Activities for Learning, Inc.

RightStart™ Mathematics

by Joan A. Cotter, Ph.D.

Level D Lessons

For Home Educators

Special thanks to Sharalyn Colvin, who converted *RightStart™ Mathematics: Grade 3 Lessons* into *RightStart™ Mathematics: Level D For Home Educators*.

Note: Rather than use the designation, K-4, to indicate a grade, levels are used. Level A is kindergarten, Level B is first grade, and so forth.

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Home Educators

RIGHTSTART™ MATHEMATICS

by Joan A. Cotter, Ph.D.

The following are items needed to complete the RightStart™ Mathematics Level D Lessons:

STATUS	ITEM	CODE
REQUIRED	***Level D Lessons*** (Third Grade)	T-D
REQUIRED	***Level D Worksheets***	W-D
REQUIRED	***Math Card Games*** book	M4
REQUIRED; choice of abacus	Classic AL Abacus - 8-1/2" x 9-1/2" hardwood frame & beads	A-CL
	Standard AL Abacus - 7-1/2" x 9-1/2" plastic frame & beads	A-ST
	Junior AL Abacus - 5-1/4" x 6" plastic frame & beads	A-JR
RECOMMENDED	Place Value Cards	P
RECOMMENDED	Abacus Tiles	AT
REQUIRED	Thousand Cubes	AC
REQUIRED	Cards, Six Special Decks needed for Games	C
RECOMMENDED	Fraction Charts	F
REQUIRED	Basic Drawing Board Geometry Set	DS
REQUIRED	Colored Tiles, apx 200 in set	RH2
REQUIRED	Casio Calculator SL-450	R4
RECOMMENDED	Math Balance (Invicta)	R7
RECOMMENDED	Centimeter Cubes, 100 in set	R8
REQUIRED	4-in-1 Ruler	R10
REQUIRED	Folding Meter Stick	R15
RECOMMENDED	Plastic Coins	R5

Note: If a child has not previously worked with the AL abacus and is just starting RightStart™ Mathematics , *RightStart Mathematics Transition Lessons* are required before starting the *RightStart Mathematics Level D Lessons* .

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RightStart™ MATHEMATICS: OBJECTIVES FOR LEVEL D

Name _____

Teacher _____ **Year** _____

	1ST QTR	2ND QTR	3RD QTR	4TH QTR
Numeration				
Can skip count 2s to 10s the first ten multiples				
Can read and write numbers to 1 million	N/A			
Addition				
Can add 2-digit numbers mentally				
Can add several 4-digit numbers				
Subtraction				
Can subtract 2-digit numbers mentally				
Can subtract 4-digit numbers				
Knows subtraction facts				
Multiplication				
Understands multiplication				
Can multiply 4-digit numbers by 1-digit numbers	N/A			
Knows multiplication facts				
Understands square numbers	N/A	N/A		
Division				
Understands division as inverse of multiplication	N/A	N/A		
Can solve division story problems with remainders	N/A	N/A		
Fractions				
Can find 1/2 and 1/4 of various quantities				
Can show the meaning of 3/4 as three 1/4s	N/A			
Can solve problems, such as 3/4 + __ = 1				
Understands fractions as a type of division	N/A	N/A	N/A	
Calculator				
Can solve multi-step problems				
Can estimate the answer				
Money				
Can make change for amounts less than one dollar				
Problem Solving				
Can solve problems in more than one way				
Geometry				
Can construct an equilateral triangle with drawing tools	N/A			
Understands line symmetry	N/A	N/A	N/A	
Knows terms polygon, hexagon, octagon, right angle, etc.	N/A	N/A	N/A	
Measurement				
Can tell time to the minute				
Can measure in inches	N/A	N/A		
Can measure in centimeters	N/A	N/A		
Can find perimeter	N/A	N/A		
Can find area in square inches or square cm	N/A	N/A		
Can construct and read bar graphs	N/A	N/A	N/A	
Patterns				
Can recognize and continue a simple pattern	N/A	N/A		
Data and Probability				
Can collect and display data	N/A	N/A	N/A	
Understands that some events are more likely than others	N/A	N/A	N/A	

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How This Program Was Developed

We have been hearing for years that Japanese students do better than U.S. students in math in Japan. The Asian students are ahead by the middle of first grade. And the gap widens every year thereafter.

Many explanations have been given, including less diversity and a longer school year. Japanese students attend school 240 days a year.

A third explanation given is that the Asian public values and supports education more than we do. A first grade teacher has the same status as a university professor. If a student falls behind, the family, not the school, helps the child or hires a tutor. Students often attend after-school classes.

A fourth explanation involves the philosophy of learning. Asians and Europeans believe anyone can learn mathematics or even play the violin. It is not a matter of talent, but of good teaching and hard work.

Although these explanations are valid, I decided to take a careful look at how mathematics is taught in Japanese first grades. Japan has a national curriculum, so there is little variation among teachers.

I found some important differences. One of these is the way the Asians name their numbers. In English we count ten, eleven, twelve, thirteen, and so on, which doesn't give the child a clue about tens and ones. But in Asian languages, one counts by saying ten-1, ten-2, ten-3 for the teens, and 2-ten 1, 2-ten 2, and 2-ten 3 for the twenties.

Still another difference is their criteria for manipulatives. Americans think the more the better. Asians prefer very few, but insist that they be imaginable, that is, visualizable. That is one reason they do not use colored rods. You can imagine the one and the three, but try imagining a brown eight–the quantity eight, not the color. It can't be done without grouping.

