You've covered quite a distance in your journey through our number system, from whole numbers through fractions, to decimals. Today's math mysteries all have to do with distance.

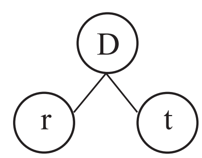
Distance problems have two parts: measuring how far something or someone traveled and measuring how fast something or someone traveled. When we travel, we usually travel faster during part of the trip and slower during other parts of the trip. We don't travel the exact same speed all through the trip.

Do you remember when you learned how to use addition and then division to find the **average** of several numbers? When we talk about the speed we travel, we are using the average speed across the trip. We can use this average as if we traveled at a fixed and steady pace. We call this using a **uniform rate**. In these problems **rate** compares one measurement, distance, with another measurement, time.

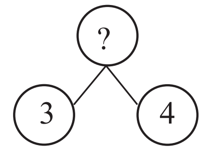
A common rate that you may have seen while riding in a car is a speed limit sign that lists the number of miles that can be traveled per hour. The driver must stay at or below that speed or rate of travel.

To find how far something or someone has traveled in a certain amount of time, we can use a set formula. A **formula** is a sentence using numbers and symbols to describe a certain relationship. In words we write it as "distance equals the rate times the time." In the formula, we use letters as abbreviations for the words: D stands for distance, r stands for rate, and t stands for time. In a formula, the times sign (x) is often dropped because it looks like the letter x. So whenever you see two letters next to each other, it means to multiply. The formula then becomes: **D=rt**.

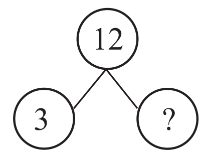
We can use the multiplication number bond to show how distance, rate, and time are related. The distance is the product of the rate multiplied by the time.



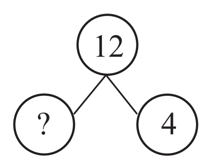
Do you remember that a multiplication bond shows a division fact as well as a multiplication fact? This number bond shows the fact 3 x 4. The answer is 12.



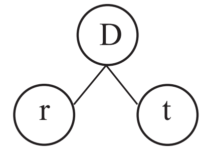
In this number bond, we know the answer (12) and the group size (3). We need to know the number of times we multiply that group size to get 12. To find that number we use the reverse operation of division. $12 \div 3 = 4$.



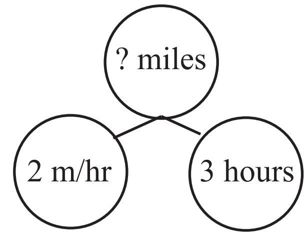
In this number bond, we also know the answer (12) and one of the numbers, but it is the number of times instead of the group size. To find the size of the group, we divide.
$12 \div 4 = 3$.



Like number bonds, a formula is useful because it helps us know what operation to use depending on which pieces of information we have.

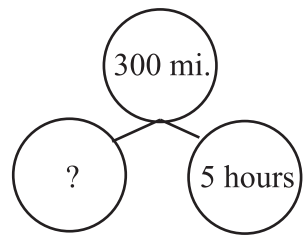


Usually when we have a distance problem, we know the speed the person or object is traveling (the rate) and the amount of time (time). We want to find the distance traveled. That is like a regular multiplication bond. Let's say Suzie is walking at a rate of 2 miles each hour. We write that as 2 mi/hr, where *mi* is the abbreviation for miles and *hr* is the abbreviation for hours. The slash stands for the word *per*, which is a synonym for *each*. Let's also say that Suzie walks for 3 hours. Now we can fill in the number bond to find the distance she walks. The distance bond shows us we need to multiply to find the answer. 2 x 3 = **6 miles**

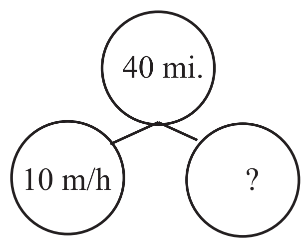


When we have a distance bond like this, **the units have to match up**. Notice that the distance is measured in miles and the time is measured in hours. That means the rate must be shown as miles per hour (mi/hr). By the way, on speed limit signs a different abbreviation is used; they show the number and then the letters *mph* (another way to abbreviate miles per hour).

