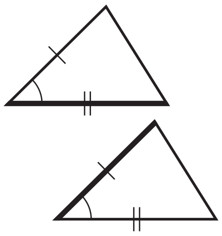
G8-11 Congruence Rules

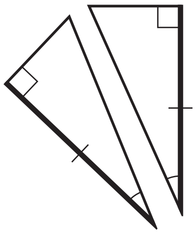
If two polygons are congruent, you can place them one on top of the other so that they match exactly.
The vertices that match are called **corresponding vertices**.
The angles that match are called **corresponding angles**.
The sides that match are called **corresponding sides**.

1. The two triangles are congruent. Are the two thick sides corresponding sides? Explain.

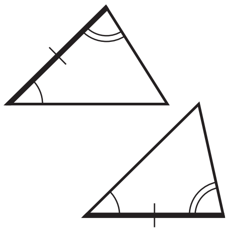
a)



b)

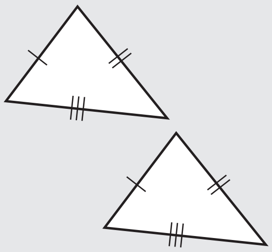


c)

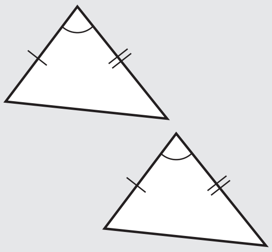


Congruence Rules for Triangles

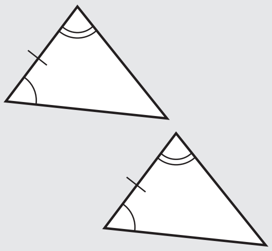
SSS (side-side-side)



SAS (side-angle-side)



ASA (angle-side-angle)

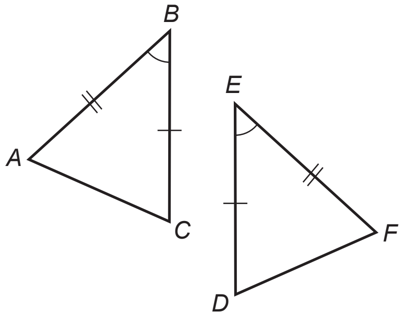


Two triangles are congruent if they have …

- three pairs of equal corresponding sides
- two pairs of equal corresponding sides and a pair of equal corresponding angles between these sides
- two pairs of equal corresponding angles and a pair of equal corresponding sides between these angles

2. Identify the congruence rule that tells you that each pair of triangles is congruent.
Write the congruence statement.

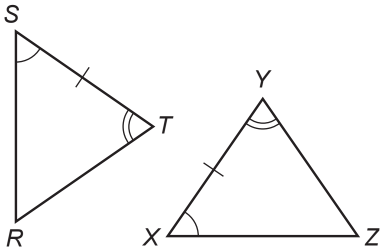
a)



Congruence rule: ________

$\triangle$ *ABC* $\cong \triangle$________

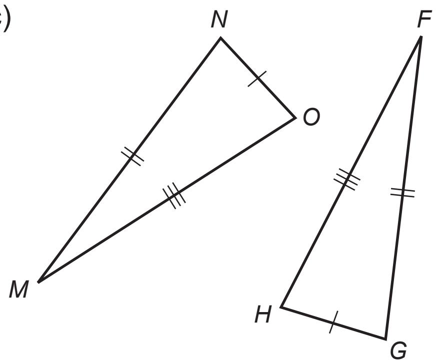
b)



Congruence rule: ________

$\triangle$________ $\cong \triangle$________

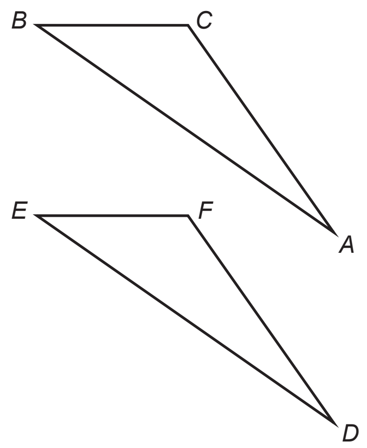
c)



Congruence rule: ________

$\triangle$________ $\cong \triangle$________

3. In the triangles in the picture, $\angle A = \angle D = 20°$, $\angle B = \angle E = 35°$, and $BC = EF$.



a) Mark the equal sides and angles on the picture.

b) Do the given equations of sides and angles

fit any congruence rule? ______

c) The sum of the angles in a triangle is ______.

$\angle C = \underline{\hspace{1cm}} - (\angle\underline{\hspace{1cm}} + \angle\underline{\hspace{1cm}}) = \underline{\hspace{1cm}}$

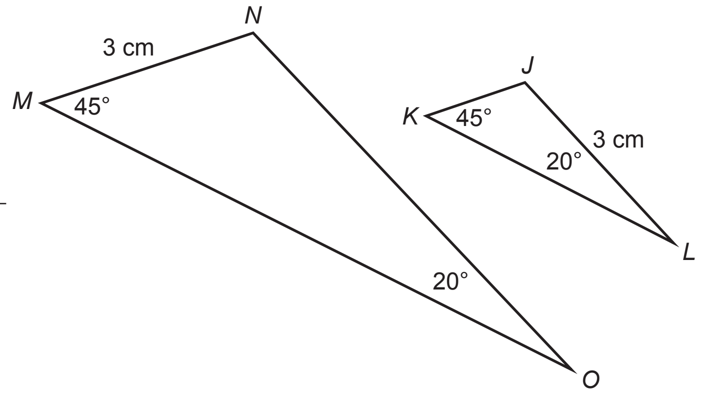
$\angle F = \underline{\hspace{1cm}} - (\angle\underline{\hspace{1cm}} + \angle\underline{\hspace{1cm}}) = \underline{\hspace{1cm}}$

d) Which congruence rule can you apply now? ______

Write the equations for the sides and angles that the congruence rule requires.

