Introduction v	Geometric recognition 54
September 1	Numeral recognition 54 Number line 58
Work with Cuisenaire® rods 3	Subtraction with
series grant general c	Cuisenaire® rods 58
Recognizing numerals 6	Inequalities 60
Counting 8	Counting 60
Recognizing number groups 8	Cuisenaire® rods 63
One-to-one correspondence 8	Addition and subtraction 65
Recognizing number groups 12	Addition and subtraction 67
Diagnostic chalkboard session 12	Summary of work 67
Counting 15	Evaluation 67
One-to-one correspondence 15	Sets 70
Work with Cuisenaire® rods 18	Word problems:
Counting 22	multiplication 76
One-to-one correspondence 22	Addition and subtraction 78
Recognizing geometric solids 27	Multiplication using
Counting 27	Cuisenaire® rods 80
Counting: tally method 30	Special work 80
Work with Cuisenaire® rods 30	Geoboards: rows and columns 84
Cuisenaire® rods 33	Logic 84
Counting 33	Writing numerals 88
One-to-one correspondence 33	Difficult problems 88
Work with Cuisenaire® rods 37	The past month 92
Work with Cuisenaire® rods 40	Number line: addition and
Addition 40	multiplication 95
Counting 44	Cuisenaire® rods: addition
Work with rods 44	and subtraction 95
	Cuisenaire® rods:
October 49	mixed operations 97
Number line 51	Number line: odd and even 100
Number line 54	Rod natterns 101

Numeral recognition: odd-even 103 Rod models: beginning use of parentheses 103

November 105 Odd-even 107 108 Addition and subtraction Addition and subtraction 109 Addition and subtraction 111 Distinguishing numeral from number Addition and subtraction 116 Fractions 118 Mixed operations Work with fractions 120 Addition and subtraction 122 Factoring 123 Number line 125 125 Fractions 128 Free play Addition 129 Work with rods 129 Mixed operations 131 Rod game 131 Rod patterns 133 The clock 135 Ratios 136 Place value 136 The clock 140 Factoring 140

December 145 Clocks 147 Computation 147 Clocks 149 Place value 149 Addition and subtraction 153 Number line 153 Addition, subtraction, and multiplication 155

Addition, subtraction, and multiplication 157
Clocks 158
Addition, subtraction, and multiplication 158
Progressions 160
Addition and subtraction 160
Odd-even 160

January 163 Addition, subtraction, and multiplication 165 Multiplication Series and progressions 166 Odd-even 169 Clock arithmetic 169 Money problems 169 Doubling and halving 173 Fractions 175 Multiplication 176 Fractions 176 Series and progressions 177 Fractions 177 Mixed operations 178 Squares 178 Series and progressions 180 Squares 183 Equalities and inequalities 185 Equalities and inequalities 188 Geometric recognition

February–June 191
Arithmetic operations 193
Counting 198
Frame arithmetic 199
Equalities and inequalities 201
Doubling and halving 202
Fractions 205
Word problems 207

## Introduction

The First-Grade Diary is a daily log of the progress of a group of 20 primary-grade children in the mathematics laboratory in 1960–61. Most of the children in this class had participated in a readiness program in kindergarten, where they had many free-play sessions with Cuisenaire® rods and other concrete materials. Often they were able to grasp concepts more rapidly and easily than is usual for first-graders and were free to move ahead to new concepts and activities.

The *Diary* was originally kept as a cumulative case history of a specific class to refine methods and procedures for the benefit of future classes. It records games, materials, sequences of developmental activities, and children's responses and insights into mathematical concepts. The learning experiences of these children throughout the school year, their successes and their failures, are described here as an example of one approach to teaching mathematics to a group of first-graders. The children's rapid progress, enthusiasm, and independence indicated that these classroom notes might be of interest and help to others.

Entries for the first part of the school year (September through January) are detailed accounts of the actual daily work of the children. They include diagrams of a variety of concrete materials and teaching aids, specific references to Cuisenaire® rods and to some of the lab sheets as they were used in the activities of the first-grade class. Many of the group activities were recorded in dialogue form so that the responses of the children—even their errors—might illustrate their thinking in the process of discovering certain mathematical concepts. In addition to the actual descriptive class notes, records of work with individual children and evaluations of pupil progress are included.

