

Comparing Fractions with the Rule of Four

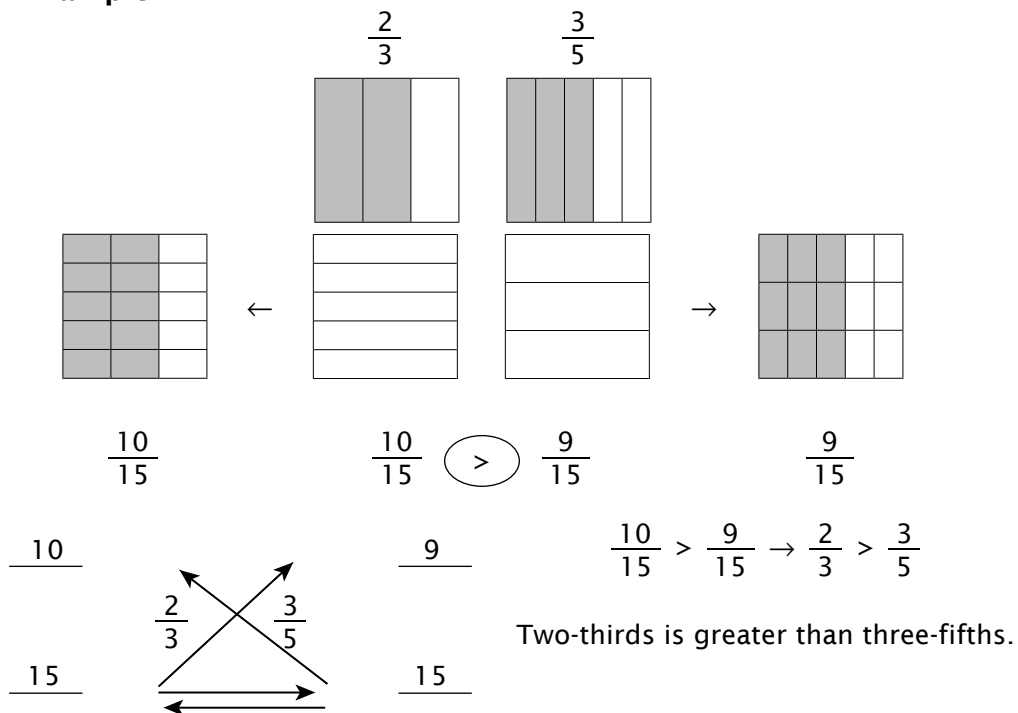
We know that the symbol “=” means “equals” or “is the same as.” In *Beta*, we introduced symbols that are used to show that one value is larger or smaller than another. As we read an equation from left to right, the symbol “>” means “is greater than,” and the symbol “<” means “is less than.” These symbols are called *inequality* symbols, and they are used in number sentences called *inequalities*. For example, “nine is greater than three” is written as $9 > 3$. “Three is less than nine” is written as $3 < 9$. Inequality symbols may also be used to compare fractions.

To remember which symbol is which, some say the open, or large, end of the symbol always points to the larger number, and the small end points to the smaller number. Some students think of the symbol as a hungry alligator with his mouth open, always trying to eat the larger number.