$\underline{\hspace{1cm}} = \underline{\hspace{1cm}}, \underline{\hspace{1cm}} = \underline{\hspace{1cm}}, \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$

4. Nina thinks that she can apply the same method as in Question 3 to the two triangles shown.



a) Are the triangles congruent? ______

b) $\angle N = \underline{\hspace{1cm}} - \angle\underline{\hspace{1cm}} - \angle\underline{\hspace{1cm}} = \underline{\hspace{1cm}}$

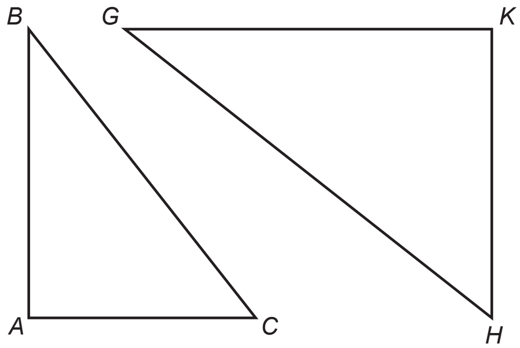
$\angle J = \underline{\hspace{1cm}} - \angle\underline{\hspace{1cm}} - \angle\underline{\hspace{1cm}} = \underline{\hspace{1cm}}$

c) Explain why the ASA congruence rule does not work for $\triangle JKL$ and $\triangle NOM$.

____

____

5. a) Measure the angles and the sides of these triangles. Write the measurements on the diagram.



b) Which angles are equal?

____

c) Which sides are equal?

____

d) Are the triangles congruent? Explain why the congruence rules do not apply here.

Angle-angle-angle (**AAA**) is not a congruence rule.

6. Draw a counterexample for this statement:

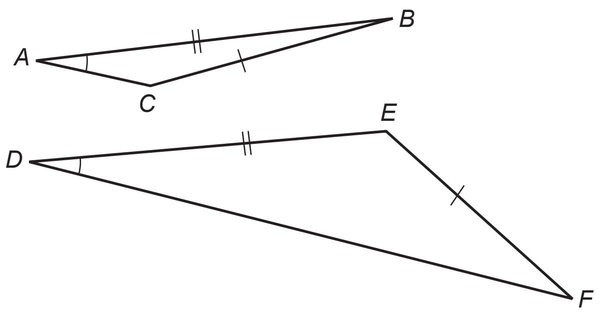
All triangles that have three pairs of corresponding equal angles are congruent.

7. a) What sides and angles in $\triangle ABC$ and $\triangle DEF$ are equal?

____

b) Can you use the SAS congruence rule? Why or why not? ______________________

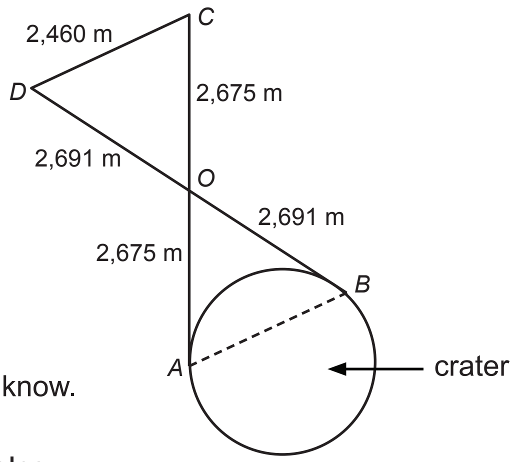
____



c) Do triangles $\triangle ABC$ and $\triangle DEF$ look congruent? ______

d) Is side-side-angle a congruence rule? Is angle-side-side a congruence rule? Explain.

8. Scientists want to measure the distance across a crater on Mars. They send a rover along the path *AOCDOB*.



a) What are the equal sides and angles in triangles $\triangle ABO$ and $\triangle OCD$?

_________ = _________ _________ = _________

_________ = _________

b) What congruence rule can you use? ______

c) What is the distance *AB* across the crater? Explain how you know.

9. Draw two triangles that are not congruent. Explain why the triangles are not congruent.

10. Sketch a counterexample to show why this statement is false.

If two triangles have two pairs of corresponding sides that are equal, the triangles are congruent.

11. $\triangle PQR$ and $\triangle XYZ$ have $PQ = XY = 5$ cm and $QR = YZ = 7$ cm. Sketch the triangles. Does the statement given ensure that $\triangle PQR \cong \triangle XYZ$? If so, note the congruence rule.

a) $\angle P = \angle Y$ b) $\angle Q = \angle Y$ c) $\angle P = \angle X$ d) $PR = XZ$

12. $\triangle BCD$ and $\triangle FGH$ have $\angle C = \angle G$ and $\angle D = \angle H$. Sketch the triangles. Does the statement given ensure that $\triangle BCD \cong \triangle FGH$? If so, note the congruence rule.

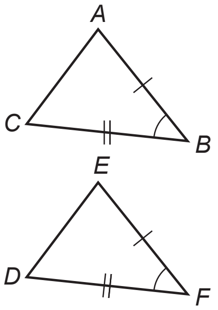
a) $\angle B = \angle F$ b) $CD = GH$ c) $BD = FH$ d) $BC = GH$

G8-12 Congruence (Advanced)

REMINDER: Congruence Rules for Triangles

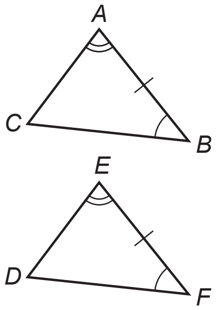
SAS (side-angle-side)

If in $\triangle ABC$ and $\triangle EFD$, $AB = EF$, $\angle B = \angle F$, and $BC = FD$, then $\triangle ABC \cong \triangle EFD$.



