Contents

Evaluating Polynomial Functions

- **Polynomial function:** The expression used to describe the function is a polynomial.

  Example: $f(x) = 2x^3 - 3x^2 + 7x + 8$

  $g(x) = -3x^4 + 5x^2 - 2$ ← Polynomials

  $f(x)$ & $g(x)$ are functions.

- **Evaluating polynomial functions**

  Example: **1.** If $f(x) = 2x^3 + 1$, find $f(2)$ and $f(-1)$.

  $f(2) = 2(2)^3 + 1 = 16 + 1 = \mathbf{17}$ Replace x with 2.

  $f(-1) = 2(-1)^3 + 1 = -2 + 1 = \mathbf{-1}$ Replace x with -1.

  2. If $R(x) = -8x^3 + x^2 + 2$, find $R(0)$ and $R\left(\frac{1}{2}\right)$.

  $R(0) = -8(0)^3 + (0)^2 + 2 = \mathbf{2}$ Replace x with 0.

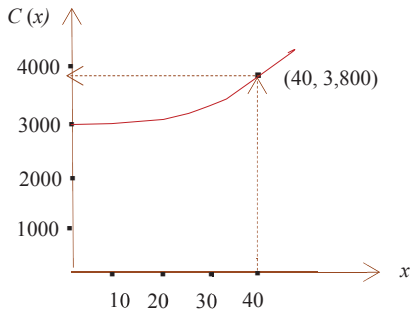
  $R\left(\frac{1}{2}\right) = -8\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^2 + 2$ Replace x with $\frac{1}{2}$.

  $= -1 + \frac{1}{4} + 2 = \mathbf{\frac{5}{4}}$

  Example: The polynomial function $C(x) = 3{,}000 + 0.5x^2$ can be used to determine the total cost (in dollars) of producing x laptops in an electronics firm.

  1. What is the total cost of producing 10 laptops?

  2. Use the following graph to estimate $C(40)$.



  Solution: **1.** $C(10) = 3{,}000 + 0.5(10)^2$

  $= \mathbf{\$3{,}050}$

  $C(x) = 3{,}000 + 0.5x^2$, replace x with 10.

  x = Number of laptops

  2. $C(40)$: locate $x = 40$ on the x axis, move vertically to the graph, and then move horizontally to the $C(x)$ axis. Thus $\mathbf{C(40) \approx \$3{,}800}$.

Adding and Subtracting Polynomials

- **Adding or subtracting polynomials**

Example: Find the sum of $2x^3 - 3x^2 + x - 4$ and $x^3 + 4x^2 + 2x + 1$.

Steps	Solution
	$(2x^3 - 3x^2 + x - 4) + (x^3 + 4x^2 + 2x + 1)$
- Regroup like terms.	$= (2x^3 + x^3) + (-3x^2 + 4x^2) + (x + 2x) + (-4 + 1)$
- Combine like terms.	$= \mathbf{3x^3 + x^2 + 3x - 3}$

Example: Find the difference of $5x^2 + 4x - 2$ and $2x^2 - 3x + 13$.

Steps	Solution
	$(5x^2 + 4x - 2) - (2x^2 - 3x + 13)$
- Remove parentheses. (Reverse each sign in second parentheses.)	$= 5x^2 + 4x - 2 - 2x^2 + 3x - 13$
- Regroup like terms.	$= (5x^2 - 2x^2) + (4x + 3x) + (-2 - 13)$
- Combine like terms.	$= \mathbf{3x^2 + 7x - 15}$

- **Column method**

Example: Find the sum of $3x^3 - 5x^2 + 7x - 3$ and $2x^3 + 3x + 5$.

Steps **Solution**

- Line up like terms in columns.
- Add.

$$\begin{array}{rl} & 3x^3 - 5x^2 + 7x - 3 \\ + & 2x^3 \quad\quad + 3x + 5 \\ \hline & \mathbf{5x^3 - 5x^2 + 10x + 2} \end{array}$$

Leave space for the missing term.

Example: Find the difference of $(5x^2 - 2x + 3) - (2x^2 - 5)$.

Steps **Solution**

- Line up like terms in columns:
- Change signs in minuend and add: (Leave space for the missing term.)

$$\begin{array}{rl} & 5x^2 - 2x + 3 \quad \leftarrow \text{Subtrahend} \\ + & -2x^2 \quad\quad + 5 \quad \leftarrow \text{Minuend} \\ \hline & \mathbf{3x^2 - 2x + 8} \quad \leftarrow \text{Difference} \end{array}$$

- **The opposite of the polynomial:** $-p$: the opposite of the polynomial
  p: polynomial

$p + (-p) = 0$

Example: Write two expressions for the opposite of the polynomial.

$$\mathbf{7a^4b^2 - 3a^3b - 4a^2}$$

Solution: opposite expression: $\mathbf{-(7a^4b^2 - 3a^3b - 4a^2)}$

or $\mathbf{-7a^4b^2 + 3a^3b + 4a^2}$

Replace each term with its opposite.

Factoring Polynomials by Grouping

Steps for factoring by grouping:

Steps	Example: $8y^2 - 2y + 12y - 3$
- Group terms with the GCF.	$8y^2 - 2y + 12y - 3 = (8y^2 - 2y) + (12y - 3)$
- Factor out the GCF from each group.	$= 2y\,(4y - 1) + 3(4y - 1)$
- Factor out the GCF again from the last step.	$= (4y - 1)(2y + 3)$

Factoring completely: Continue factoring until no further factors can be found.

Example: Factor the following completely.

1. $6ab^2 - 3a^2b + 2b - a$

$= (6ab^2 - 3a^2b) + (2b - a)$ — Group terms with the GCF.

$= 3ab\,(2b - a) + (2b - a) \cdot 1$ — Factor out the GCF ; $(2b - a) = (2b - a) \cdot 1$

$= (2b - a)\,(3ab + 1)$ — Factor out the GCF again.

2. $2ab + bc - 2bc + 4ab$

$= (2ab + 4ab) + (bc - 2bc)$ — Rearrange and group terms with the same pattern.

$= 6ab - bc$ — Combine like terms.

$= b(6a - c)$ — Factor out the GCF.

3. $x^3 - xy^2 - x^2y + y^3$

$= (x^3 - x^2y) - (xy^2 - y^3)$ — Group

$= x^2\,(x - y) - y^2\,(x - y)$ — Factor out the GCF.

$= (x - y)\,(x^2 - y^2)$ — $a^2 - b^2 = (a + b)(a - b)$

$= (x - y)\,(x + y)(x - y)$ — Keep factoring until cannot factor any further.

$= (x - y)^2\,(x + y)$

Tip: Recognize factoring patterns, such as $2b - a$, $x - y$, …

4. $32x^3y - 2xy^3$

$= 2xy\,(16x^2 - y^2)$ — Factor out the GCF.

$= 2xy\,[(4x)^2 - y^2)]$ — $a^2 - b^2 = (a + b)(a - b)$

$= 2xy\,(4x + y)\,(4x - y)$ — $(4x + y)$ and $(4x - y)$ cannot be factored further.