Here's a story for you. *Tom drove 300 miles in five hours. How fast was he traveling?* This time we know the distance (D) and the time (t), and we need to know the rate (r). In this case, the distance bond looks like a division bond. We can see that we have to divide the distance by the time to find the rate. 300 ÷ 5 = **60 mi/hr**



Here's yet another story. Owen rode his four-wheeler 40 miles at 10 mi/hr. How long did he ride? This time we know the distance (D) and the rate (r), and we need to find the time (t). In this case, the distance bond looks like the other kind of division bond. We can see that we have to divide the distance by the rate. 40 ÷ 10 = **4 hours**



Together, the distance bonds and the formula tell us how to solve distance problems:

- ✓ to find the distance (D), multiply the rate (r) and the time (t)
- ✓ to find the rate (r), divide the distance (D) by the time (t)
- ✓ to find the time (t), divide the distance (D) by the rate (r)

Practice

Solve these distance mysteries using the distance bonds given in the lesson. Show all your work, including the units.

(1) Reagan traveled for 6 hours at 45 mi/hr. How far did he go?

(2) Anna and Jack hiked 12 miles in six hours. How fast were they walking?

(3) Quinn drove for 360 miles at 60 mi/hr. How long did it take her?

(4) Shane and Miles drove the four-wheelers at 18 mi/hr. for two hours. How far did they travel?

(5) The beetle traveled 85 feet at a rate of 5 feet per minute. How long did it take the beetle to travel that distance?

(6) Lucy rode her bike along the 24-mile path for 6 hours. How fast did she ride?

(7) The train from Jacksonville to Mason travels 55 mi/hr in 7 hours. How far apart are the two cities?

(8) Nathan swam 200 meters in 5 minutes. How fast did he swim?

UNIT SIX – DECIMALS **LESSON 169 – PROBLEM-SOLVING**

In the last lesson you learned the basic formula for finding distance: D = rt. You saw how a multiplication bond can be used to show the relationship of the different parts of the formula—distance, rate, and time. We called this new visual a distance bond.

You were able to solve three different types of distance problems using this distance bond. In each case, though, you were only dealing with one person, one group of people, or one object doing the traveling. Today we're going to look at what happens when we have two people or objects traveling in different ways. In effect, we will be solving two basic distance problems within one math mystery.

When comparing two distances, such as how far two people or two objects travel, there are other details we need to pay attention to besides the rate and the speed.

1. Do the two people or objects leave at the same time?
2. Do they go in the same direction or in opposite directions?
3. Do they travel at the same speed?
4. Do they leave from the same place?

Leaving from different places is a bit more involved, so we'll cover that in another level. We've dealt with the other issues in earlier lessons, but it's good to review them again.

Example #1: same place, same time, same direction, different speeds

Jesse and Janice leave the store at the same time, riding their bicycles down the road in the same direction. Jesse travels at 5 miles an hour and Janice travels at 7 miles an hour. How far apart will they be in 10 hours?

We can see there will be some distance between them because one is traveling a bit faster. We also know the amount of time that goes by: 10 hours. Before we can find how far apart they will be after that amount of time, we need to see how far each girl travels in the given time. From the formula, we know we need to multiply the miles they travel in one hour times the number of hours traveled. Since the two girls headed in the same direction, we can draw this picture of what happened.

Jesse: 5 x 10 = 50 miles

Janice: 7 x 10 = 70 miles

70 - 50 = **20 miles**

From the drawing and the numbers, we can easily see that Janice went farther. Since we need to find out how far apart they are, we need to find the difference. So we subtract. Final answer: the two girls will be **20 miles apart**.

Example #2: same place, same time, opposite directions, different speeds

What if Jesse (traveling at 5 mi/hr) and Janice (traveling at 7 mi/hr) leave in opposite directions and we want to know how far apart they will be after 8 hours?

We already know we have to find the distance each girl travels over the 8 hours. But do we subtract as we did in the example above? To see for sure, we can draw a picture of what is happening. They start at the same place but go in opposite directions. Now we can see that we have to add instead to find how far apart they are.

Jesse: 5 x 8 = 40 miles

Janice: 7 x 8 = 56 miles

40 + 56 = **96 miles**

Our final answer: the two girls will be **96 miles apart**.

Example #3: same place, different time, same direction, different speeds

A bus leaves the station at noon and averages 55 miles an hour (55 mi/hr). At 1 pm, another bus leaves the same station along the same road as the first bus. The second bus averages 65 miles an hour (65 mi/hr). How long will it take the second bus to catch up to the first bus?