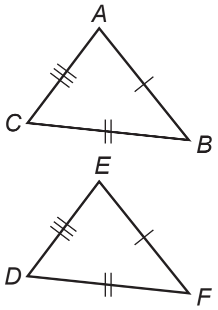
ASA (angle-side-angle)

If in $\triangle ABC$ and $\triangle EFD$, $\angle A = \angle E$, $AB = EF$, and $\angle B = \angle F$, then $\triangle ABC \cong \triangle EFD$.

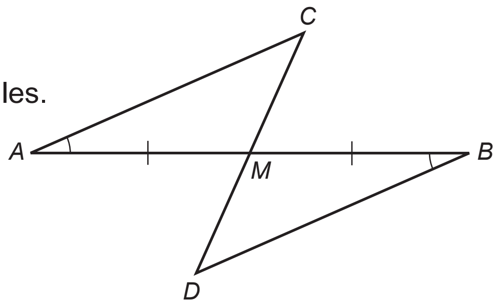


SSS (side-side-side)

If in $\triangle ABC$ and $\triangle EFD$, $AB = EF$, $AC = ED$, and $BC = FD$, then $\triangle ABC \cong \triangle EFD$.



1. In the diagram, $CM = 7$ cm. Fill in the blanks to find the length of DM.



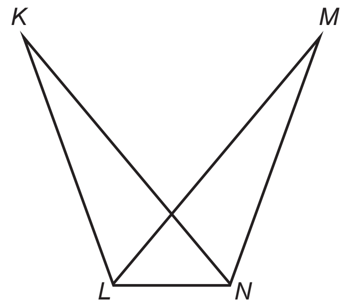
a) $AM =$ ______ and $\angle A =$ ______.

b) $\angle AMC =$ __________ because they are ________________ angles.

c) $\triangle AMC \cong \triangle$__________ by the __________ congruence rule.

This means $DM =$ ______ = ______ cm.

2. In the diagram, $KL = MN = 42$ cm, and $KN = LM = 50$ cm. $\angle K = 20°$ and $\angle KNL = 50°$. Fill in the blanks to find $\angle M$ and $\angle MLN$.



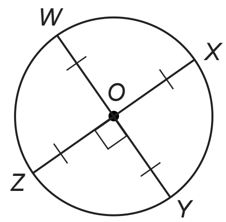
a) Which sides are equal in triangles KLN and LMN?

____

b) $\triangle KLN \cong \triangle$__________ by the __________ congruence rule.

So, $\angle M = \angle$__________ = ______ and $\angle MLN = \angle$__________ = ______.

3. A school wants to install a square rope climbing structure over a large circular sandbox. A contractor places a pole in the center O of the box and places rope anchors at the opposite sides of the pit in points W, X, Y, and Z.



a) Draw the quadrilateral $WXYZ$.

b) The contractor claims that $WXYZ$ is a square. Is he correct? ______

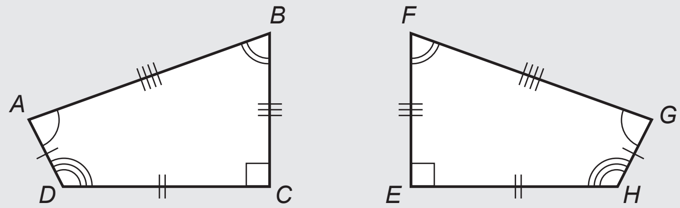

c) Use congruent triangles to explain your answer in part b).

To prove that two polygons are congruent, imagine placing them one on top of the other, so that they match exactly. If the polygons have equal corresponding sides and equal corresponding angles, they are congruent. Example:

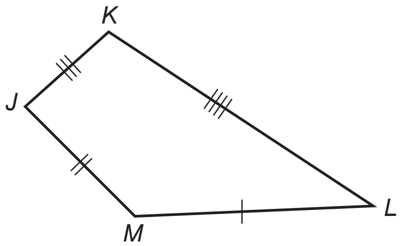
$\angle A = \angle G$, $\angle B = \angle F$, $\angle C = \angle E$, $\angle D = \angle H$

$AB = GF$, $BC = FE$, $CD = EH$, $DA = HG$

So $ABCD \cong GFEH$.



4. The equal sides are marked on the quadrilaterals $JKLM$ and $NOPQ$. Also, $\angle J = \angle N$ and $\angle L = \angle P$.



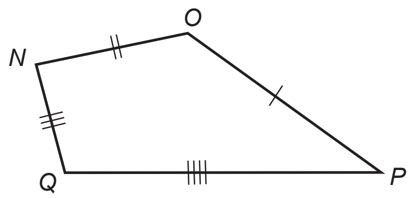
a) Mark the equal angles on the diagram.

b) Draw line segments KM and OQ.

c) $\Delta JKM \cong \Delta$______ by the ______ congruence rule.

This means $\angle JKM = \angle$______. Label both these angles w.

This also means $\angle JMK = \angle$______. Label both these angles x.



d) $\Delta KLM \cong \Delta$______ by the ______ congruence rule.

This means $\angle LKM = \angle$______. Label both these angles y.

This also means $\angle LMK = \angle$______. Label both these angles z.

e) Express the measure of $\angle JKL$ and the angle equal to it in $NOPQ$ using w, x, y, z.

$\angle JKL = \angle$______ $=$ _______________

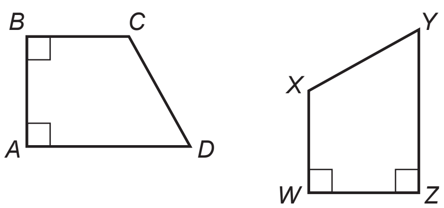
f) Express the measure of $\angle JML$ and the angle equal to it in $NOPQ$ using w, x, y, z.

