(4x-3)}(2x+3)} \cdot \frac{3x\cancel{(4x-3)}}{\cancel{(2x+5)}(x-4)} = \frac{4}{2x+3} \cdot \frac{3x}{x-4} =$$
$$\frac{12x}{2x^2-5x-12}; x \neq -\tfrac{5}{2}, -\tfrac{3}{2}, \tfrac{3}{4}, 4, 0$$

$$\frac{10x^2+35x}{18x^3-12x^2} \div \frac{4x^2+14x}{6x^2-4x} =$$
$$\frac{10x^2+35x}{18x^3-12x^2} \cdot \frac{6x^2-4x}{4x^2+14x} =$$
$$\frac{5\cancel{x}\cancel{(2x+7)}}{6x^{\cancel{2}}\cancel{(3x-2)}} \cdot \frac{\cancel{2x}\cancel{(3x-2)}}{\cancel{2x}\cancel{(2x+7)}} =$$
$$\frac{5}{6x}; x \neq -\tfrac{7}{2}, 0, \tfrac{2}{3}$$

$$\frac{3x^2-14x-24}{4x^2-81} \div \frac{4x^2-25x+6}{8x^2+34x-9} =$$
$$\frac{3x^2-14x-24}{4x^2-81} \cdot \frac{8x^2+34x-9}{4x^2-25x+6} =$$
$$\frac{(3x+4)\cancel{(x-6)}}{(2x-9)\cancel{(2x+9)}} \cdot \frac{\cancel{(2x+9)}\cancel{(4x-1)}}{\cancel{(x-6)}\cancel{(4x-1)}} =$$
$$\frac{3x+4}{2x-9}; x \neq -\tfrac{9}{2}, \tfrac{1}{4}, \tfrac{9}{2}, 6$$

④ Solve. Remember to state any exclusions.

$$\frac{3x}{4x-5} + \frac{4}{5x+2} =$$
$$\frac{3x(5x+2)}{(4x-5)(5x+2)} + \frac{4(4x-5)}{(4x-5)(5x+2)} =$$
$$\frac{15x^2+6x}{20x^2-17x-10} + \frac{16x-20}{20x^2-17x-10} =$$
$$\frac{15x^2+22x-20}{20x^2-17x-10}; x \neq -\tfrac{2}{5}, \tfrac{5}{4}$$

$$\frac{4x}{3x+1} - \frac{4x}{2x-7} =$$
$$\frac{4x(2x-7)}{(3x+1)(2x-7)} - \frac{4x(3x+1)}{(3x+1)(2x-7)} =$$
$$\frac{8x^2-28x}{6x^2-19x-7} - \frac{12x^2+4x}{6x^2-19x-7} =$$
$$\frac{-4x^2-32x}{6x^2-19x-7}; x \neq -\tfrac{1}{3}, \tfrac{7}{2}$$

$$\frac{8}{5x-2} + \frac{3x}{2x-1} =$$
$$\frac{8(2x-1)}{(5x-2)(2x-1)} + \frac{3x(5x-2)}{(5x-2)(2x-1)} =$$
$$\frac{16x-8}{10x^2-9x+2} + \frac{15x^2-6x}{10x^2-9x+2} =$$
$$\frac{15x^2+10x-8}{10x^2-9x+2}; x \neq \tfrac{2}{5}, \tfrac{1}{2}$$

$$\frac{3x^2}{2x-3} - \frac{2x}{x+5} =$$
$$\frac{3x^2(x+5)}{(2x-3)(x+5)} - \frac{2x(2x-3)}{(2x-3)(x+5)} =$$
$$\frac{3x^3+15x^2}{2x^2+7x-15} - \frac{4x^2-6x}{2x^2+7x-15} =$$
$$\frac{3x^3+11x^2+6x}{2x^2+7x-15}; x \neq -5, \tfrac{3}{2}$$

Teaching Tips, Cont.

- Tell the students that it is important that the learn to identify quadratic equations quickly and accurately because they will have to use this information in upcoming Lessons.
- Complete the Classwork exercises. Have some students work the problems on the board for the class and explain their answers. All students should work the problems in their books.
- Review for Test 9 using worksheets 41-45. These worksheets were assigned in previous lessons.