This time we not only have different speeds, we have different times. And timing is everything. We don't want to jump right into the problem to figure out whether we add or subtract. We need to stop to think about what must happen in order for the second bus to catch up.

First, we have to think about the difference in their speeds. Both buses are traveling a certain amount of miles in one hour. In one hour bus 1 travels 55 miles. In one hour bus 2 travels 65 miles. So we subtract to find how much farther the second bus will travel each hour:

$$\begin{array}{r} 65 \text{ miles} \\ -\ 55 \text{ miles} \\ \hline 10 \text{ miles} \end{array}$$

Then we have to think about the head start the first bus had. It left at noon or 12 pm, and the second bus left at 1 pm, so the first bus had a one-hour head start. In that one hour, bus 1 traveled 55 miles. This means it had a 55-mile head start. We also know that because of the difference in their speeds, bus 2 goes an extra 10 miles each hour. So with each hour that passes, bus #2 will make up 10 of the 55-mile head start that bus 1 had. That means we need to know how many "groups" of the 10 miles there are in that head start distance. So we divide.

$$\begin{array}{r} 5 \\ 10\overline{)55} \\ \underline{50} \\ 5 \end{array}$$

When we divide 10 into 55, we can see that we are going to have a remainder. How do we explain the reminder? We don't, at least not with whole numbers only. Here's an example of why other types of numbers were added to our number system.

$$\begin{array}{r} 5.5 \\ 10\overline{)55.\mathbf{0}} \\ \underline{50} \\ 5\,0 \\ \underline{5\,0} \\ 0 \end{array}$$

We can use what we know about equivalent decimals to find the answer. We know that 55 and 55.0 show the same amount. So we can add the decimal point and a zero to the dividend so we can keep dividing. When we bring down the added zero, we can divide 10 into 50 five times. We put the decimal point in the quotient and put the 5 above the zero in the TENTHS place. This time when we subtract, there is no remainder.

Our final answer is the second bus will catch up **5.5 hours**.

Practice

Solve these distance mysteries using the formula given in the lesson. Show all your work, including the units and the drawings.

(1) James and John head out from the same park, but go in opposite directions. James travels at 2 miles an hour and John travels 4 miles an hour. How far apart will they be at the end of 3 hours?

(2) What if James and John head out from the park in the same direction, and we want to know how far apart they will be after 6 hours?

(3) Frank and Fred are truck drivers for the Make Things Move Trucking Company. Both Frank and Fred head out from the company headquarters down Highway 66. Frank is hauling heavy equipment, so he travels at 55 miles an hour. Fred has an empty truck because he is going to pick up some parts to deliver to a factory. He is traveling at 70 miles per hour. How much farther will Fred be after 12 hours?

(4) Jacob did an experiment to see which was faster, a snail or a slug. In order for the experiment to work, he knew he had to measure both in the same amount of time and in the same unit of distance. Since both travel very slowly, he measured the distance in feet and timed it for one minute. They left from the same place at the same time, in the same direction. He then multiplied by 60 minutes to find out how fast each would travel in an hour. The snail averaged 32 feet per hour and the slug averaged 12 feet per hour. At that rate, how far apart will they be after 24 hours?

(5) A person walking quickly can travel 5 miles in an hour. A turtle can travel 3 miles an hour. If a turtle and a person leave at the same time and walk in opposite directions, how far apart will they be after 2 hours?

(6) One man travels at a rate of 15 miles in 5 days. Another man travels at the rate of 20 miles in 2 days. How much farther will the one man travel than the other in one day?

(7) Two boys are riding the same bike path. One boy goes 10 miles while another goes 7 miles. When the first boy has gone 90 miles, how far has the second boy gone?

(8) Two trucks are hauling cars from the factory to the dealer. The distance is 240 miles. The first hauler makes the trip in 6 hours. The second hauler runs into lots of traffic and makes the same trip in 8 hours. How much faster was the average speed of the first hauler than the average speed of the other hauler?

Challenge Question

(9) The Sharks baseball team is going to a sports camp. They are taking two cars. The first car leaves at 8 am and travels 45 miles an hour. The second car has to wait two hours for one of the team members. They travel the same route at 55 miles an hour. How long will it take the second car to catch up?