$\angle JML = \angle$______ $=$ _______________

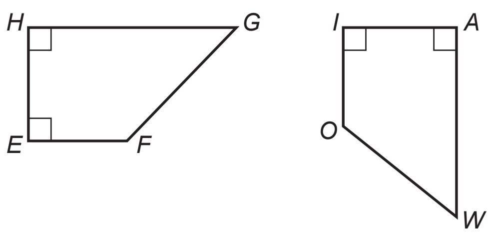
g) Write the congruence statement for the quadrilaterals. _______________

5. Are the quadrilaterals congruent? Use a ruler and a protractor to check. If yes, write the pairs of corresponding equal sides and corresponding equal angles. Then write the congruence statement.

a)



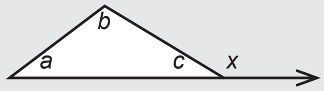
b)



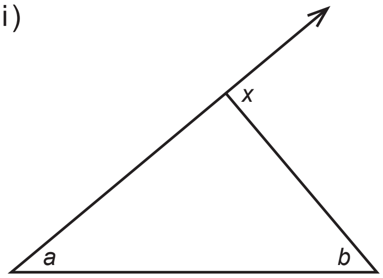
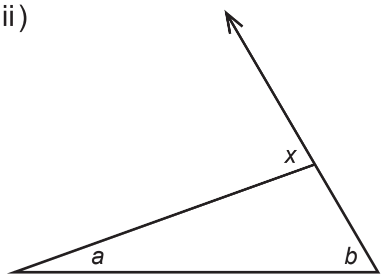
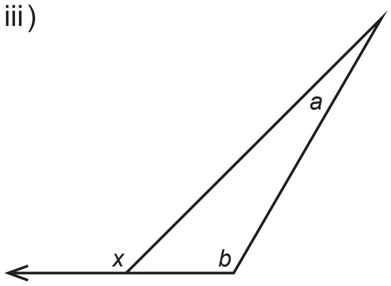
Bonus ▶ In parallelograms $ABCD$ and $PQRS$, we have $AB = QR$, $BC = PQ$, and $\angle B = \angle Q$. Jake thinks the parallelograms are congruent. Is that correct? Explain.

G8-13 Exterior Angles of a Triangle

An **exterior angle** is formed by extending a side of a triangle.
Angle *x* is an exterior angle.

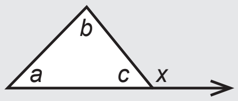


1. a) Measure the angles with a protractor and fill in the table.

	i) 	ii) 	iii) 
∠*a*			
∠*b*			
∠*x*			

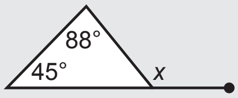
b) Express the measure of ∠*x* using the measures of ∠*a* and ∠*b*. ____________________

An exterior angle of a triangle is equal to the sum of the two opposite angles in the triangle ($x = a + b$).

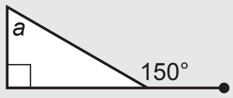


Examples:

$x = 88° + 45°$
$= 133°$

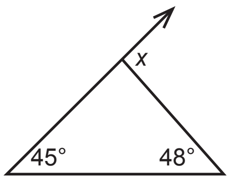


$a + 90° = 150°$
$a = 150° - 90° = 60°$



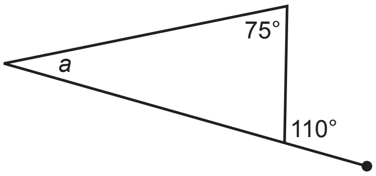
2. Find the missing angle. Show your work.

a)



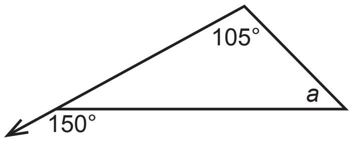
∠*x* = ______

b)



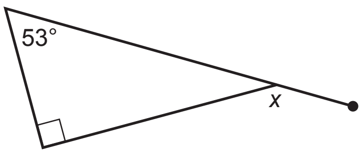
∠*a* = ______

c)

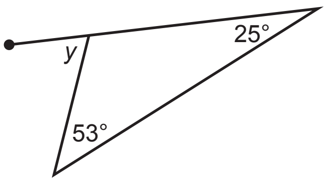


∠*a* = ______

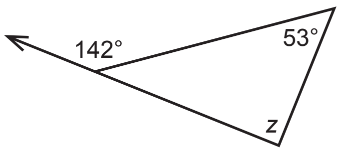
d)



e)

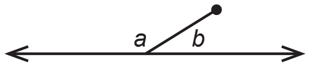


f)



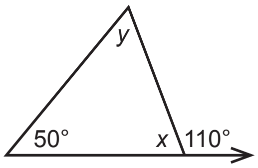
REMINDER: Angles in a triangle add to 180°.

Two angles that combine to make a straight line add to 180°. $a + b = 180°$.



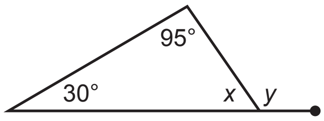
3. Find the missing angles. Show your work.

a)



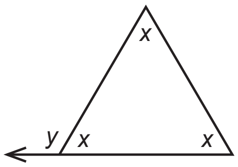
$\angle x =$ ______, $\angle y =$ ______

b)



$\angle x =$ ______, $\angle y =$ ______

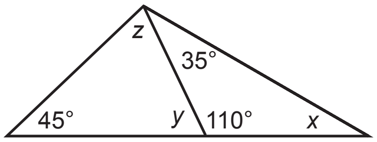
c)



$\angle x =$ ______, $\angle y =$ ______

4. Find the missing angles.

a)