Assignments

- Complete Lesson 90, Activities 2-4.
- Study for Test 9 (Lessons 78-87).

Test 9

Testing Objectives

The student will:

- Simplify rational expressions
- Add rational expressions
- Subtract rational expressions
- Multiply rational expressions
- Divide rational expressions
- Simplify complex numbers

Materials Needed

- Test 9
- *It's College Test Prep Time!* from the Student Book
- Exploring Math through… Ice Hockey from Student Book

Teaching Tips

- Administer Test 9, allowing the students 30-40 minutes to complete the test.

Test 9

① Simplify. Remember to state any exclusions. **7 points**

$$\frac{20x^2-12x}{5x-3}$$
$$\frac{4x\cancel{(5x-3)}}{\cancel{5x-3}}=4x;x\neq\tfrac{3}{5}$$

$$\frac{3x^2+16x-12}{9x^2-4}$$
$$\frac{(x+6)\cancel{(3x-2)}}{(3x+2)\cancel{(3x-2)}}=\frac{x+6}{3x+2};x\neq-\tfrac{2}{3},\tfrac{2}{3}$$

$$\frac{9x^2+26x+16}{9x^2+17x+8}$$
$$\frac{\cancel{(9x+8)}(x+2)}{\cancel{(9x+8)}(x+1)}=\frac{x+2}{x+1};x\neq-\tfrac{8}{9},-1$$

② Solve. Remember to state any exclusions. **32 points**

$$\frac{4}{x+2}+\frac{6}{x+2}=$$
$$\frac{4+6}{x+2}=\frac{10}{x+2};x\neq-2$$

$$\frac{3x+1}{4x+3}+\frac{3x-2}{4x+3}=$$
$$\frac{(3x+1)+(3x-2)}{4x+3}=$$
$$\frac{6x-1}{4x+3};x\neq-\tfrac{3}{4}$$

$$\frac{24x+42}{56x^2+16x}\cdot\frac{28x^2+8x}{4x^2-9x-28}=$$
$$\frac{{}_3\cancel{6}\cancel{(4x+7)}}{{}_2\cancel{8x}\cancel{(7x+2)}}\cdot\frac{\cancel{4x}\cancel{(7x+2)}}{\cancel{(4x+7)}(x-4)}=$$
$$\frac{3}{x-4};x\neq-\tfrac{7}{4},-\tfrac{2}{7},0,4$$

$$\frac{3x+9}{x^2+5x}\div\frac{x^2-4x-21}{x^2-7x}=$$
$$\frac{3x+9}{x^2+5x}\cdot\frac{x^2-7x}{x^2-4x-21}=$$
$$\frac{3\cancel{(x+3)}}{\cancel{x}(x+5)}\cdot\frac{\cancel{x}\cancel{(x-7)}}{\cancel{(x+3)}\cancel{(x-7)}}=$$
$$\frac{3}{x+5};x\neq-5,-3,0,7$$

$$\frac{1}{x+3}+\frac{4}{5}=$$
$$\frac{(1)(5)}{(x+3)(5)}+\frac{(4)(x+3)}{(5)(x+3)}=$$
$$\frac{5+4x+12}{5x+15}=\frac{4x+17}{5x+15};x\neq-3$$

$$\frac{8x}{4x-5}-\frac{3x}{4x-5}=$$
$$\frac{8x-3x}{4x-5}=\frac{5x}{4x-5};x\neq\tfrac{5}{4}$$

$$\frac{23x-7}{9x-5}-\frac{5x+3}{9x-5}=$$
$$\frac{(23x-7)-(5x+3)}{9x-5}=\frac{18x-10}{9x-5}=$$
$$\frac{2\cancel{(9x-5)}}{\cancel{9x-5}}=2;x\neq\tfrac{5}{9}$$

$$\frac{5x^2-23x-10}{2x^2+9x+9}\cdot\frac{8x^2+10x-3}{20x^2+3x-2}=$$
$$\frac{(x-5)\cancel{(5x+2)}}{\cancel{(2x+3)}(x+3)}\cdot\frac{\cancel{(2x+3)}\cancel{(4x-1)}}{\cancel{(5x+2)}\cancel{(4x-1)}}=$$
$$\frac{x-5}{x+3};x\neq-3,-\tfrac{3}{2},-\tfrac{2}{5},\tfrac{1}{4}$$