Another important difference is the emphasis on non-counting strategies for computation. Japanese children are discouraged from counting; rather they are taught to see quantities in groups of fives and tens.

For example, when an American child wants to know 9 + 4, most likely the child will start with 9 and count up 4. In contrast, the Asian child will think that if he takes 1 from the 4 and puts it with the 9, then he will have 10 and 3, or 13.

Unfortunately, very few American first-graders at the end of the year even know that 10 + 3 is 13.

I decided to conduct research using some of these ideas in two similar first grade classrooms. The control group studied math in the traditional workbook-based manner. The other class used the lesson plans I developed. The children used that special number naming for three months.

They also used a special abacus I designed, based on fives and tens. I asked 5-year-old Stan how much is 11 + 6. Then I asked him how he knew. He replied, "I have the abacus in my mind."

The children were working with thousands by the sixth week. They figured out how to add four-place numbers on paper after learning how to do it on the abacus.

Every child in the experimental class, including those enrolled in special education classes, could add numbers like 9 + 4, by changing it to 10 + 3.

I asked the children to explain what the 6 and 2 mean in the number 26. Ninety-three percent of the children in the experimental group explained it correctly while only 50% of third graders did so in another study.

I gave the children some base ten rods (none of them had seen them before) that looked like ones and tens and asked them to make 48. Then I asked them to subtract 14. The children in the control group counted 14 ones, while the experimental class removed 1 ten and 4 ones. This indicated that they saw 14 as 1 ten and 4 ones and not as 14 ones. This view of numbers is vital to understanding algorithms, or procedures, for doing arithmetic.

I asked the experimental class to mentally add 64 + 20, which only 52% of nine-year-olds on the 1986 National test did correctly; 56% of those in the experimental class could do it.

Since children often confuse columns when taught traditionally, I wrote 2304 + 86 = horizontally and asked them to find the sum any way they liked. Fifty-six percent did so correctly, including one child who did it in his head.

This following year I revised the lesson plans and both first grade classes used these methods. I am delighted to report that on a national standardized test, both classes scored at the 98th percentile.

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Some General Thoughts on Teaching Mathematics

1. Only five percent of mathematics should be learned by rote; 95 percent should be understood.
2. Teaching with understanding depends upon building on what the child already knows. Teaching by rote does not care.
3. The role of the teacher is to encourage thinking by asking questions, not giving answers. Once you give an answer, thinking usually stops.
4. It is easier to understand a new model after you have made one yourself. For example, a child needs to construct graphs before attempting to read ready-made graphs.
5. Good manipulatives cause confusion at first. If the new manipulative makes perfect sense at first sight, it wasn't needed. Trying to understand and relating it to previous knowledge is what leads to greater learning, according to Richard Behr and others.
6. Lauren Resnick says, "Good mathematics learners expect to be able to make sense out of rules they are taught, and they apply some energy and time to the task of making sense. By contrast, those less adept in mathematics try to memorize and apply the rules that are taught, but do not attempt to relate these rules to what they know about mathematics at a more intuitive level."
7. According to Arthur Baroody, "Teaching mathematics is essentially a process of translating mathematics into a form children can comprehend, providing experiences that enable children to discover relationships and construct meanings, and creating opportunities to develop and exercise mathematical reasoning."
8. Mindy Holte puts learning the facts in proper perspective when she says, "In our concern about the memorization of math facts or solving problems, we must not forget that the root of mathematical study is the creation of mental pictures in the imagination and manipulating those images and relationships using the power of reason and logic."
9. The only students who like flash cards are those who don't need them.
10. Mathematics is not a solitary pursuit. According to Richard Skemp, solitary math on paper is like reading music, rather than listening to it; "Mathematics, like music, needs to be expressed in physical actions and human interactions before its symbols can evoke the silent patterns of mathematical ideas (like musical notes), simultaneous relationships (like harmonies) and expositions or proofs (like melodies)."
11. "More than most other school subjects, mathematics offers special opportunities for children to learn the power of thought as distinct from the power of authority. This is a very important lesson to learn, an essential step in the emergence of independent thinking." (A quote from *Everybody Counts)*

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12. Putting thoughts into words helps the learning process.
13. The difference between a novice and an expert is that an expert catches errors much more quickly. A violinist adjusts pitch so quickly that the audience does not hear it.
14. Europeans and Asians believe learning occurs not because of ability, but primarily because of effort. In the ability model of learning, errors are a sign of failure. In the effort model, errors are natural. In Japanese classrooms, the teachers discuss errors with the whole class.
15. For teaching vocabulary, be sure either the word or the concept is known. For example, if a child is familiar with six-sided figures, we can give him the word, hexagon. Or, if he has heard the word, multiply, we can tell him what it means. It is difficult to learn a new concept and the term simultaneously.
16. Introduce new concepts globally before details. This lets the children know where they are headed.
17. Informal mathematics should precede paper and pencil work. Long before a child learns how to add fractions with unlike denominators, she should be able to add one half and one fourth mentally.
18. Some pairs of concepts are easier to remember if one of them is thought of as dominant. Then the non-dominant concept is simply the other one. For example, if even is dominant over odd; an odd number is one that is not even.
19. Worksheets should also make the child think. Therefore, they should not be a large collection of similar exercises, but should present a variety.
20. In Japan students spend more time on fewer problems. Teachers do not concern themselves with attention spans as is done in the U.S.
21. In Japan the goal of the math lesson is that the student has understood a concept, not necessarily has done something (a worksheet).
22. The calendar should show the entire month, so the children can plan ahead. The days passed can be crossed out or the current day circled.
23. A real mathematical problem is one in which the procedures to find the answer or answers are not obvious. It is like a puzzle, needing trial and error. Emphasize the satisfaction of solving problems and like puzzles, of not giving away the solution to others.
24. Keep math time enjoyable. A person who dislikes math will avoid it. We store our emotional state along with what we've learned. A child under stress stops learning. If a lesson is too hard, end it and play a game. Try again another day.