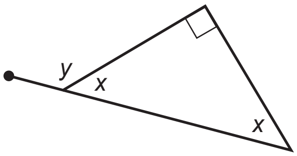


$\angle x =$ ______

$\angle y =$ ______

$\angle z =$ ______

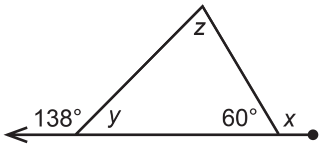
b)



$\angle x =$ ______

$\angle y =$ ______

c)

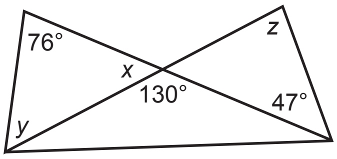


$\angle x =$ ______

$\angle y =$ ______

$\angle z =$ ______

d)

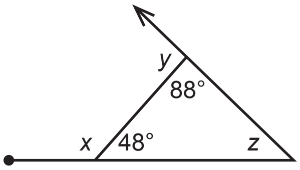


$\angle x =$ ______

$\angle y =$ ______

$\angle z =$ ______

e)

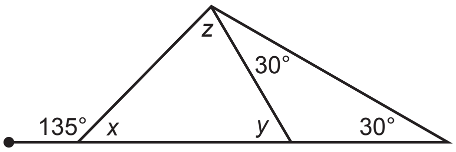


$\angle x =$ ______

$\angle y =$ ______

$\angle z =$ ______

f)

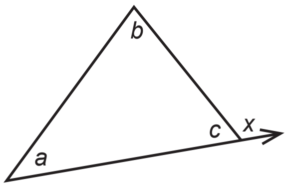


$\angle x =$ ______

$\angle y =$ ______

$\angle z =$ ______

5. Prove that $\angle x = \angle a + \angle b$ using the sum of the angles in a triangle and supplementary angles (angles that add to 180°).



a) What do you know about the measures of $\angle a$, $\angle b$, and $\angle c$? Explain.

b) What do you know about the measures of $\angle x$ and $\angle c$? Explain.

c) Write two expressions for the measure of $\angle c$ using your answers in parts a) and b).

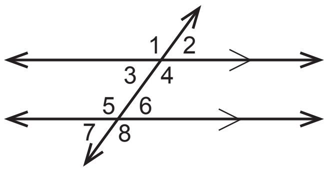
d) Use the expressions in part c) to explain why $\angle x = \angle a + \angle b$.

G8-14 Corresponding Angles and Parallel Lines

Corresponding angles create a pattern like in the letter F shown: 

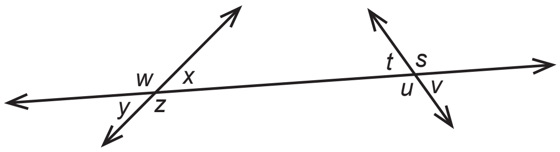
These pairs of angles are corresponding angles for lines *m* and *n*:

1. List the corresponding angles.

a) 

∠1 and ______, ∠2 and ______

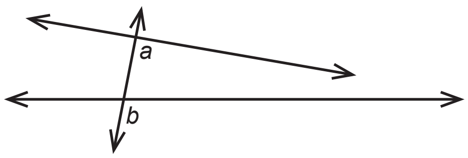
∠3 and ______, ∠4 and ______

b) 

∠*w* and ______, ∠*x* and ______

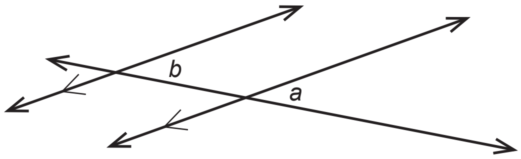
∠*y* and ______, ∠*z* and ______

2. Measure the corresponding angles *a* and *b*. Are they equal or not equal?

a) 

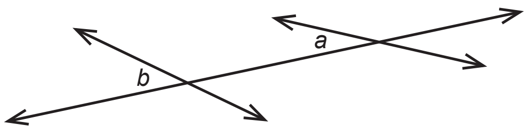
∠*a* = ______, ∠*b* = ______

The corresponding angles are ____________.

b) 

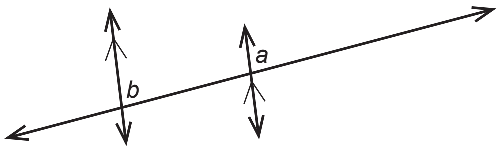
∠*a* = ______, ∠*b* = ______

The corresponding angles are ____________.

c) 

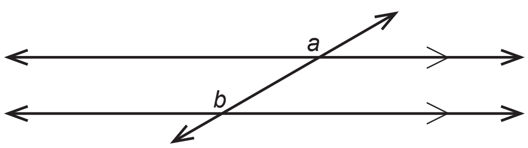
∠*a* = ______, ∠*b* = ______

The corresponding angles are ____________.

d) 

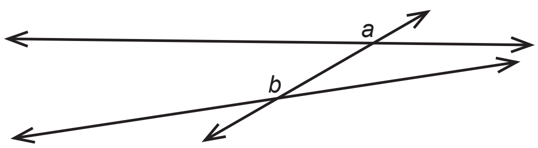
∠*a* = ______, ∠*b* = ______

The corresponding angles are ____________.

e) 

∠*a* = ______, ∠*b* = ______

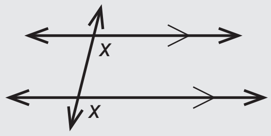
The corresponding angles are ____________.

f) 

∠*a* = ______, ∠*b* = ______

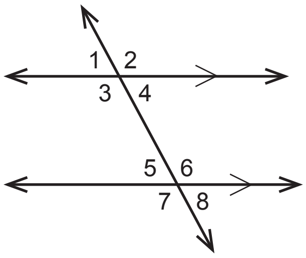
The corresponding angles are ____________.

When the lines are parallel, corresponding angles are equal.