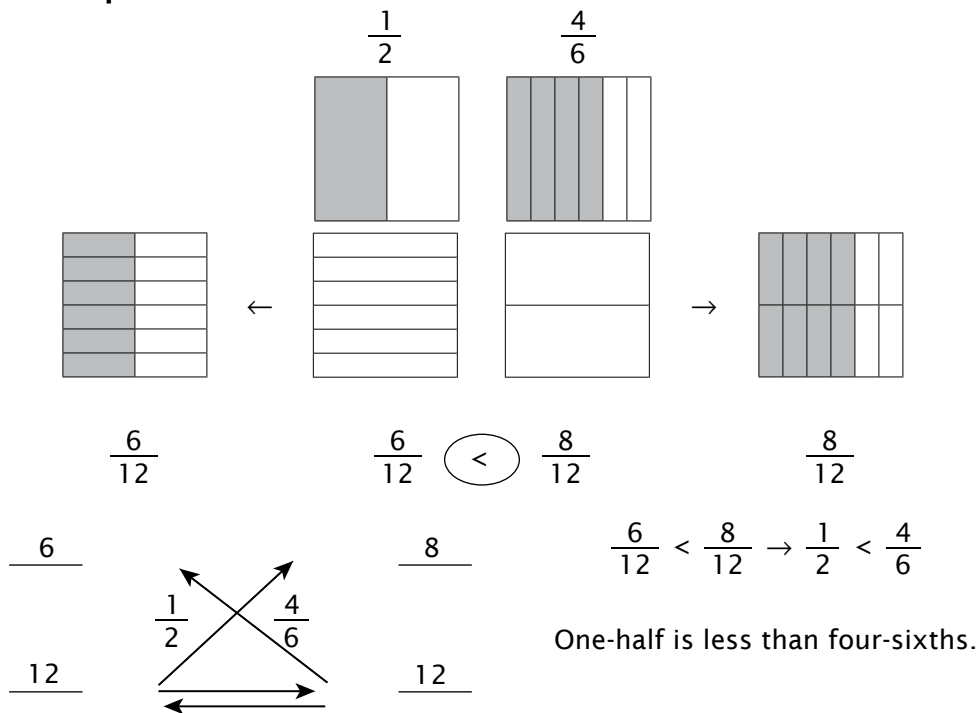
Inequalities with fractions fall into one of two categories: both fractions have the same denominator, or each fraction has a different denominator. If the denominators of two fractions are the same, you simply compare the numerators. It is easy to see that three-fourths is greater than one-fourth ($\frac{3}{4} > \frac{1}{4}$).

Comparing $\frac{2}{3}$ and $\frac{3}{5}$ is more difficult. Which is larger? When we added two fractions with different denominators, we first found common denominators (same kind), and then combined the numerators. In the same way, it is easier to compare fractions with different denominators when we first rewrite them with a common denominator. Once the fractions have the same denominator, we compare the numerators. Study the examples on the next page.

