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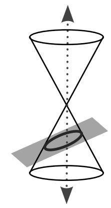
9.1

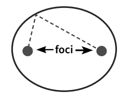
Conic Sections and Analytic Geometry: Ellipses

The intersection of a cone and a plane that is neither parallel to the surface of the cone nor perpendicular to its axis creates an *ellipse*. A circle is the intersection of a plane with the axis of a cone; a parabola is a slice of a cone that is parallel to a side.

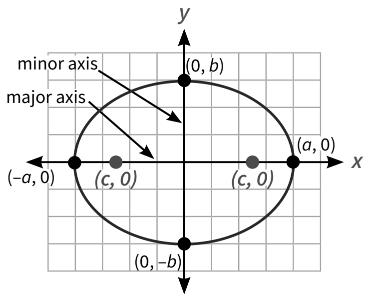


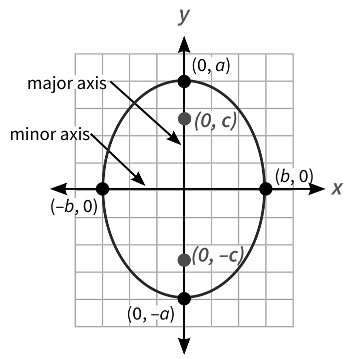
Geometric Definition of an Ellipse

- An ellipse is the set of points in a plane such that the sum of the distances from two fixed points to any point on the ellipse is a constant.



An ellipse has several parts. Each ellipse has two axes of symmetry perpendicular to each other. The longer axis is the major axis, the shorter is the minor axis. The two foci, distance c from the center, are always on the major axis. Vertices are the two points where the major axis intersects the ellipse, both at distance a from the center. Co-vertices are the two points where the minor axis intersects the ellipse, both at distance b from the center.





The algebraic equation for an ellipse, like that for the circle and the parabola, is derived by applying the distance formula to an ellipse drawn in the Cartesian plane. Following are the equations for both a horizontal and a vertical ellipse, with centers at the origin.

Algebraic Equation for an Ellipse

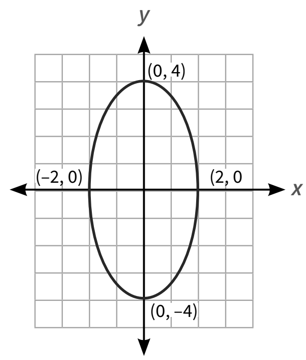
- The standard equations for an ellipse centered at the origin and with $a > b > 0$ are:

Horizontal	**Vertical**
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$

- Note that the two equations differ only by the exchanging of a and b.

Noting which axis is longer reveals whether an ellipse is horizontal or vertical and, therefore, which equation to use. The variable with the larger denominator in the equation indicates the direction of the longest axis.

Example 1 **Graph the ellipse defined by the equation $\frac{x^2}{4} + \frac{y^2}{16} = 1$.**



Since the denominator under the y^2 variable is larger than the denominator under the x^2 variable, this is a vertical ellipse, $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$. Therefore, in the equation $16 = a^2$ and $4 = b^2$.

Because $a = 4$, plot the vertices for the major axis at (0, 4) and (0, –4).

Because $b = 2$, plot the co-vertices at (–2, 0) and (2, 0). Then sketch the ellipse.

Example 2 **Write the equation for an ellipse centered at the origin with a vertex (major axis) at (9, 0) and a covertex (minor axis) at (0, 3).**

The vertex is on the x-axis, so this is a horizontal ellipse, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. $a = 9$ and $b = 3$

Substituting a and b gives the equation $\frac{x^2}{81} + \frac{y^2}{9} = 1$

Note: The length of the major axis is $2a$ and the length of the minor axis is $2b$. So if the length of either is given, divide by 2 to obtain the coordinate distance from the center.

Example 3 **Write the equation for a vertical ellipse centered at the origin with a major vertex of length 10, and a minor vertex of length 6.**

Because the major axis = $2a$, let $10 = 2a$. Therefore, $a = 5$.

Likewise, the minor axis = $2b$, so let $6 = 2b$. Therefore, $b = 3$.