$$\frac{6x+36}{8x^2+32x}\div\frac{x^2+2x-24}{4x^2+16x}=$$
$$\frac{6x+36}{8x^2+32x}\cdot\frac{4x^2+16x}{x^2+2x-24}=$$
$$\frac{{}_3\cancel{6}\cancel{(x+6)}}{{}_2\cancel{8x}\cancel{(x+4)}}\cdot\frac{\cancel{4x}\cancel{(x+4)}}{\cancel{(x+6)}(x-4)}=$$
$$\frac{3}{x-4};x\neq-6,-4,0,4$$

$$\frac{x}{3x+2}-\frac{x}{4}=$$
$$\frac{(x)(4)}{(3x+2)(4)}-\frac{(x)(3x+2)}{(4)(3x+2)}=$$
$$\frac{4x-3x^2-2x}{12x+8}=\frac{-3x^2+2x}{12x+8};x\neq-\tfrac{2}{3}$$

Test 9

③ Solve. Remember to state any exclusions. **12 points**

$$\frac{5}{x+3}+\frac{2}{x-1}=$$
$$\frac{5(x-1)}{(x+3)(x-1)}+\frac{2(x+3)}{(x+3)(x-1)}=$$
$$\frac{5x-5}{x^2+2x-3}+\frac{2x+6}{x^2+2x-3}=$$
$$\frac{7x+1}{x^2+2x-3};x\neq-3,1$$

$$\frac{1}{2x+1}+\frac{2}{x-1}=$$
$$\frac{1(x-1)}{(2x+1)(x-1)}+\frac{2(2x+1)}{(2x+1)(x-1)}=$$
$$\frac{x-1}{2x^2-x-1}+\frac{4x+2}{2x^2-x-1}=$$
$$\frac{5x+1}{2x^2-x-1};x\neq-\tfrac{1}{2},1$$

$$\frac{x}{x+2}-\frac{x}{x-5}=$$
$$\frac{x(x-5)}{(x+2)(x-5)}-\frac{x(x+2)}{(x+2)(x-5)}=$$
$$\frac{x^2-5x}{x^2-3x-10}-\frac{x^2+2x}{x^2-3x-10}=$$
$$\frac{-7x}{x^2-3x-10};x\neq-2,5$$

$$\frac{3}{x+4}-\frac{4}{x+2}=$$
$$\frac{3(x+2)}{(x+4)(x+2)}-\frac{4(x+4)}{(x+4)(x+2)}=$$
$$\frac{3x+6}{(x+4)(x+2)}-\frac{4x+16}{(x+4)(x+2)}=$$
$$\frac{-x-10}{x^2+6x+8};x\neq-4,-2$$

④ Simplify the complex fractions. **9 points**

$\dfrac{\frac{1}{3}}{8}$ LCD = 3.

$$\frac{\cancel{3}\left(\frac{1}{\cancel{3}}\right)}{3(8)}=\frac{1}{24}$$

$\dfrac{\frac{5}{8}}{7}$ LCD = 8.

$$\frac{\cancel{8}\left(\frac{5}{\cancel{8}}\right)}{8(7)}=\frac{5}{56}$$

$\dfrac{\frac{4}{9}}{10}$ LCD = 9.

$$\frac{\cancel{9}\left(\frac{4}{\cancel{9}}\right)}{9(10)}=\frac{4}{90}=\frac{2}{45}$$

$\dfrac{5}{\frac{1}{6}}$ LCD = 6.

$$\frac{6(5)}{\cancel{6}\left(\frac{1}{\cancel{6}}\right)}=\frac{30}{1}=30$$

$\dfrac{8}{\frac{3}{7}}$ LCD = 7.

$$\frac{7(8)}{\cancel{7}\left(\frac{3}{\cancel{7}}\right)}=\frac{56}{3}$$

$\dfrac{6}{\frac{5}{11}}$ LCD = 11.

$$\frac{11(6)}{\cancel{11}\left(\frac{5}{\cancel{11}}\right)}=\frac{66}{5}$$

$\dfrac{\frac{3}{4}}{\frac{2}{5}}$ LCD = $5(4)=20$.