Example 1



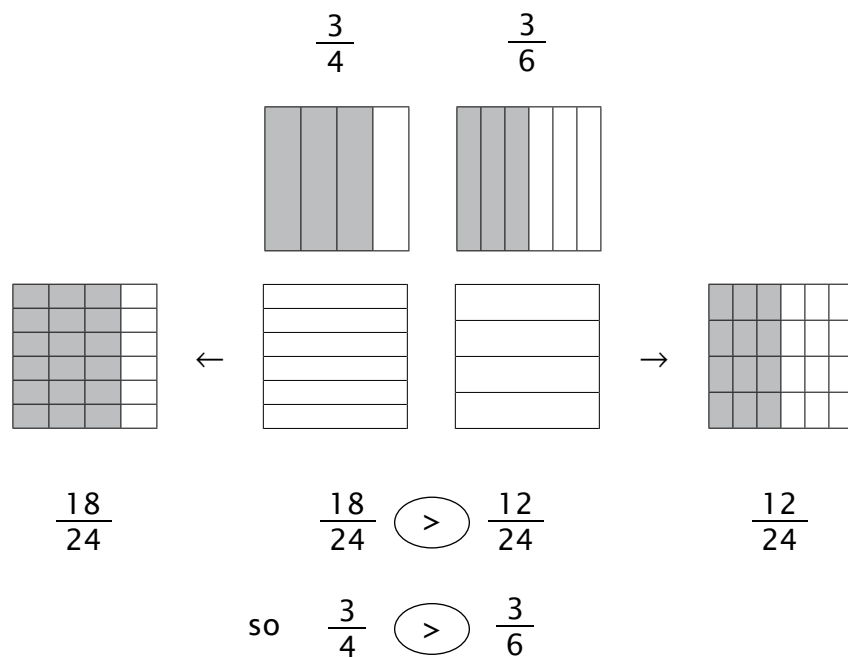
Example 2



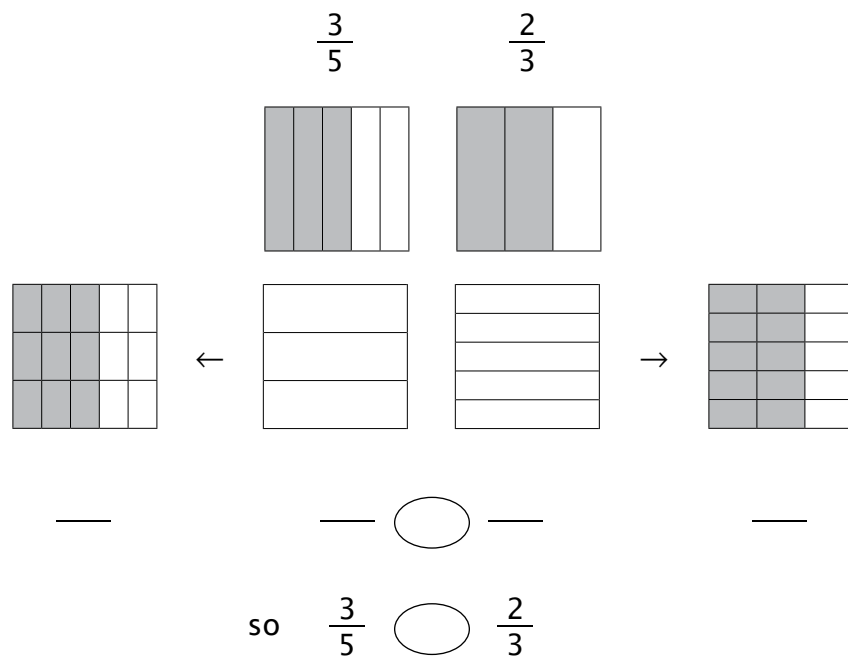
LESSON PRACTICE

Build the problems and draw lines to show what you did, and then compare the fractions. Write $>$, $<$, or $=$ in the ovals. The first one is done for you.

1.



2.



Build, and then compare using the rule of four. The first one is done for you.

3.

| | | | | | | | | | | |
|-------------------|---------------|---------------|-------------------|---------------|--------------|---------------|----|---------------|--------------|---------------|
| $\frac{4}{\quad}$ | $\frac{1}{2}$ | $\frac{2}{4}$ | $\frac{4}{\quad}$ | $\frac{4}{8}$ | $\bigcirc =$ | $\frac{4}{8}$ | so | $\frac{1}{2}$ | $\bigcirc =$ | $\frac{2}{4}$ |
| $\frac{\quad}{8}$ | | | $\frac{\quad}{8}$ | | | | | | | |

4.

| | | | | | | | | | | |
|-----------------------|---------------|---------------|-----------------------|-----------------------|------------|---------------|----|-----------------------|------------|---------------|
| $\frac{\quad}{\quad}$ | $\frac{3}{5}$ | $\frac{4}{6}$ | $\frac{\quad}{\quad}$ | $\frac{\quad}{\quad}$ | \bigcirc | $\frac{3}{5}$ | so | $\frac{\quad}{\quad}$ | \bigcirc | $\frac{4}{6}$ |
| $\frac{\quad}{\quad}$ | | | $\frac{\quad}{\quad}$ | | | | | | | |

5.

| | | | | | | | | | | |
|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|------------|---------------|----|-----------------------|------------|---------------|
| $\frac{\quad}{\quad}$ | $\frac{\quad}{\quad}$ | $\frac{\quad}{\quad}$ | $\frac{\quad}{\quad}$ | $\frac{\quad}{\quad}$ | \bigcirc | $\frac{2}{3}$ | so | $\frac{\quad}{\quad}$ | \bigcirc | $\frac{3}{4}$ |
| $\frac{\quad}{\quad}$ | | | | | | | | | | |

6.

| | | | | | | | | | | |
|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|------------|---------------|----|-----------------------|------------|---------------|
| $\frac{\quad}{\quad}$ | $\frac{\quad}{\quad}$ | $\frac{\quad}{\quad}$ | $\frac{\quad}{\quad}$ | $\frac{\quad}{\quad}$ | \bigcirc | $\frac{2}{5}$ | so | $\frac{\quad}{\quad}$ | \bigcirc | $\frac{1}{3}$ |
| $\frac{\quad}{\quad}$ | | | | | | | | | | |

LESSON PRACTICE

Compare using the rule of four. Fill in the blanks and ovals. All of these can be built with the fraction overlays. The first one is done for you.

1.

 $\frac{3}{6}$ $\frac{2}{3}$

$$\frac{3}{6} \bigcirc \frac{2}{3} \text{ so } \frac{1}{2} \bigcirc \frac{1}{3}$$

one-half is greater than one-third

2.

$$\frac{2}{3} \bigcirc \frac{5}{6}$$

two-thirds is _____ five-sixths

3.

$$\frac{2}{3} \bigcirc \frac{3}{6}$$

two-thirds is _____ three-sixths



4.

 $\frac{1}{2}$ $\frac{2}{5}$

$$\frac{1}{2} \bigcirc \frac{2}{5}$$



one-half is _____ two-fifths

5.