3. List the corresponding equal angles.

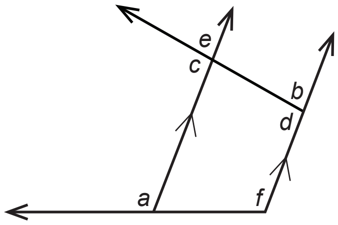
a)



∠1 = ∠5, ∠2 = ______

∠3 = ______, ∠4 = ______

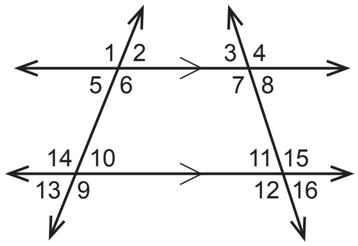
b)



∠a = ______, ∠c = ______

∠e = ______

c)

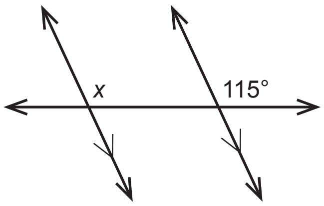


∠3 = ______, ∠4 = ______

∠5 = ______, ∠10 = ______

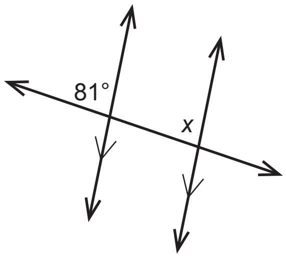
4. Give the measure of the corresponding angles.

a)



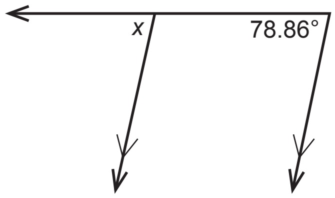
∠x = ______

b)



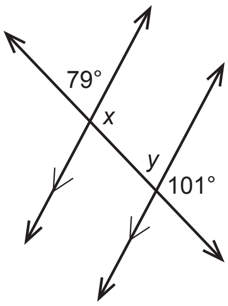
∠x = ______

c)



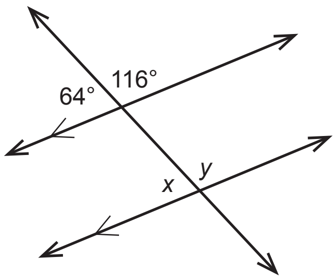
∠x = ______

d)



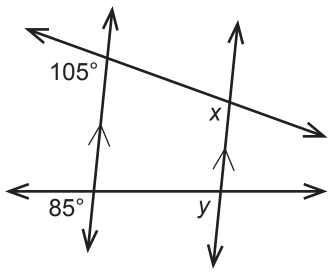
∠x = ______, ∠y = ______

e)



∠x = ______, ∠y = ______

f)

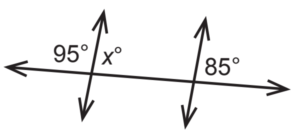


∠x = ______, ∠y = ______

When the corresponding angles are equal, the lines are parallel.

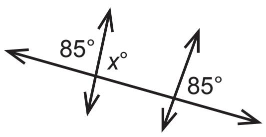
5. Find ∠x using supplementary angles. Circle the angle corresponding to ∠x. Then draw arrows to show if the lines are parallel.

a)



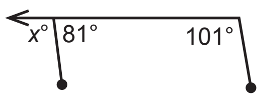
∠x = ______

b)



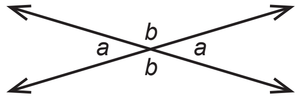
∠x = ______

c)



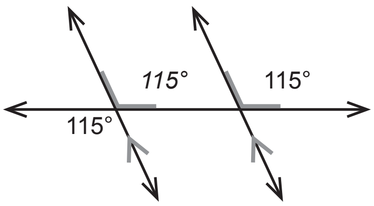
∠x = ______

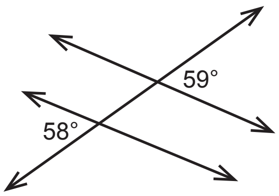
REMINDER: Vertical angles are equal.

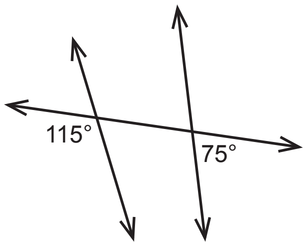


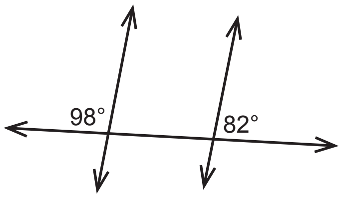
6. For each pair of lines that do not intersect in the picture …

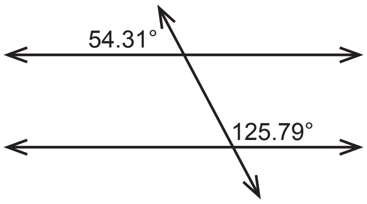
- mark a pair of corresponding angles and find their measures.
- mark parallel lines.

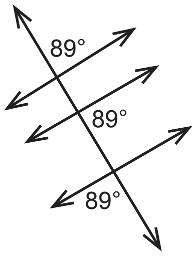
a) 

b) 

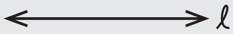
c) 

d) 

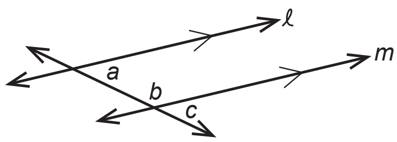
e) 

f) 

You can use small letters to label lines, too.



7. Lines ℓ and m are parallel. Use what you know about corresponding and vertical angles to explain why $\angle a + \angle b = 180°$.

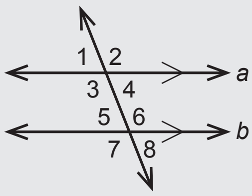


Co-interior angles create a pattern like in the letter "C":
Angles 3 and 5, and angles 4 and 6, are co-interior angles.