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RightStart™ Mathematics

There are 13 major characteristics that make this research-based program effective.

1. Refers to quantities of up to 5 as a group; discourages counting individually.
2. Uses fingers and tally sticks to show quantities up to 10; teaches quantities 6 to 10 as 5 plus a quantity, for example 6 = 5 + 1.
3. Avoids counting procedures for finding sums and remainders. Teaches five- and ten-based strategies for the facts that are both visual and visualizable.
4. Employs games, not flash cards, for practice.
5. Once quantities 1 to 10 are known, proceeds to 10 as a unit. Uses the "math way" of naming numbers for several months; for example, "1 ten-1" (or "ten-1") for eleven, "1-ten 2" for twelve, "2-ten" for twenty, and "2-ten 5" for twenty-five.
6. Uses expanded notation (overlapping) place-value cards for recording tens and ones; the ones card is placed on the zero of the tens card. Encourages a child to read numbers starting at the left and not backward by starting at the ones column.
7. Proceeds rapidly to hundreds and thousands using manipulatives and place-value cards. Provides opportunities for trading between ones and tens, tens and hundreds, and hundreds and thousands with manipulatives.
8. Only after the above work, about the fourth month of first grade, introduces the traditional English names for quantities 20 to 99 and then 11 to 19.
9. Teaches mental computation. Investigates informal solutions, often through story problems, before learning procedures.
10. Teaches four-digit addition on the abacus, letting the child discover the paper and pencil algorithm. This occurs in Level B. Four-digit subtraction is mastered in Level C.
11. Introduces fractions with a linear visual model.
12. Approaches geometry through drawing boards and tools.
13. Teaches short division (where only the answer is written down) for single-digit divisors, before long division. Both are taught in Level E.

Some Pointers

Kindergarten. Most of the kindergarten lesson plans have two distinct topics, which can be taught on alternate days.

Transition Lessons. These lessons are designed for children starting Levels C, D, or E (grades 2, 3, or 4) who have not been doing RightStart™ Mathematics previously. The lessons need to be studied before the regular manual, except where noted. The manual tells which lessons to use for the particular grade.

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Counting. Counting needs to be discouraged because it is slow and inaccurate. It also interferes with understanding quantity and learning place-value.

Warm-up. The warm-up time is the time for quick review, memory work, and sometimes an introduction to the day's topics. The drawing board makes an ideal slate for quick responses.

Place value. In order to understand addition algorithms, place-value knowledge is essential. From the very beginning, the children are helped to see quantities grouped in fives and tens. Children can understand place value in first grade and even in kindergarten when it is approached as it is in this program.

Worksheets. The worksheets are designed to give the children a chance to think about and to practice the day's lesson. Some lessons, especially in the early grades, have no worksheet.

Games. Games, not worksheets or flash cards, are used for practice. They can be played as many times as necessary until memorization takes place. Games are as important to math as books are to reading.

Some games are incorporated in this manual. Extra games, found in the book, *Math Card Games,* are suggested in the Review and Practice lessons in Levels C to E. There are games for the child needing extra help, as well as for the advanced child.

Teaching. Establish with the children some indication when you want a quick response and when you want a more thoughtful response. Research shows that the quiet time for thoughtful response should be about three seconds. Avoid talking during this quiet time; resist the temptation to rephrase the question. This quiet time gives the slower child time to think. It also gives a quicker child time to think more deeply.

Encourage the child to develop perseverance. Avoid giving answers too quickly. Children tend to stop thinking once they hear the answer.

Help the children realize that it is their responsibility to ask questions when they do not understand. Do not settle for "I don't get it."

Number of lessons. It is not necessary that each lesson be done in one day. Sometimes two days may be more appropriate. However, do complete each manual before going to the next one.

Visualization. The ability to imagine or visualize is an important skill to develop in mathematics and other subjects as well. Often you are called upon to suggest to the children that they imagine a particular topic.

Questions. I really want to hear how this program is working. Please let me know any improvements and suggestions that you may have.

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Lesson 1

The Months of the Year

OBJECTIVES

1. To name, write, and compare the months of the year
2. To write dates several ways
3. To discuss reading the year

MATERIALS

Math journal (found at the end of the worksheets) or the Dry Erase drawing board
A book about calendars or seasons to be read to the child, optional

WARM-UP

Ask the child to recite the months in order and say "Thirty Days has September."

> **Note:** The warm-up activities have several purposes. They provide review and practice of concepts previously learned.

Thirty Days has September
Thirty days has September,
April, June, and November.
The rest have thirty-one to carry,
But only twenty-eight for February,
Except in leap year, that's the time
When February has twenty-nine.

ACTIVITIES

Days in a month. Ask, How many months are in a year? [12] Then ask the child to write the numbers 1 to 12 in a column in her math journal and to write the months in order in another column. You might also use this opportunity to discuss the spelling of the months, all of which are phonetic when pronounced clearly. Also ask her to write the number of days in the month in the third column as shown below.

> **Note:** Discussing spelling in math class can help the child integrate the various subjects.

1	January	31
2	February	28
3	March	31
4	April	30
5	May	31
6	June	30
7	July	31
8	August	31
9	September	30
10	October	31
11	November	30
12	December	31

The number of the month, the month, and the number of days in the month.