—  — so $\frac{1}{3}$  $\frac{2}{6}$

one-third is _____ to two-sixths

6.

—  — so $\frac{2}{4}$  $\frac{1}{5}$

two-fourths is _____ one-fifth

7. One-half of the students voted for Trisha as class president, while two-fifths of them voted for Tom. Which person ended up with the most votes?

8. Mike ran $\frac{2}{6}$ of a mile and Donald ran $\frac{2}{3}$ of a mile. Which person ran the greater distance?

LESSON PRACTICE

7C

Compare using the rule of four. Fill in the blanks and ovals. Not all of these can be built with the fraction overlays.

1.

— — so $\frac{4}{5}$ $\frac{4}{6}$

four-fifths is _____ four-sixths

2.

— — so $\frac{4}{6}$ $\frac{2}{2}$

four-sixths is _____ two halves

3.

— — so $\frac{3}{8}$ $\frac{4}{7}$

three-eighths is _____ four-sevenths

4.

— — so $\frac{2}{9}$ $\frac{1}{3}$

two-ninths is _____ one-third

Use the rule of four to compare the fractions, and then write the correct symbol in the oval.

5. $\frac{3}{4}$ $\frac{5}{6}$

6. $\frac{3}{6}$ $\frac{2}{4}$

7. $\frac{1}{2}$ $\frac{3}{10}$

8. $\frac{4}{5}$ $\frac{6}{7}$

9. Shirley ate one-fourth of a pizza and Andrea ate one-sixth of a pizza. Which girl ate more pizza?
10. Jeremiah had $\frac{3}{5}$ of an acre of land on the east side of the road and $\frac{7}{12}$ of an acre on the west side. Which was the larger piece of land?

SYSTEMATIC REVIEW

7D

Use the rule of four to compare the fractions, and then write the correct symbol in the oval.

1. $\frac{1}{3}$ ○ $\frac{3}{6}$

2. $\frac{5}{8}$ ○ $\frac{1}{2}$

3. $\frac{3}{12}$ ○ $\frac{1}{4}$

Add or subtract.

4. $\frac{2}{4} + \frac{1}{6} = \underline{\hspace{1cm}}$

5. $\frac{6}{10} - \frac{3}{8} = \underline{\hspace{1cm}}$

6. $\frac{2}{9} + \frac{5}{7} = \underline{\hspace{1cm}}$

Fill in the missing numbers in the numerators or denominators to make equivalent fractions.

7. $\frac{6}{8} = \frac{\hspace{1cm}}{16} = \frac{\hspace{1cm}}{24} = \frac{24}{\hspace{1cm}}$

Solve.

8. $\frac{1}{2}$ of 6 = $\underline{\hspace{1cm}}$

9. $\frac{3}{6}$ of 42 = $\underline{\hspace{1cm}}$

10. $\frac{3}{8}$ of 24 = $\underline{\hspace{1cm}}$



QUICK REVIEW

When the final remainder of a division problem doesn't divide evenly, you may divide it into a fraction by writing it over the divisor. Add the resulting fraction to your answer to make a mixed number. Look carefully at the one that is done for you.

Divide fully. The first one is done for you.

$$11. \quad 4 \overline{) 26} \quad \frac{2}{4}$$

$$\begin{array}{r} 6 \\ 4 \overline{) 26} \\ \underline{24} \\ 2 \end{array}$$

$$12. \quad 5 \overline{) 23}$$

$$13. \quad 7 \overline{) 59}$$

14. Alaina had 17 yards of fabric. She divided it into four equal parts to make curtains. How many yards of fabric does she have for each curtain? Include a fraction in your answer if the problem does not divide evenly.
15. Brad has completed $\frac{2}{7}$ of the chores that needed to be done, and Penny has done $\frac{5}{8}$ of them. Which person has done the most chores? What part of the chores remains to be finished?
16. If Brad and Penny had a total of 56 chores to do, how many actual chores remain to be done? (See #15.)
17. One-fourth of a cup of brown sugar is needed for one recipe, and one-third of a cup is needed for another. How much brown sugar is needed in all?
18. During the first storm $\frac{1}{3}$ of an inch of rain fell. The second storm gave us $\frac{7}{8}$ of an inch of rain. How much more rain fell during the second storm than during the first?

