UNIT 8

Robert Boyle

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8.1

Factoring Completely

Some polynomials may require more than one step to factor them completely. Sometimes the first set of factors can be factored again. This may happen when the GCF is factored out of a polynomial.

Example 1 **Factor the polynomial: $12x^3 + 36x^2 + 27x$.**

$12x^3 + 36x^2 + 27x$	Original polynomial.
$3x(4x^2 + 12x + 9)$	GCF factored out, leaving a perfect square trinomial.
$3x(2x + 3)(2x + 3)$	Perfect square trinomial factored.

Sometimes the first set of binomial factors can be factored.

Example 2 **Factor the polynomial: $x^4 - 18x^2 + 81$.**

$x^4 - 18x^2 + 81$	Original polynomial—a perfect square trinomial.
$(x^2 - 9)(x^2 - 9)$	Trinomial factored, leaving differences of squares.
$(x - 3)(x + 3)(x - 3)(x + 3)$	Differences of squares factored.
$(x - 3)^2 (x + 3)^2$	Answer simplified with exponents.

In some cases, the second and even succeeding binomials can be factored.

Example 3 **Factor the polynomial: $x^{16} - 1$.**

$x^{16} - 1$	Original polynomial—a difference of squares.
$(x^8 + 1)(x^8 - 1)$	Binomial factored, leaving another difference of squares.
$(x^8 + 1)(x^4 + 1)(x^4 - 1)$	Binomial factored, leaving another difference of squares.
$(x^8 + 1)(x^4 + 1)(x^2 + 1)(x^2 - 1)$	Binomial factored, leaving another difference of squares.
$(x^8 + 1)(x^4 + 1)(x^2 + 1)(x + 1)(x - 1)$	Final difference of squares factored.

Math in History

The son of a wealthy landholder, twelve-year-old Robert Boyle's education included touring Europe with his older brother, Francis, and their tutor. At that time young people with financial means often followed the tradition of such a "Grand Tour" to visit classical sites in Greece and Italy. Robert and Francis spent much of their tour in Geneva, Switzerland.

Steps To Completely Factor Polynomials

- Factor any GCF from the expression.
- Look for the following special patterns and factor accordingly:
  - A difference of two squares.
  - A perfect square trinomial.
- Factor any other trinomials, if possible.
- Look for the possibility of factoring by grouping.
- After each step, check the answer to see if it can be broken down any further.

Today's Lesson

Factor the polynomials completely.