These angles are sometimes called **same-side interior angles**.

If the lines a and b are parallel, co-interior angles add to 180°.

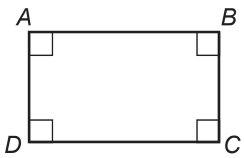
If the co-interior angles add to 180°, the lines are parallel.



8. All angles in the quadrilateral *ABCD* are right angles.

a) Identify a pair of co-interior angles for lines *AB* and *CD*.
Identify a pair of co-interior angles for lines *AD* and *BC*.

b) Use the angle measures of co-interior angles to explain that these pairs of sides are parallel.



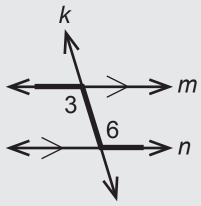
c) I have just proven that a rectangle is a __________________.

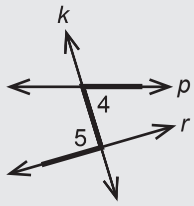
G8-15 Alternate Angles and Parallel Lines

Alternate angles or **alternate interior angles** make a pattern like the letter Z:

$\angle 3$ and $\angle 6$ are alternate angles for parallel lines *m* and *n*.

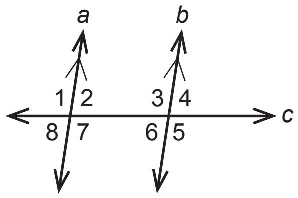
$\angle 4$ and $\angle 5$ are alternate angles for lines *p* and *r*.





1. List the pairs of alternate angles.

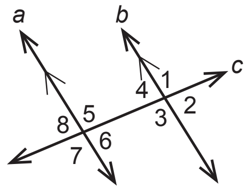
a)



$\angle 2$ and ______

$\angle 3$ and ______

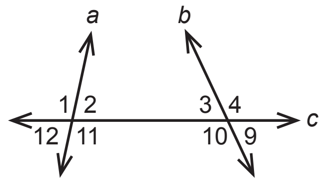
b)



$\angle 3$ and ______

$\angle 4$ and ______

c)

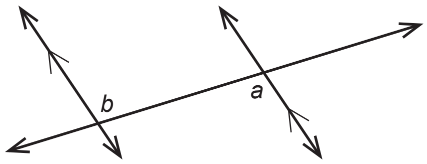


$\angle 2$ and ______

$\angle 11$ and ______

2. Measure the alternate angles *a* and *b*. Are they equal or not equal?

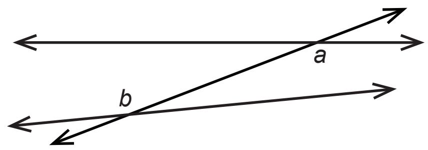
a)



$\angle a =$ ______, $\angle b =$ ______

The alternate angles are ______________.

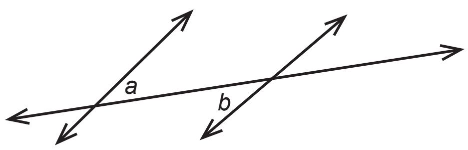
b)



$\angle a =$ ______, $\angle b =$ ______

The alternate angles are ______________.

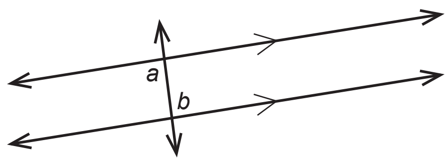
c)



$\angle a =$ ______, $\angle b =$ ______

The alternate angles are ______________.

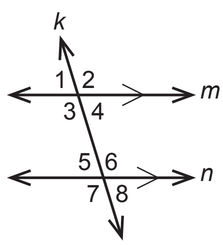
d)



$\angle a =$ ______, $\angle b =$ ______

The alternate angles are ______________.

3. Fill in the blanks to prove that alternate angles for parallel lines are equal.



$\angle 3$ and $\angle$______ are alternate angles.

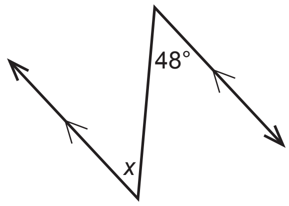
Lines *m* and *n* are ______________________.

$\angle 3$ and $\angle 7$ are ______________________ angles, so $\angle 3 = \angle 7$.

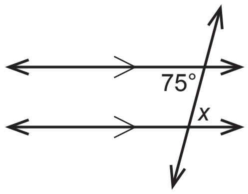
$\angle 6$ and $\angle 7$ are ______________________ angles, so $\angle 6 =$ ______.

This means $\angle 3 =$ ______.

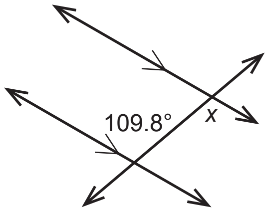
4. Find the missing alternate angles.

a) 

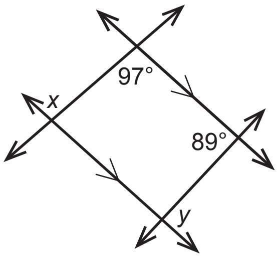
$\angle x =$ ______

b) 

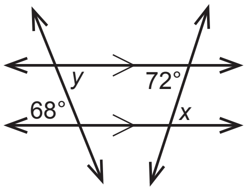
$\angle x =$ ______

c) 

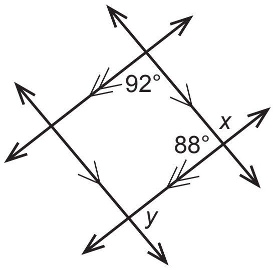
$\angle x =$ ______

d) 