SYSTEMATIC REVIEW

7E

Use the rule of four to compare the fractions, and then write the correct symbol in the oval.

1. $\frac{3}{5}$ ○ $\frac{1}{3}$

2. $\frac{2}{3}$ ○ $\frac{1}{6}$

3. $\frac{9}{10}$ ○ $\frac{7}{12}$

Add or subtract.

4. $\frac{1}{2} + \frac{2}{5} = \underline{\hspace{1cm}}$

5. $\frac{2}{4} - \frac{1}{3} = \underline{\hspace{1cm}}$

6. $\frac{3}{8} + \frac{3}{5} = \underline{\hspace{1cm}}$

Fill in the missing numbers in the numerators or denominators to make equivalent fractions.

7. $\frac{1}{10} = \underline{\hspace{1cm}} = \underline{\hspace{1cm}} = \frac{4}{\underline{\hspace{1cm}}}$

Solve.

8. $\frac{7}{8}$ of 32 = $\underline{\hspace{1cm}}$

9. $\frac{2}{7}$ of 21 = $\underline{\hspace{1cm}}$

10. $\frac{3}{4}$ of 20 = $\underline{\hspace{1cm}}$

Divide. Include a fraction in the answer if the problem does not divide evenly.

11. $6 \overline{) 32}$

12. $8 \overline{) 19}$

13. $5 \overline{) 48}$

Estimate, and then multiply to find the exact answer.

14.
$$\begin{array}{r} 21 \\ \times 16 \\ \hline \end{array}$$

15.
$$\begin{array}{r} 34 \\ \times 29 \\ \hline \end{array}$$

16.
$$\begin{array}{r} 75 \\ \times 12 \\ \hline \end{array}$$

17. One-sixth of the cars that Valerie saw on her vacation were red and one-seventh of them were blue. What part of the cars that she saw was either red or blue?
18. Luke's team won $\frac{4}{7}$ of the games they played this season. If they played 28 games, how many did they win?
19. Evan's rectangular lawn measures 8 yards by 10 yards. He planted a hedge along $\frac{1}{4}$ of the perimeter. How long was his hedge?
20. Last week's storm gave us one-half foot of snow. This week we had a storm that dropped three-eighths of a foot. Write a comparison of the two storms using $>$, $<$, or $=$.

SYSTEMATIC REVIEW

Use the rule of four to compare the fractions, and then write the correct symbol in the oval.

1. $\frac{5}{10}$ ○ $\frac{6}{12}$

2. $\frac{2}{7}$ ○ $\frac{3}{5}$

3. $\frac{1}{2}$ ○ $\frac{2}{3}$

Add or subtract.

4. $\frac{2}{3} + \frac{1}{5} = \underline{\hspace{1cm}}$

5. $\frac{4}{6} - \frac{1}{4} = \underline{\hspace{1cm}}$

6. $\frac{5}{6} + \frac{1}{9} = \underline{\hspace{1cm}}$

Fill in the missing numbers in the numerators or denominators to make equivalent fractions.

7. $\frac{3}{4} = \underline{\hspace{1cm}} = \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$

Solve.

8. $\frac{3}{5}$ of 10 = $\underline{\hspace{1cm}}$

9. $\frac{1}{4}$ of 12 = $\underline{\hspace{1cm}}$

10. $\frac{4}{6}$ of 24 = $\underline{\hspace{1cm}}$

Divide. Include a fraction in the answer if the problem does not divide evenly.

11. $3 \overline{) 13}$

12. $4 \overline{) 39}$

13. $9 \overline{) 58}$

Estimate, and then multiply to find the exact answer.

14.
$$\begin{array}{r} 64 \\ \times 51 \\ \hline \end{array}$$

15.
$$\begin{array}{r} 45 \\ \times 19 \\ \hline \end{array}$$

16.
$$\begin{array}{r} 82 \\ \times 37 \\ \hline \end{array}$$

17. What is the perimeter of a triangle with sides of 8 feet, 9 feet, and 10 feet?
18. Kiley answered $\frac{5}{6}$ of the test questions correctly, while Casey answered $\frac{4}{5}$ of the questions on the same test correctly. Write a comparison showing who got the larger fraction correct.
19. If there were 30 questions on the test in #18, how many questions did each girl answer correctly? Write another comparison using the actual number of questions correct. Does it agree with the comparison you wrote for #18?
20. Faith has finished $\frac{5}{8}$ of her chores, and Colleen has finished $\frac{3}{4}$ of hers. Write a comparison of the progress of the two girls in finishing chores.