Substitute these a and b values in the vertical ellipse equation: $\frac{x^2}{9} + \frac{x^2}{25} = 1$.

Today's Lesson

Graph the ellipse defined by the equation.

1. $\frac{x^2}{16} + \frac{y^2}{4} = 1$

2. $\frac{x^2}{4} + \frac{y^2}{9} = 1$

Write equations for ellipses centered at the origin.

3. vertex = (5, 0), co-vertex = (0, 2)

4. vertical major axis = 8, minor axis = 4

5. vertex = (0, 8), co-vertex = (4, 0)

6. horizontal major axis = 10, minor axis = 6

REVIEW

Calculate the requested sum (if possible). 8.14

7. $-7, -14, -28, -56 \ldots S_\infty$

8. $a_1 = 100, r = \frac{1}{4}, S_\infty$

Histograms. 8.13

9. A church wanted to visualize the age of its attendees using histograms. One histogram was made using bin widths of 10 years, while the other histogram was made using bin widths of 20 years.

Bins	Frequency
0-19	34
20-39	29
40-59	17
60-79	14
80-100	12

Bins	Frequency
0-9	25
10-19	8
20-29	12
30-39	18
40-49	1
50-59	16
60-69	9
70-79	5
80-89	10
90–99	2

a. Construct separate histograms using the two data tables.

b. How does changing the bin width from 10 years to 20 years change the shape of the histogram?

c. Which age group has fewer people, 0-19 years or 40-79 years?

d. What percentage of the congregation is fifty or older?

Determine the vertex, focus, and directrix for the horizontal parabola defined by the given equation. Then sketch the parabola. 8.11

10. $(y - 2)^2 = -12\,(x - 3)$

State whether the function is increasing or decreasing. Then graph. 8.6

11. $y = \left(\frac{1}{2}\right)^{x+1}$

Use the Gauss-Jordan method to solve the systems. 8.1, 8.12

12. $\begin{cases} x + 4y = -12 \\ 3x - 4y = -4 \end{cases}$

13. $\begin{cases} 3x - 2y = -7 \\ 5x + 2y = 15 \end{cases}$

14. $\begin{cases} x + 2y + 6z = 1 \\ 2x + y + 4z = -1 \\ 2x + 2y + 7z = -2 \end{cases}$

Find the answers. 7.6

15. A seed salesman receives a $1,000 a month salary, as well as a 2.5% commission on all sales he makes over $5,000. If he sells $96,000 worth of seeds in March, how much will he make that month?

Solve the systems using Cramer's Rule. 7.1

16. $\begin{cases} x - 4y = -19 \\ 2x + 3y = 6 \end{cases}$

17. $\begin{cases} x - 2y = -3 \\ 4x + y = -21 \end{cases}$

18. $\begin{cases} x - 2y - 2z = -4 \\ y + z = 2 \\ 2x - 3y - 2z = -9 \end{cases}$

Name the signs of the sine, cosine, and tangent for angles whose terminal rays contain the following points. 6.7

19. $(3, 1)$

20. $(-5, -6)$

21. $(9, -2)$

Find the reference angle, the sine, cosine and tangent of the following angles. 6.7

22. $184°$

23. $70°$

Simplify. 6.1

24. $\sqrt{4\sqrt{2}}$

25. $\sqrt{2\sqrt{2}}$

26. $\sqrt{5\sqrt[3]{5}}$

Determine the distance and midpoint of the line segments with the following endpoints. 5.3

27. $(6, 7)(7, 6)$

28. $(-12, -2)(-6, 4)$

Extra Practice

Graph the ellipse defined by the equation.

29. $\frac{x^2}{4} + \frac{y^2}{1} = 1$

30. $\frac{x^2}{25} + \frac{y^2}{16} = 1$

31. $\frac{x^2}{12.25} + \frac{y^2}{14} = 1$

Write equations for ellipses centered at the origin.

32. vertex = (3,0), co-vertex = (0,1)

33. horizontal major axis = 4, vertical minor axis = 3

34. vertex = (0,5), co-vertex = (2,0)