How many months have 30 days? [4] What are they? [April, June, September, and November] How many months have 31 days? [7] What are they? [January, March, May, July, August, October, and December] Have we named all the months? [no, not February] Ask her if she can explain about February. [28 days until leap year when it has 29 days]

Writing dates. Ask the child to write today's date in her journal or on the drawing board. Discuss writing the month first, followed by the day, and the year. See the example on the next page.

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September 1, 2008	Sep. 1, 2008	9-1-08

Various ways to write the date.

Also ask her to write the date by using abbreviations. Explain that the abbreviation usually is the first three letters of the month. See the second example above.

Next explain that often we use a number for the months. Ask her to look at her list and find the number for September. [9] Demonstrate writing the date as shown above on the right. (Slashes, "/," are sometimes used, but may be confused with a 1.)

For practice in writing dates, ask the child to write the following dates all three ways, or choose other suitable dates:

1. The date for the first day of the next month
2. The date for the last day in September
3. The date for the next day
4. The date for one week from today
5. The date of her last birthday
6. The date for the next New Year's Day

Reading the year. Write 1900 and ask the child for two ways to read the number. [1 thousand 9 hundred, or 19 hundred] Ask how she knows it is 19 hundred. [2 zeroes after the 19] Write the year, for example, 1998, and explain that it really is 19 *hundred* 98, but most of the time we skip the word *hundred* and just say *nineteen ninety-eight.* The year 2000 is pronounced *two thousand* and 2005 is *two thousand five.*

Point to the year and ask the child to read it as a number. [for example, 1998 is 1 thousand 9 hundred ninety-eight]

Tell her that people have used other ways of naming days and years. Ask her what a year is. [the time that the earth takes to go around the sun] Read, if available, a book about calendars or the seasons.

ENRICHMENT Tell the child that in some countries, like Sweden and Germany, calendars begin on Monday, not Sunday. Also in Germany, the numbers go *down,* not across.

Also explain that in other countries, including Europe, and sometimes in the United States, people write the day of the month before the month as shown below.

1 September 2008	1 Sep 2008	1-9-08

The way Europeans write the date.

That makes sense to them because they write in order, from the smallest unit to the largest: day, month, and year.

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Lesson 2

Calendar for One Year

OBJECTIVES

1. To use a calendar to gain information
2. To solve calendar problems

MATERIALS

Math journal
A calendar (immediately preceding Worksheet 1-1)

WARM-UP

Ask the child to count by 2s to 20, 5s to 50, and 10s to 100.

Ask the child to recite the months in order and to sing Thirty Days has September."

Thirty Days has September
Thirty days has September,
April, June, and November.
The rest have thirty-one to carry,
But only twenty-eight for February,
Except in leap year, that's the time
When February has twenty-nine.

Ask the child to say the days of the week.

ACTIVITIES

Introducing the year's calendar. Ask the child to find the current month on the calendar. Give him time to study it. Ask him to circle today's date. Also ask him to find other dates of importance to him, such as his birthday, his family members' birthdays, and holidays.

Note: In RightStart first grade, the child made calendars, so calendars are not new.

Ask him what *S M T W T F S* means. [the days of the week] Ask him to write the days of the week. You might want to talk about the spelling. Tell him that in English, the months always start with a capital letter, but that is not true in all languages.

Pronouncing the last syllable as *day,* rather than *dee* helps in spelling. Sunday and Friday are simple. In Monday the first vowel sound is spelled with an *o,* the same as in *month.* In Thursday and Saturday, the "er" sound is spelled with "ur." In Tuesday the vowel sound is made with "ue." Wednesday has two silent letters, the first *d,* and the second *e.*

Ask if any months are the same. [In non-leap years, January and October are identical; also several pairs start on the same day: February & March & November, April & July, and September & December. In leap years, January & July are identical with April also starting the same day.]

In leap years pairs starting on the same day include February & August, March & November, September & December and January & April & July.

Problem 1. What day of the week is New Year's Day of the next year? How do you know? [It is the next day after December 31st.]

Problem 2. Tell the child that Thanksgiving in the United States is the fourth Thursday in November; ask him to find the date.

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Then tell him that in Canada, Thanksgiving is the second Monday in October. What is that date?

Problem 3. Tell the child the scouts (or other club) meet on the first Monday of the month. How many times will the club meet in the year? [12] Ask him to list all the dates for the year. Encourage him to use all numbers for the dates.

Problem 4. Tell the child that another club decided to meet on the fourth Tuesday of the month. How many times will the club meet in the year? [12] Ask him to write the dates.

Problem 5. Next tell the child that another club decided to meet on the fifth Wednesday of the month. How many times will the club meet in the year? [It varies.] Ask him to write the dates.

Problem 6. What is the date one week after July 4th [July 11th], What is the date two weeks after July 4th? [July 18th] How could you tell without looking at the calendar? [add 7 or 14]

Problem 7. Tell the child that Memorial Day is the last Monday in May and Labor Day is the first Monday in September. You might tell him that many people think that Memorial Day is the beginning of summer and Labor Day is the end of summer. You could also ask what the holidays mean to him. The problem is: How many days are between Memorial Day and Labor Day? [98 days when Memorial Day falls on May 26 to 31 and 105 days when Memorial Day falls on May 25]

Note: The calendar will be used in the next lesson.

When he has finished, ask him to show his solution and explain how he arrived at that solution. Ask if he could solve it another way. This problem, for example, can be solved by counting the days, or by counting the weeks by 7s, or by adding the number of days in June, July, and August and then adding the extra days.