APPLICATION & ENRICHMENT

7G

Decide whether you should add or subtract for each problem below and on the reverse side of the page. Follow the directions to put the correct letters in the blanks.

| | | | | | | |
|----------|----------|-----------|-----------|-----------|-----------|-----------|
| <u>1</u> | <u>2</u> | <u>3</u> | <u>4</u> | <u>5</u> | <u>6</u> | <u>7</u> |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| <u>8</u> | <u>9</u> | <u>10</u> | <u>11</u> | <u>12</u> | <u>13</u> | <u>14</u> |
| 8 | 9 | 10 | 11 | 12 | 13 | 14 |

1. A fraction of the job was done before lunch, and another fraction was finished after lunch. What part of the job has been finished?

For addition, put R in blanks 1 and 7.

For subtraction, put S in blank 1.

2. Jim ate one part of the pizza, and Bob ate another part. What part of the pizza has been eaten?

For addition, put E in blanks 2 and 9.

For subtraction, put T in blank 2.

3. Ava painted a fraction of the fence yesterday. When she finished work today, all of the fence was painted. What part of the fence was painted today?

For addition, put O in blanks 3 and 6.

For subtraction, put A in blanks 3 and 10.

4. One part of the books was biographies, and another part was poetry. What part of the books was either biography or poetry?

For addition, put D in blank 4.

For subtraction, put P in blank 4.

5. Yesterday it rained a fraction of an inch. Today it rained another fraction of an inch. How much more rain did we get today than we got yesterday?

For addition, put R in blank 7 and 8.

For subtraction, put F in blank 5.

6. Michael grew a fraction of a foot taller last year. Gabriel grew a different fraction of a foot taller the same year. What is the difference in the amount they grew?

For addition, put F in blank 5.

For subtraction, put O in blank 6.

7. Amy needs a fraction of a cup of honey for bread and a fraction of a cup for cookies. How much honey does she need altogether?

For addition, put M in blank 8.

For subtraction, put E in blanks 9 and 12.

8. Micah walked a fraction of a mile, and Brandon walked a larger fraction of a mile. How much farther did Brandon walk?

For addition, put V in blank 10.

For subtraction, put N in blanks 11 and 13.

9. Sammy ate a fraction of the cookies, and Tom ate another fraction of the cookies. What fraction tells the total part of the cookies that were eaten?

For addition, put I in blank 12.

For subtraction, put I in blank 11.

10. Kim did part of the job on Monday and another part on Tuesday. What part of the job has she completed?

For addition, put G in blank 14.

For subtraction, put W in blank 13 and leave blank 14 empty.

LESSON TEST

Use the rule of four to compare the fractions and write the correct symbol in the oval.

1. $\frac{1}{4}$ ○ $\frac{3}{7}$

2. $\frac{3}{8}$ ○ $\frac{1}{2}$

3. $\frac{4}{5}$ ○ $\frac{2}{9}$

4. $\frac{6}{11}$ ○ $\frac{2}{3}$

5. $\frac{5}{9}$ ○ $\frac{6}{7}$

6. $\frac{3}{4}$ ○ $\frac{6}{8}$

Add or subtract.

7. $\frac{1}{8} + \frac{4}{9} = \underline{\hspace{2cm}}$

8. $\frac{2}{3} - \frac{1}{5} = \underline{\hspace{2cm}}$

9. $\frac{4}{5} + \frac{1}{6} = \underline{\hspace{2cm}}$

Fill in the missing numbers in the numerators or denominators to make equivalent fractions.

10. $\frac{2}{5} = \underline{\hspace{1cm}} = \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$

Divide. Include a fraction in the answer if the problem does not divide evenly.

11. $2 \overline{) 19}$

12. $6 \overline{) 51}$

13. $5 \overline{) 39}$

Estimate, and then multiply to find the exact answer.

