UNIT 5

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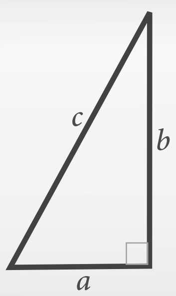
The Pythagorean Theorem



The Pythagorean theorem, $a^2 + b^2 = c^2$, states the geometric relationship between the sides of a right triangle. The legs of the triangle are represented by a and b, and the hypotenuse by c, as shown here. Because of the Pythagorean relationship, knowing the lengths of any two sides of a right triangle makes it possible to calculate the length of the third side.

Example 1 **Find the hypotenuse of a triangle whose legs measure 8 inches and 10 inches.**

$a^2 + b^2 = c^2$	Pythagorean Theorem.
$(8)^2 + (10)^2 = c^2$	Measures of two legs substituted for a and b.
$64 + 100 = c^2$	Arithmetic simplified.
$164 = c^2$	Addition simplified.
$\sqrt{164} = c$	Square root of both sides taken.
$c \approx 12.8$	Answer approximated.

The length of the hypotenuse is approximately 12.8 inches.

Example 2 **Find the second leg of a triangle with one leg measuring 13 inches and hypotenuse measuring 15 inches.**

$a^2 + b^2 = c^2$	Pythagorean Theorem.
$\mathrm{a}^2 + (13)^2 = (15)^2$	Measures of two legs substituted for a and b.
$\mathrm{a}^2 + 169 = 225$	Arithmetic simplified.
$a^2 = 56$	Addition simplified.
$a = \sqrt{56} \approx 7.5$	Square root of both sides taken.

The length of the second leg is approximately 7.5 inches.

In these examples, at least one of the sides was an irrational number. Right triangles with whole number measures for all three sides do exist.

3-4-5   5-12-13   8-15-17

Triangles with these measures are called *Pythagorean triples*. Triangles whose measures are multiples of these, such as 6-8-10 (a multiple of 3-4-5) are also Pythagorean triples.

Today's Lesson

Find the hypotenuse of the triangles that have the following leg measurements.

1. $a = 6, b = 3$ **2.** $a = 10, b = 10$ **3.** $a = 13, b = 5$

Find the measurement of the missing leg for the following triangles.

4. $a = 1, c = 8$ **5.** $a = 6, c = 10$ **6.** $a = 20, c = 30$

REVIEW

Solve the equations. Note any extraneous solutions. 4.14

7. $\frac{x+3}{x+5} = \frac{-4}{x^2+12x+35} + \frac{2}{x+7}$ **8.** $\frac{a+9}{6} = \frac{2a}{3}$

Sketch this polynomial. 4.13

9. $x^3 + 2x^2 - 3x$

Find the roots of the following equations. 4.11

10. $x^3 + 3x^2 + 6x + 8 = 0$ **11.** $x^3 - 3x^2 + 4 = 0$ **12.** $2x^3 - 5x^2 + 14x - 6 = 0$

Divide. 4.6

13. $\frac{7}{5i}$ **14.** $\frac{(5+7i)}{2i}$ **15.** $\frac{(2+4i)}{(1+3i)}$

Factor completely. 4.1

16. $10x^3 - 5x^2 - 15x$ **17.** $42xy + 12x + 84y + 24$ **18.** $18x^3 - 72x$

Solve the systems using elimination. 3.11

19. $\begin{cases} 3x - 4y + z = -1 \\ -3x + 2y - 3z = -9 \\ x + 3y + z = 3 \end{cases}$ **20.** $\begin{cases} x + y = -3 \\ -3x - 4y = 2 \end{cases}$

Simplify. 1.4

21. $x^5 \bullet x \bullet x^4$ **22.** $(m^2n^4o)^3$ **23.** $\left(\frac{2x^3y^4}{6xy^3}\right)^2$ **24.** $m^{10} \div m^5$

Change each function to vertex form by completing the square. Graph number 18. 4.4

25. $y = x^2 + 6x + 17$ **26.** $y = -x^2 - 8x - 15$

Evaluate each function for the given values. 3.3

27. $f(x) = 8x + 1$ for $x = 3, -4, 1$

28. $f(x) = 6x^3$ for $x = 2, -2, -1$

Solve the quadratic equations. 2.11

29. $(x + 1)^2 = 1$

30. $x^2 = 64$

31. $2x^2 - 5x = 0$

Rationalize the denominators. 4.3

32. $\frac{18}{\sqrt[3]{4}}$

33. $\frac{5}{2\sqrt[3]{9}}$

34. $\frac{\sqrt[3]{6}}{\sqrt[3]{9}}$

Solve using the quadractic formula. 3.7

35. $3x^2 - x - 1 = 0$

36. $x^2 - 3x + 1 = 0$

Extra Practice

Find the missing leg or hypotenuse of the following triangle.

37. $a = 7, b = 3$

38. $a = 3, c = 9$

39. $a = 6, b = 5$