$$\frac{{}_5\cancel{20}\left(\frac{3}{\cancel{4}}\right)}{{}_4\cancel{20}\left(\frac{2}{\cancel{5}}\right)}=\frac{15}{8}$$

$\dfrac{\frac{1}{4}}{\frac{7}{8}}$ LCD = 8.

$$\frac{{}_2\cancel{8}\left(\frac{1}{\cancel{4}}\right)}{\cancel{8}\left(\frac{7}{\cancel{8}}\right)}=\frac{2}{7}$$

$\dfrac{\frac{2}{5}}{\frac{3}{8}}$ LCD = $5(8)=40$.

$$\frac{{}_8\cancel{40}\left(\frac{2}{\cancel{5}}\right)}{{}_5\cancel{40}\left(\frac{3}{\cancel{8}}\right)}=\frac{16}{15}$$

60 points total

It's College Test Prep Time!

1. Given (x, y) ♎ z is defined as $\frac{xy^z - x^z y}{xyz}$ for all nonzero numbers x, y, and z, what is the value of (4, 5) ♎ 3?

   A. $\frac{41}{3}$ Substitute the given values in the formula. $\frac{4(5)^3 - 4^3(5)}{4(5)(3)}$

   B. 3 $\frac{4(125)-64(5)}{20(3)} = \frac{500-320}{60} = \frac{180}{60} = 3$

   C. $\frac{1}{3}$

   D. 0

   E. $-\frac{13}{3}$

2. If a and b are both positive real numbers and $\frac{2a}{b} = c$, what is the value of $\frac{2a+3}{c+1}$?

   A. $\frac{2ab+3b}{2a+b}$ Substitute for c and simplify.

   B. $\frac{2a+3}{2a+1}$ $\frac{2a+3}{\frac{2a}{b}+1} = \frac{b(2a+3)}{b\left(\frac{2a}{b}+1\right)} = \frac{2ab+3b}{2a+b}$

   C. $\frac{3}{b+1}$

   D. $\frac{2ab+3b}{2a+1}$

   E. $\frac{a+3}{b+1}$

Exploring Math through... Ice Hockey

Ice hockey is a game that involves just about every facet of math imaginable. Without even thinking about it, players must do math calculations in their heads. Oftentimes, these calculations are done in a fraction of a second. Other calculations must be so precise that they cannot be rushed, and they may be computed more than once to ensure accuracy.

When a hockey rink is lined, a few thin layers of ice are put down first. The markings are then painted on the ice and topped with several more layers of ice, making the entire iced area about one inch thick. If even one measurement is done incorrectly, the entire rink will have to be redone because you cannot melt just a portion of the rink, and you cannot remove paint from ice without removing the ice.

During game play, players are tasked with the laws of physics and principles of geometry and trigonometry. It is said that the wall, also known as the boards, serves as extra men on a hockey team. This is especially true when the players have a working knowledge of the properties of angles. A key mathematical concept that applies to every game of ice hockey is that the angle of incidence equals the angle of reflection. As it applies to hockey, the angle the path of a puck forms with the wall at the moment the puck hits the wall is equal to the angle the path of the puck forms with the wall as it is leaving the wall.

Players must also consider speed and distance when determining how much force to use to hit the puck. Too much force can cause the puck to overshoot the target, and too little force could leave the puck in an undesired area. In either case, the player has just given the opponent an advantage.

There are numerous other ways in which math affects ice hockey including the angle the player's skates make with the ice to the degree to which a player bends his knees when controlling the puck. No matter what positions players have in the game, math will affect the way they play the game

Teaching Tips, Cont.

- When all students are finished taking the test, introduce *It's College Test Prep Time* from the student book. This page may be completed in class or assigned as homework.
- Have students read the Exploring Math feature for Lessons 91-100.

Assignments

- Complete *It's College Test Prep Time!*
- Read Exploring Math through... Ice Hockey

Alpha Omega Publications®
804 N. 2nd Ave. E.
Rock Rapids, IA 51246-1759
800-622-3070
www.aop.com

JMT080 - June '12 Printing
ISBN 978-0-7403-2554-0
9 780740 325540