14. $\begin{array}{r} 39 \rightarrow \\ \times 24 \\ \hline \end{array}$

15. $\begin{array}{r} 72 \rightarrow \\ \times 15 \\ \hline \end{array}$

16. $\begin{array}{r} 68 \rightarrow \\ \times 43 \\ \hline \end{array}$

17. Five-eighths of the trees in my yard are maples. If there are 16 trees in my yard, how many are maple trees?
18. Christa ate $\frac{1}{5}$ of the chocolates in the box and Douglas ate $\frac{3}{10}$ of them. Write a comparison showing who ate the most chocolates.
19. If there were 20 chocolates in the box in #18, how many chocolates did each person eat? Write another comparison using the actual number of chocolates eaten. Does it agree with the comparison you wrote for #18?
20. Yesterday we got $\frac{2}{10}$ of an inch of rain. Today we had $\frac{8}{10}$ of an inch of rain. How much more rain did we get today?

13. $(70) \times (20) = (1,400)$
 $67 \times 18 = 1,206$
14. $(30) \times (40) = (1,200)$
 $32 \times 39 = 1,248$
15. $15 \times \$33 = \495
16. $80 \div 10 = 8$
 $8 \times 9 = 72$ posts
17. $\frac{3}{6} + \frac{1}{6} = \frac{4}{6}$ gone
 $\frac{6}{6} - \frac{4}{6} = \frac{2}{6}$ left
18. $\frac{1}{6} - \frac{1}{12} = \frac{12}{72} - \frac{6}{72} = \frac{6}{72}$ ft
19. 500
20. $6 + 6 + 6 + 6 = 24$ yd

Systematic Review 6F

1. $\frac{20}{30} + \frac{6}{30} = \frac{26}{30}$
2. $\frac{6}{12} + \frac{4}{12} = \frac{10}{12}$
3. $\frac{22}{77} + \frac{21}{77} = \frac{43}{77}$
4. $\frac{5}{10} - \frac{2}{10} = \frac{3}{10}$
5. $\frac{8}{12} - \frac{3}{12} = \frac{5}{12}$
6. $\frac{18}{72} - \frac{16}{72} = \frac{2}{72}$
7. $\frac{5}{6} = \frac{10}{12} = \frac{15}{18} = \frac{20}{24}$
8. $\frac{1}{10} = \frac{2}{20} = \frac{3}{30} = \frac{4}{40}$
9. $28 \div 7 = 4$;
 $4 \times 3 = 12$
10. $54 \div 6 = 9$; $9 \times 1 = 9$
11. $8 \div 8 = 1$; $1 \times 4 = 4$
12. $(70) \times (90) = (6,300)$
 $73 \times 89 = 6,497$
13. $(30) \times (90) = (2,700)$
 $26 \times 91 = 2,366$
14. $(50) \times (10) = (500)$
 $47 \times 11 = 517$
15. $18 \times 12 = 216$ eggs
16. $312 \times 3 = 936$ mi

17. $\frac{4}{9} + \frac{3}{6} = \frac{24}{54} + \frac{27}{54} = \frac{51}{54}$ of a loaf
18. $60 \div 6 = 10$
 $5 \times 10 = 50$ min
19. 300
20. $13 + 18 + 13 + 18 = 62$ in
 $2 \times 24 = 48$ in
 $62 \text{ in} > 48 \text{ in}$; no

Lesson Practice 7A

1. done
2. $\frac{9}{15} < \frac{10}{15}$ so $\frac{3}{5} < \frac{2}{3}$
3. done
4. $\frac{18}{30} < \frac{20}{30}$ so $\frac{3}{5} < \frac{4}{6}$
5. $\frac{8}{12} < \frac{9}{12}$ so $\frac{2}{3} < \frac{3}{4}$
6. $\frac{6}{15} > \frac{5}{15}$ so $\frac{2}{5} > \frac{1}{3}$

Lesson Practice 7B

1. done
2. $\frac{12}{18} < \frac{15}{18}$ so $\frac{2}{3} < \frac{5}{6}$
 less than
3. $\frac{12}{18} > \frac{9}{18}$ so $\frac{2}{3} > \frac{3}{6}$
 greater than
4. $\frac{5}{10} > \frac{4}{10}$ so $\frac{1}{2} > \frac{2}{5}$
 greater than
5. $\frac{6}{18} = \frac{6}{18}$ so $\frac{1}{3} = \frac{2}{6}$
 equal
6. $\frac{10}{20} > \frac{4}{20}$ so $\frac{2}{4} > \frac{1}{5}$
 greater than
7. $\frac{5}{10} > \frac{4}{10}$ so $\frac{1}{2} > \frac{2}{5}$
 Trisha got more votes.
8. $\frac{6}{18} < \frac{12}{18}$ so $\frac{2}{6} < \frac{2}{3}$
 Donald ran farther.