1. $5x^2 - 20x - 105$

2. $6axy - 3bxy$

3. $12x^2 - 46x + 14$

4. $3x^4 - 48$

5. $12x^2 + 96x + 192$

6. $x^8 - 1$

7. $2x^5 - 10x^3 + 8x$

8. $32x^4 - 40x^3 - 12x^2$

9. $6x^2y - xy - y$

REVIEW

Divide. 7.14

10. $(12x^2 + 7x - 10) \div (4x + 5)$

11. $(42x^2 + x - 1) \div (1 + 6x)$

Simplify the radicals. 5.9

12. $\sqrt{12x^2y^6}$

13. $\sqrt{54y^3}$

14. $\sqrt[3]{16r^{12}s^{16}}$

Simplify the expressions. 6.12

15. $-4x^2(x^2 - 5x - 2)$

16. $-8x(x - 4) - x(2x + 4)$

Multiply the sum and difference binomials. 5.11

17. $(\sqrt{5x} - 2)(\sqrt{5x} + 2)$

18. $(\frac{1}{3} - x^2y)(\frac{1}{3} + x^2y)$

19. $(xyz^2 - 6)(xyz^2 + 6)$

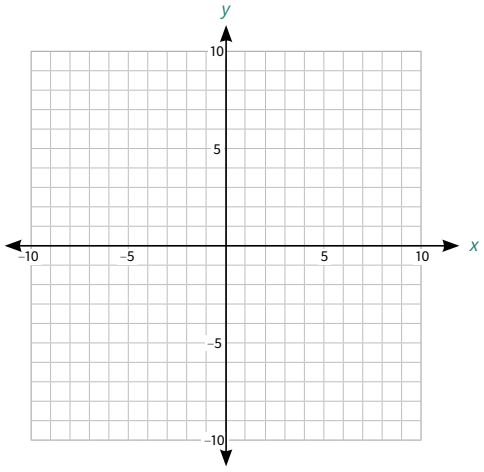
Solve. 3.11

20. $\frac{2x}{2} - \frac{x}{3} + 4 = \frac{3x}{2} - \frac{1}{6}$

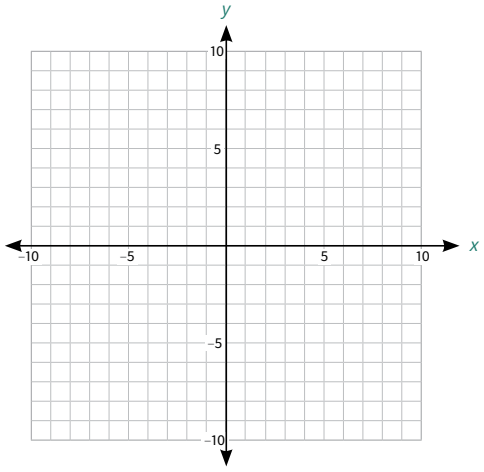
21. $0.02n - 1.0 = 0.1 - 0.2n$

Graph the equations using slope-intercept form. 5.4

22. $y = \frac{5}{6}x + 2$

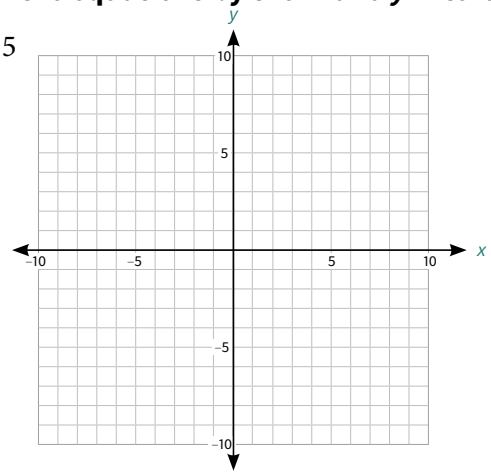


23. $5x + y = 5$

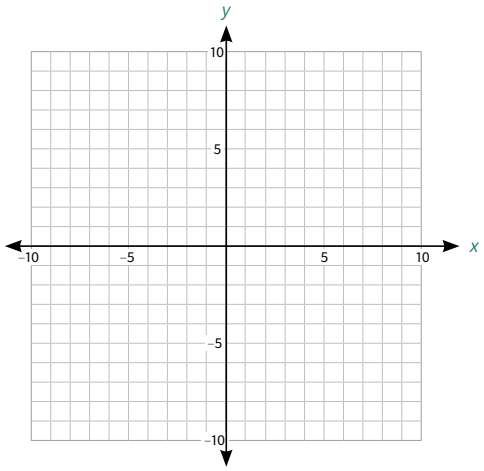


Graph each of the equations by the *x*- and *y*-intercepts. 6.1

24. $5x - 2y = 5$



25. $5x - 6y = -15$



Write equations for the lines passing through these points. 7.9

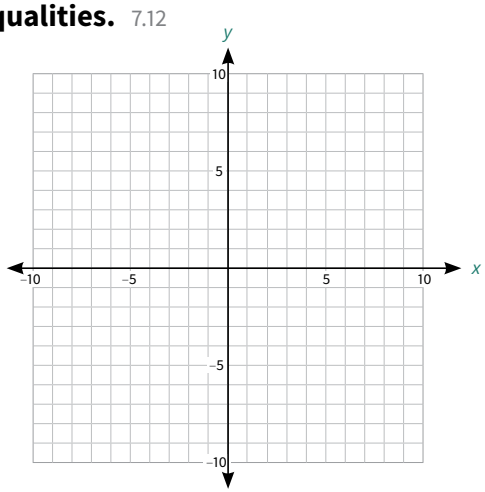
26. (–8, –1), (0, 3)

27. (4, 18), (–1, 3)

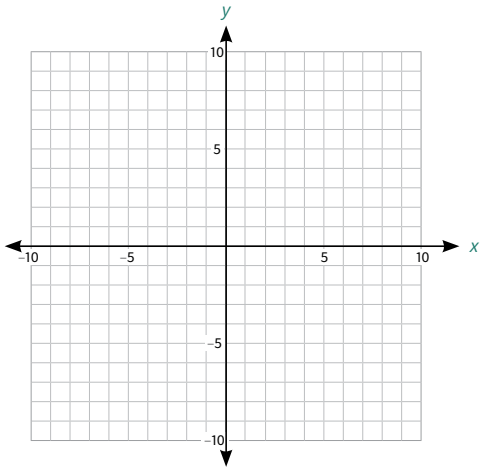
28. (4, –5), (–3, 6)

Graph the inequalities. 7.12

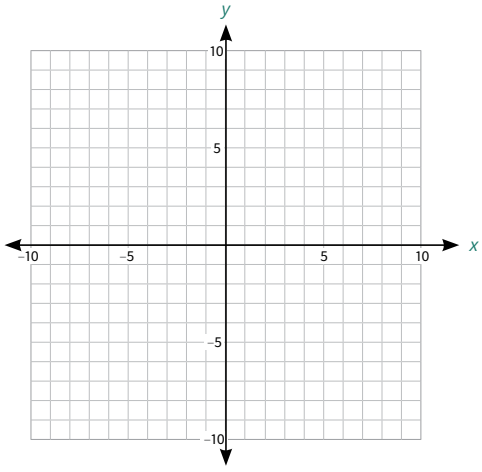
29. $2x - 8y < 4$



30. $x + 2y > 16$



31. $-\frac{1}{2}x + 2y \geq 10$



Solve by using inverse variation. 7.13

32. If q varies inversely as p, and q is 26 when p is 3, what is q when p is 4?

33. Time is inversely proportional as speed. A train traveling at 70 mph normally completes a certain trip in 7.5 hours. How fast must the train travel to complete it in 7 hours?

Use the five steps to set up and solve the equations. 5.8

34. In Norwood's coffee shop, he mixed coffee beans selling for $3.99 per pound with a coffee selling for $4.79 per pound to make a 50-pound mix that he sold for $4.49 per pound. How many pounds of each coffee were in the new mix?

Solve using formulas. 7.11

35. James and Stan each grow and sell produce for a portion of their income. At the end of one year, James calculated his profits to be 50% of his expenses while Stan's was 45%. If Stan's expenses were $1,500 more than James' and their combined profits were $8,750, what was each man's expenses and profits? (Hint: Use the interest formula with expenses as the principal and profit as the interest.)

Today's Lesson

Factor the polynomials completely.

36. $2k^3 - 12k^2 + 18k$

37. $2x^4y - 162y$

38. $54xy^2 - 18xy + 9y^2 - 3y$

Extra Practice

Factor the polynomials completely.

39. $18mn + 6m - 45n - 15$

40. $15xy - 10x + 9y - 6$