ENRICHMENT

Problem 8: golden birthdays. Explain that some people celebrate golden birthdays. A golden birthday happens when a person's age is the same as the day of the month as their birthday. For example, a person whose birthday is September 9 has a golden birthday on their 9th birthday.

At what age will a baby born on July 2 have a golden birthday? [age 2] If the baby was born July 2 of this year, when will the baby have a golden birthday? [2 years] What year will a person born February 22, 1990, have a golden birthday? [1990 + 22 = 2012] Encourage the child to solve it mentally. [1990 + 10 = 2000 +12 = 2012] Ask if he has already had his golden birthday. Ask him to find the year of his golden birthday.

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Lesson 3

Calendars for the Following Years

OBJECTIVES

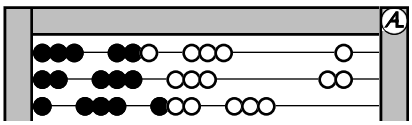
1. To become aware of similarities and differences of calendars for two consecutive years
2. To solve more calendar problems

MATERIALS

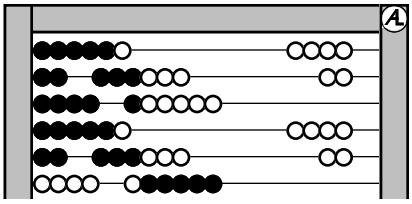
Math journal
AL abacus, optional
2 calendars, current year and the following year (immediately preceding Worksheet 1-1)
Worksheet 1-2, "Calendar Problems"
Worksheet 1-1, "Reviewing Skip Counting"

WARM-UP

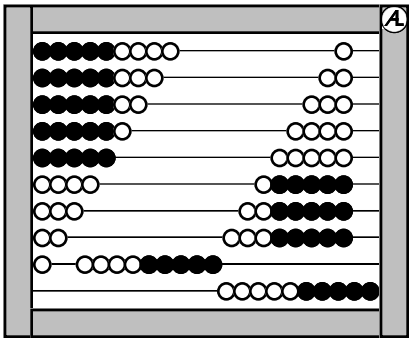
Ask the child to count by 3s to 30, 6s to 60, and 9s to 90. If desired, ask him to move the beads on the abacus as he counts. See below. The 6s are every other 3; combine 2 groups of 3s to make the 6s. The 9s are every third 3.



Counting by 3s.



Counting by 6s.



Counting by 9s.

Ask the child to recite "Thirty Days has September" (Lesson 2).

Ask the child to recite the months in order and to say the number of days it has. You might want her to write this information also.

ACTIVITIES

The next year's calendar. Ask the child to find the next year's calendar. How are they the same? [Each month has the same number of days, except possibly February.] How are they different? [The months start on a different day.] Ask him the following or similar questions:

1. Is either year a leap year?
2. Can 2 years in a row be leap years? [No, we have leap years only every 4 years.]
3. What day of the week is New Year's Day in both years?
4. What day of the week is July 4th in both years?
5. Find today's date in the next year. What happens to the day of the week?
6. What days are her birthdays in both years?
7. Ask her to mark the school days, or her vacation days.
8. What day of the week is Thanksgiving this year? Next year?

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[Thursday] What is this year's date? Next year's date? How does it change? What would happen in the next several years? Remind them that it must be the fourth Thursday of November.

9. What day of the week is Thanksgiving in this year? What is the date? How does that compare to this year's date?

Problem solving. To solve a problem, ask the child to read it carefully, at least twice. Next she needs to be sure she knows what is being asked. Give her three minutes of uninterrupted time to work on it. Then she can discuss it if she wants. Be sure she can explain her solution.

Problem 1. How many days are left in this year? Find the answer without counting.

Problem 2. How many weeks are in a year? [52 weeks and 1 or 2 days left over] How many weeks are left in this year? If each month had exactly 4 weeks, like February does most of the time, how many months would be in a year? [13, 40 weeks would be 10 months and 12 more weeks would be 3 more months]

Problem 3. How many days from your birthday this year to your birthday the next year? [365, or 366]

Note: The child will need this worksheet in future lessons.

Problem 4. If a baby is 3 months old, how many days ago was it born? [around 90] Discuss that *3 months* often means about 3 months.

Worksheet 1-2. In second grade the children worked extensively with skip counting. This worksheet asks for skip counting in a format that emphasizes the patterns. They are given below.

Multiples of 2

2	**4**	**6**	**8**	**10**
12	**14**	**16**	**18**	**20**

Multiples of 3

3	**6**	**9**
12	**15**	**18**
21	**24**	**27**
30		

Multiples of 4

4	**8**	**12**	**16**	**20**
24	**28**	**32**	**36**	**40**

Note: Children having difficulties with these skip counting patterns should play Skip Counting Memory (P2) found in *Math Card Games* by Joan A. Cotter.

Multiples of 7

7	**14**	**21**
28	**35**	**42**
49	**56**	**63**
70		

Multiples of 6

6	**12**	**18**	**24**	**30**
36	**42**	**48**	**54**	**60**

Multiples of 8

8	**16**	**24**	**32**	**40**
48	**56**	**64**	**72**	**80**

Multiples of 5

5	**10**
15	**20**
25	**30**
35	**40**
45	**50**

Multiples of 9

9	**18**	**27**	**36**	**45**
90	**81**	**72**	**63**	**54**

Note: Nines are written with the second line in reverse to show the digits reversing.