$\angle x =$ ______, $\angle y =$ ______

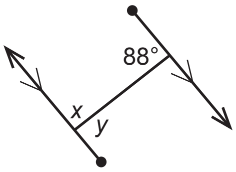
e) 

$\angle x =$ ______, $\angle y =$ ______

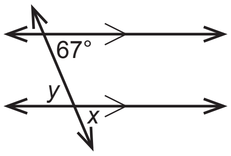
f) 

$\angle x =$ ______, $\angle y =$ ______

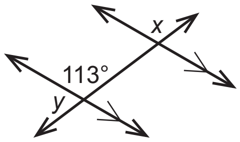
5. Find the missing alternate, corresponding, supplementary, or vertical angles.

a) 

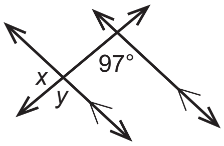
$\angle x =$ ______, $\angle y =$ ______

b) 

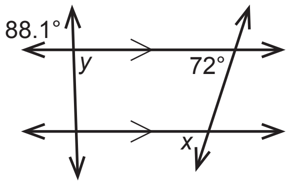
$\angle x =$ ______, $\angle y =$ ______

c) 

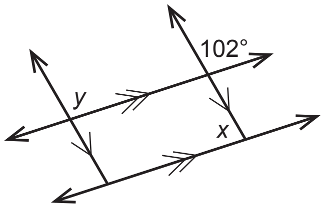
$\angle x =$ ______, $\angle y =$ ______

d) 

$\angle x =$ ______, $\angle y =$ ______

e) 

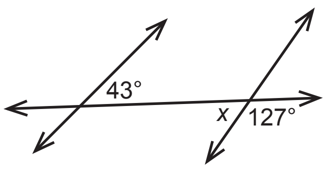
$\angle x =$ ______, $\angle y =$ ______

f) 

$\angle x =$ ______, $\angle y =$ ______

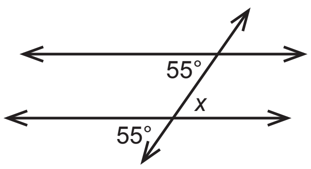
If the alternate angles are equal, the lines are parallel.

6. Find $\angle x$ using supplementary or vertical angles. Circle the angle that is alternate to $\angle x$. Then decide if the lines or rays are parallel.

a) 

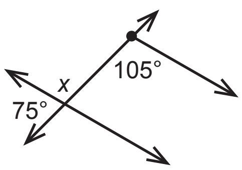
$\angle x =$ ______

parallel not parallel

b) 

$\angle x =$ ______

parallel not parallel

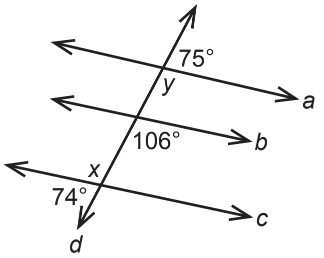
c) 

$\angle x =$ ______

parallel not parallel

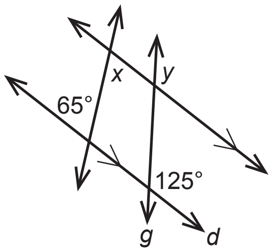
7. Fill in the missing angles. Use arrows to mark parallel lines if they are not marked already.

a)



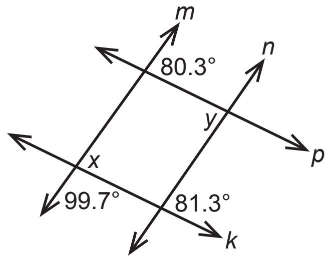
$\angle x =$ ______, $\angle y =$ ______

b)



$\angle x =$ ______, $\angle y =$ ______

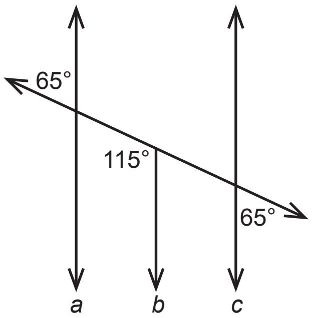
c)



$\angle x =$ ______, $\angle y =$ ______

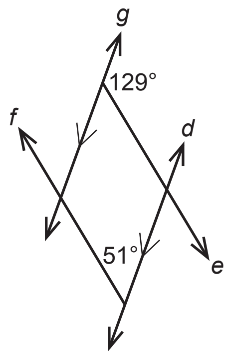
8. Fill in all the missing angles. Decide which lines or rays are parallel.

a)



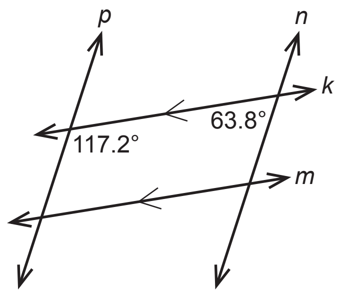
____

b)



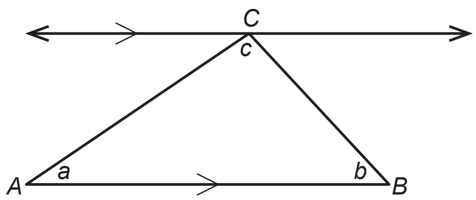
____

c)



____

9. Sharon wants to prove (using logic) that the sum of the angles in a triangle is 180°.



a) Sharon draws a triangle *ABC* and labeled the angle measures *a*, *b*, and *c*. Write an equation using *a*, *b*, *c* that she needs to prove. ______________________

b) Sharon draws a line parallel to *AB* through *C*. Label the alternate angles on the diagram with their measures.

c) Which angles on the diagram make a straight angle? What do their measures add to? Write an equation. ______________________

d) Did you prove what you needed to prove? ______________________

Bonus ▶

a) Find the measure of $\angle ECN$. Explain how you know.

b) If you extend the lines *ME* and *CN*, will they intersect? Explain.