By the end of January, the children spent most of their time working independently on written materials. The class notes for February through June have been brought together under topics so that instruction for any particular subject can be pursued as far as children require.

The *Diary* is not intended to serve as a manual in the traditional sense, and the teacher is cautioned against following it as a daily guide. It is hoped that

teachers will be inspired to develop their own methods of presenting the materials. Since no two groups of children are exactly alike and no two teachers are exactly alike in their style and technique, no two first-grade classes should be expected to follow the same path in learning mathematics.

Teachers who have used the *Diary* report that it has served as a ready source of stimulating ideas. Many of those who teach grades higher than the first have found useful ideas for remedial and review work. We hope that teachers will be inspired to build on this diary, to keep diaries of their own, and to develop their own personal approaches to the Mathematics Laboratory Materials.

# September

# Summary of Topics

Order
Inequalities
Establishing unit
Building rod patterns
Counting
Numeral recognition
Recognizing number groups
Ordering numbers
One-to-one correspondence
Tallying
Arithmetic operations
Fractions
Geometric recognition

# Work with Cuisenaire® rods

#### I

#### Free play with rods (10 minutes)

Piles of assorted rods were placed on each table. The children were free to build with the rods. (The majority of the children had attended the kindergarten at Miquon School and had been exposed to the Cuisenaire® rods previously.)

- 1 Four boys built a "water plant," a "piping system," and an "oil refinery." Other children built pyramids, flat patterns and designs, houses, and boats.
- **2** Free play ended with a trip around the room to inspect each other's buildings.

#### H

#### Group activity:

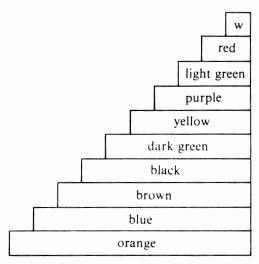
#### Discovering one-to-one correspondence

A pile of assorted rods was placed on each table.

1 Lore picked a yellow demonstration rod and held it up in front of the class. The children responded by each picking a yellow rod from his collection of rods. Then Lore held up a red demonstration rod and the children held up a red rod. This procedure continued until the one-to-one correspondence between the demonstration rods and Cuisenaire® rods was established. (The demonstration rods are homemade and cut on an inch scale, while the Cuisenaire® rods are cut on a centimeter scale.)

2 Lore picked a white demonstration rod and gave the following direction: Start with the white rod and build a color stairs.

The children built color stairs with Cuisenaire® rods. (Only two children had "gaps" in their stairs.)



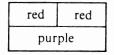
The color stairs

#### Ш

#### Game: "Half of"

Children entering first grade seem to understand the concept of *half*. This probably stems from early experience in sharing with other children—breaking a cookie "in half" (two equal or nearly equal pieces).

- 1 Lore: Find the rod that is "half of" the purple rod. Find "half of" the red rod. Find "half of" the dark green rod. Find "half of" the orange rod.
- 2 The red rod is half the length of the purple rod.



The white rod is "half of" the red rod; the light green rod is "half of" the dark green rod; the yellow rod is "half of" the orange rod. (The children appeared to grasp this concept easily.)

#### IV

#### A close look at the white cube

Directions from Lore and children's responses:

**1** Lore: Select one of the little white blocks from your collection of rods. This is called a *cube*.



2 Lore: Press the face of the cube against the fleshy part of your thumb or your arm. The shape you see is called a square.



**3** Lore: Make a stamp of the edge of the cube on your hand. What do you see?

Children: A line.

4 Lore: Press the corner of the cube against the fleshy part of your thumb or arm. What do you see?

Children: A dot. A point. •

- 5 Lore: How many "faces" does the white cube have? [Children's answers ranged between four and seven.]
- 6 Lore then held up a white cube and asked the children to count the faces.

## V

#### Comments

Jeff and Anne were able to measure with a ruler (centimeter scale) and read off the heights of the pyramids they built during free play. Jeff found not only that his pyramid was 21 centimeters high, but also that it was made up of 21 layers of rods.

About half of the children participated easily in some oral responses to these problems:  $3 + 3 = \square$ ;  $4 + 2 = \square$ ;  $5 + 1 = \square$ . These were introduced in the course of the discussion about the rods.