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Name _____

Date _____

1. How many days are left in this year? Find the answer without counting.

2. How many weeks are in a year? How many weeks are left in this year? If each month had exactly 4 weeks, like February does most of the time, how many months would be in a year?

3. How many days from your birthday this year to your birthday the next year?

4. If a baby is 3 months old, how many days ago was it born?

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Name _____

Date _____

Write the skip counting patterns for 2.

_____ _____ _____ _____ _____

_____ _____ _____ _____ _____

Write the skip counting patterns for 4.

_____ _____ _____ _____ _____

_____ _____ _____ _____ _____

Write the skip counting patterns for 6.

_____ _____ _____ _____ _____

_____ _____ _____ _____ _____

Write the skip counting patterns for 8.

_____ _____ _____ _____ _____

_____ _____ _____ _____ _____

Write the skip counting patterns for 9.

_____ _____ _____ _____ _____

_____ _____ _____ _____ _____

Write the skip counting patterns for 5.

_____ _____

_____ _____

_____ _____

_____ _____

_____ _____

Write the skip counting patterns for 3.

_____ _____ _____

_____ _____ _____

_____ _____ _____

_____

Write the skip counting patterns for 7.

_____ _____ _____

_____ _____ _____

_____ _____ _____

_____

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Lesson 4

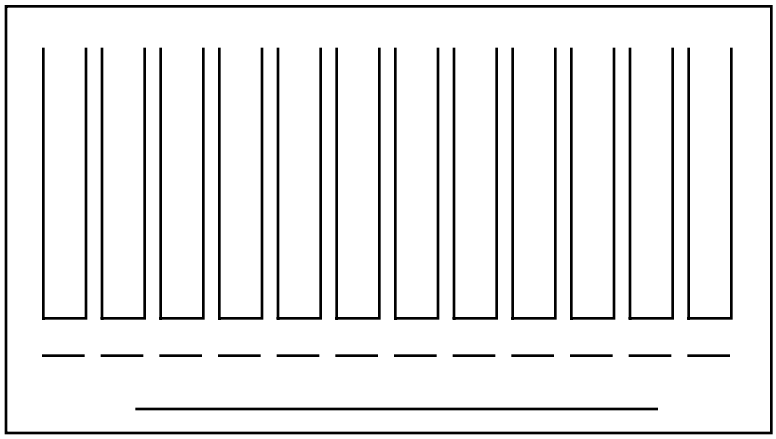
Birthday Graphs

OBJECTIVES

1. To construct a bar graph
2. To review an inch

MATERIALS

Paper, at least 36 inches wide and 24 inches long for making a large graph. Draw vertical columns 2 inches wide with ¾ inch between the columns as shown below. Leave space at the bottom for writing the months of the year and for a title. See below.

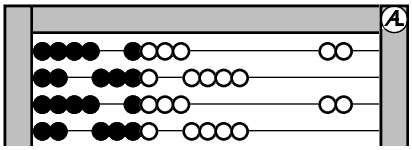


A large piece of paper for graphing the number of birthdays in each month.

Ruler
Crayons

WARM-UP

Say the months in order. Name the months with 31 days. [January, March, May, July, August, October, and December]



Counting by 4s.

Write the following and ask the child to write the answers:

35 + 7 = [42] 73 + 8 = [81] 55 + 10 = [65] 55 + 9 = [64]

Discuss how he found his answers. Discourage counting.

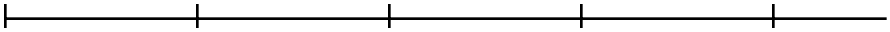
Ask him to count by 4s to 40. See the figure at the left.

ACTIVITIES

One inch on a ruler. Ask the child to draw on a piece of paper a line that he thinks is one inch long. Take out the ruler and discuss where he can find an inch on the ruler. Emphasize it is the space between the long marks near any 2 numbers. Ask if his lines are shorter or longer than an inch.

Note: Emphasizing the space between any two numbers may help the child realize what he is measuring.

Draw a horizontal line about 6 inches long and ask the child to mark it into inches. Note if the spaces between the marks look the same. [They should.]



A horizontal line marked in inches.

Repeat for a vertical line.

Building a graph. Ask the child which month has the greatest number of family and friends' birthdays and the least number of birthdays. Discuss if he could find out by using a graph. This could be a family or homeschool group project, which can be added to over the period of several days.

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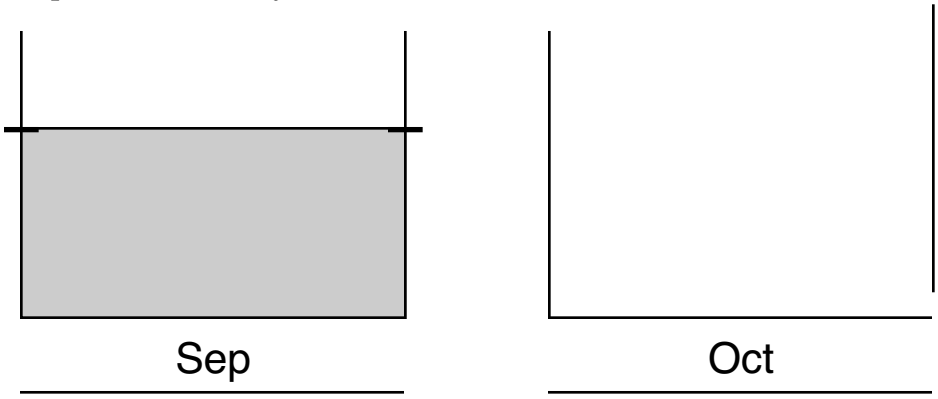
Show the child the prepared graph paper. Ask him to write the months of the year or their abbreviations below each column as shown below. Help him decide on a good title. Suggest he use one inch of space to show a birthday for a particular month. Then he can color in the rectangle.

Demonstrate with your own birthday month. Measure 1 inch along both sides, draw a short tick mark, and draw the horizontal line. A September birthday is shown below.

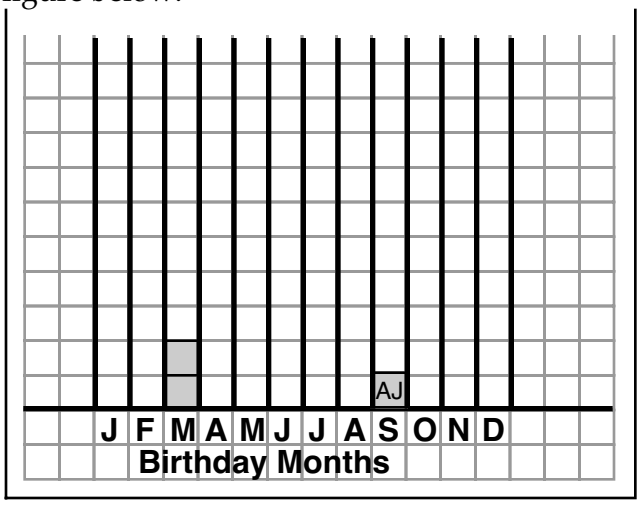


Demonstrating recording a birthday for the month of September.

Ask the child to enter his birthday in the appropriate month. Ask him to enter other birthdays. They could include his family. They could also include his friends. If he does not know the birthday months of some friends, tell him he can enter them the following days. Other family members may also add birthdays.

Individual graphs. Next the child can make an individual graph in his math journal. Starting on a clean page, he needs to draw a horizontal line 2 squares from the bottom and the vertical lines for the columns.

There is space to write only the first letter of the month below the columns. Ask him to record the birthday months of his family and friends. He might want to write the initials in the squares. See the figure below.



A birthday month graph done in a math journal.

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Lesson 9

Addition Practice

OBJECTIVE 1. To review adding multidigit numbers

MATERIALS Worksheet 3, "Addition Practice"

WARM-UP Ask the child what is 8 + 8. [16] What is 16 + 8? [24] Continue with 24 + 8 [32] to 72 + 8. [80] Then ask her to say the multiples of 8 (skip counting by 8s) and ask them to write them in her journal. What even and odd pattern do you see? [all even] Read only the ones; what is the pattern? [counting by 2s backward] Read only the tens; what is the pattern? [adding 1 each time in the row]

8 16 24 32 40
48 56 64 72 80

ACTIVITIES ***Reviewing tens, hundreds, thousands.*** Write

700

and ask the child to read it. [7 hundred] Ask how she knows it is 7 **hundred.** [2 zeroes after the 7] Annex another zero and ask the same questions. [7 thousand, because of the 3 zeroes]

Note: When a 0 is attached to the end of a number, the term is *annex*. If a zero were added, the number wouldn't change.

Ask what is 6 tens plus 6 tens. [12 tens] Ask the child to write the problem. Then ask for another name for 12 tens. [1 hundred twenty] (If she says 120 initially, ask how many tens it is. [12 tens])

Note: To continue to develop their mathematical abilities, the child MUST understand the procedures she uses.

```
  60
+ 60
 120
```

Adding several numbers. Write the following numbers and ask the child to copy them and add.

```
 326
 489
+ 216
```

Note: It is appropriate to use the word *carry*, according to the dictionary. However, "rename" is not in the dictionary and "regroup" often has a different meaning.

[1031] Ask her to show the process. Explain that you are going to ask lots of questions. For example, after she adds the ones [21], ask how she knows where to write the 2 and the 1. Ask why we can't put 21 in the ones place. Ask why we carry the 2 over to the tens. [We are trading 20 ones for 2 tens because 10 ones is 1 ten.]

```
  2
 326
 489
+ 216
   1
```

Ask the child to add the next column. Ask what the 2 and 8 are; ask if they are billions, bananas, or more ones. [tens] After they are added, ask why only the 3 is written with the tens. [13 tens is 1 hundred and 3 tens] Exclaim that the last time she put a 2 on top and this time a 1 is put on top.

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```
  12
 326
 489
+216
  31
```

Ask her to add the next column. Ask what we are adding this time. [hundreds] When the sum of 10 hundreds is reached, ask what that is. [1 thousand] It is not necessary to write the 1 in the thousands place. After she writes the 1 and the 0, ask, Why do we need a 0; isn't zero nothing? [It means no hundreds. Without the 0, it would be 1 hundred, not 1 thousand.]

```
  12
 326
 489
+216
1031
```

Now tell the child that Kevin (avoid the name of a friend or family member) added it a different way. Ask her to watch and to see if it is right. Explain that you are going to first add the hundreds, 3 hundred plus 4 hundred plus 2 hundred is 9 hundred. Write 9 hundred as shown below.

```
 326      326      326      326
 489      489      489      489
+216     +216     +216     +216
 900      900      900      900
          110      110      110
                    21       21
                             1031
```

Next add the tens in the same way, 2 tens plus 8 tens plus 1 ten equals 11 tens. Write it on the next line as shown above. Continue with the ones, writing 21 on the third line. Lastly draw a line and add all quantities as shown in the fourth column above.

Ask the child for her opinion, It is correct?

Worksheet. Tell the child on the top half of page she is to explain how to add the numbers shown to a second grader. She must explain all the steps, everything she does and why. The lower half is practice. The answers are shown below.

1760

1416	**8420**	**9841**	**9802**
1460	**5720**	**1633**	**9857**

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Name _____

Date _____

Add the following problem. Explain all you steps as if you were telling another person how and why you do what you do.

$$\begin{array}{r} 946 \\ 78 \\ +736 \\ \hline \end{array}$$

Add the following problems.

$$\begin{array}{r} 848 \\ +568 \\ \hline \end{array} \qquad \begin{array}{r} 4546 \\ +3874 \\ \hline \end{array} \qquad \begin{array}{r} 6043 \\ +3798 \\ \hline \end{array} \qquad \begin{array}{r} 5827 \\ +3975 \\ \hline \end{array}$$

$$\begin{array}{r} 541 \\ 876 \\ +\ 43 \\ \hline \end{array} \qquad \begin{array}{r} 862 \\ 4768 \\ +\ 90 \\ \hline \end{array} \qquad \begin{array}{r} 497 \\ 972 \\ +\ 164 \\ \hline \end{array} \qquad \begin{array}{r} 5647 \\ 2976 \\ +\ 1234 \\ \hline \end{array}$$

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Lesson 26

Skip Counting Patterns

OBJECTIVES
1. To study the skip counting patterns
2. To learn the term *digit*

MATERIALS Worksheet 19, "Skip Counting Patterns"

WARM-UP Read following problem and ask the child to write it in his journal and add: $39.59 + $3.87. [$43.46]

Write

$$26 + 26 = \_\_$$

Ask the child to solve it in his head. How did you do it? Ignore any counting strategies, or ask if it could be done without counting. Some methods include: knowing 25 + 25 = 50, so must be 52; adding 26 + 20, giving 46, then adding 6 to get 52; and taking 4 from second 26, giving 30 and 22.

ACTIVITIES ***Worksheet patterns.*** Ask the child to fill in the blanks for the skip counting patterns on the worksheet. See below.

Note: These formats were emphasized in earlier grades.

2	4	6	8	10
12	14	16	18	20

4	8	12	16	20
24	28	32	36	40

6	12	18	24	30
36	42	48	54	60

8	16	24	32	40
48	56	64	72	80

3	6	9
12	15	18
21	24	27
30		

7	14	21
28	35	42
49	56	63
70		

5 10
15 20
25 30
35 40
45 50

9	18	27	36	45
90	81	72	63	54

Discuss the patterns. What is the last number in every pattern? [2s end in 2 tens, 3s end in 3 tens, and so forth.]

TWOS. What patterns do you see? [They are the even numbers. The second row is 10 plus the first row. The ones repeat in the second row.]

FOURS. What patterns do you see? [The second row is 20 more than the first row. The multiples are every other even number. The ones repeat in the second row.]

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SIXES. What patterns do you see? [The first row is the even 3s. Second row is 30 more than the first row. The ones repeat in the second row.]

EIGHTS. What patterns do you see? [In each row the ones are the even numbers backward. The second row is 40 more than the first row, also every other 4. The ones repeat in the second row.]

THREES. What patterns do you see? [Consider the tens: the first row is below 10; the second row is in the teens; the third row is in the twenties. Next consider the ones: they are the numbers 0 to 9 starting at the lower left with 0 (30) and continuing up the column and over to the bottom of the next column. The numbers alternate between even and odd.]

SEVENS. What patterns do you see? [Within each row the tens increase by 1 (0, 1, 2; 2, 3, 4; and 4, 5, 6. The ones are the numbers 0 to 9 starting at the upper right (21) and continuing down the column and over to the next column on the left. The numbers alternate between even and odd.]

FIVES. What patterns do you see? [The ones are either 5 or 0, 5 in first column and 0 in the second column. Also the tens go up by 1 in both columns.]

NINES. What patterns do you see? [The sum of the digits in all cases is 9. The ones go down by 1 while the tens go up by 1. Also the second row has the digits of the first row reversed, as indicated by the arrow. The numbers alternate even and odd.]

Writing about patterns. Ask the child to write a sentence in his journal for each set of multiples, giving a way to help him learn the multiples.

Digits. Explain that a *digit* is a number from 0 to 9 that we use to make all our numbers. What are the digits are in the number 81? [8 and 1] What are the digits in 365? [3, 6, and 5] What are the digits in 1000? [1 and three 0s]

Worksheet. For homework give him the worksheet. The solutions are given below:

2, 4, 6, 8

none

8

6

7

3, 7, 9

9

Ask the child to practice the multiples outside of lesson time with another family member.

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Name _____

Date _____

Fill in the missing blanks.

2	4	6	8	10
1_	1_	1_	1_	20

4	8	_2	_6	_0
_4	_8	_2	_6	_0

6	1_	1_	2_	3_
3_	4_	4_	5_	60

8	1_	2_	3_	4_
4_	5_	6_	7_	8_

_	_	_
1_	1_	1_
2_	2_	2_
30		

_	1_	2_
2_	3_	4_
4_	5_	6_
70		

5	_0
1_	_0
2_	_0
3_	_0
4_	_0

9	1_	2_	3_	4_
9_	8_	7_	6_	5_

Which multiples have all even numbers? _____

Which multiples have all odd numbers? _____

In which multiple do the ones count by twos backward? _____

In which multiple is the second row 30 more than the first row? _____

In which multiple is the difference between numbers next to each other always 7? _____

In which multiples are all the numbers from 0 to 9 used in the ones place? _____

In which multiple do the digits always add up to 9? _____

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