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Evaluating Polynomial Functions

- **Polynomial function:** The expression used to describe the function is a polynomial.

Example: $f(x) = 2x^3 - 3x^2 + 7x + 8$

$g(x) = -3x^4 + 5x^2 - 2$

Polynomials

$f(x)$ & $g(x)$ are functions.

- **Evaluating polynomial functions**

Example: 1. If $f(x) = 2x^3 + 1$, find $f(2)$ and $f(-1)$.

$f(2) = 2(2)^3 + 1 = 16 + 1 = 17$

Replace x with 2.

$f(-1) = 2(-1)^3 + 1 = -2 + 1 = -1$

Replace x with -1.

2. If $R(x) = -8x^3 + x^2 + 2$, find $R(0)$ and $R\left(\frac{1}{2}\right)$.

$R(0) = -8(0)^3 + (0)^2 + 2 = 2$

Replace x with 0.

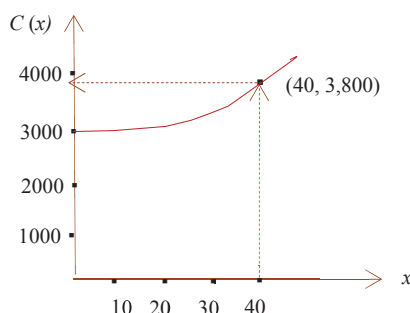
$R\left(\frac{1}{2}\right) = -8\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^2 + 2$

Replace x with $\frac{1}{2}$.

$= -1 + \frac{1}{4} + 2 = \frac{5}{4}$

Example: The polynomial function $C(x) = 3,000 + 0.5x^2$ can be used to determine the total cost (in dollars) of producing x laptops in an electronics firm.

1. What is the total cost of producing 10 laptops?
2. Use the following graph to estimate $C(40)$.



Solution: 1. $C(10) = 3,000 + 0.5(10)^2$
 $= \$3,050$

$C(x) = 3,000 + 0.5x^2$, replace x with 10.

x = Number of laptops

2. $C(40)$: locate $x = 40$ on the x axis, move vertically to the graph, and then move horizontally to the $C(x)$ axis. Thus $C(40) \approx \$3,800$.

Adding and Subtracting Polynomials

• Adding or subtracting polynomials

Example: Find the sum of $2x^3 - 3x^2 + x - 4$ and $x^3 + 4x^2 + 2x + 1$.

Steps

- Regroup like terms.
- Combine like terms.

Solution

$$\begin{aligned} & (2x^3 - 3x^2 + x - 4) + (x^3 + 4x^2 + 2x + 1) \\ &= (2x^3 + x^3) + (-3x^2 + 4x^2) + (x + 2x) + (-4 + 1) \\ &= 3x^3 + x^2 + 3x - 3 \end{aligned}$$

Example: Find the difference of $5x^2 + 4x - 2$ and $2x^2 - 3x + 13$.

Steps

- Remove parentheses.
(Reverse each sign in second parentheses.)
- Regroup like terms.
- Combine like terms.

Solution

$$\begin{aligned} & (5x^2 + 4x - 2) - (2x^2 - 3x + 13) \\ &= 5x^2 + 4x - 2 - 2x^2 + 3x - 13 \\ &= (5x^2 - 2x^2) + (4x + 3x) + (-2 - 13) \\ &= 3x^2 + 7x - 15 \end{aligned}$$

• Column method

Example: Find the sum of $3x^3 - 5x^2 + 7x - 3$ and $2x^3 + 3x + 5$.

Steps

- Line up like terms in columns.
- Add.

Solution

$$\begin{array}{r} 3x^3 - 5x^2 + 7x - 3 \\ + \quad 2x^3 \quad \quad + 3x + 5 \\ \hline 5x^3 - 5x^2 + 10x + 2 \end{array} \quad \text{Leave space for the missing term.}$$

Example: Find the difference of $(5x^2 - 2x + 3) - (2x^2 - 5)$.

Steps

- Line up like terms in columns:
- Change signs in minuend and add:
(Leave space for the missing term.)

Solution

$$\begin{array}{r} 5x^2 - 2x + 3 \\ - \quad 2x^2 \quad \quad + 5 \\ \hline 3x^2 - 2x + 8 \end{array} \quad \begin{array}{l} \longleftarrow \text{Subtrahend} \\ \longleftarrow \text{Minuend} \\ \longleftarrow \text{Difference} \end{array}$$

- **The opposite of the polynomial:**
 - p : the opposite of the polynomial
 - p : polynomial

$$p + (-p) = 0$$

Example: Write two expressions for the opposite of the polynomial.

$$7a^4b^2 - 3a^3b - 4a^2$$

Solution: opposite expression: $-(7a^4b^2 - 3a^3b - 4a^2)$

$$\text{or } -7a^4b^2 + 3a^3b + 4a^2$$

Replace each term with its opposite.

Factoring Polynomials by Grouping

Steps for factoring by grouping:

Steps

- Group terms with the GCF.
- Factor out the GCF from each group.
- Factor out the GCF again from the last step.

Example: $8y^2 - 2y + 12y - 3$

$$\begin{aligned}
 8y^2 - 2y + 12y - 3 &= (8y^2 - 2y) + (12y - 3) \\
 &= \mathbf{2y} (4y - 1) + \mathbf{3} (4y - 1) \\
 &= \mathbf{(4y - 1)(2y + 3)}
 \end{aligned}$$

Factoring completely: Continue factoring until no further factors can be found.

Example: Factor the following completely.

$$\begin{aligned}
 1. \quad 6ab^2 - 3a^2b + 2b - a &= (6ab^2 - 3a^2b) + (2b - a) \\
 &= 3ab(\mathbf{2b - a}) + (\mathbf{2b - a}) \cdot 1 \\
 &= \mathbf{(2b - a)(3ab + 1)}
 \end{aligned}$$

Group terms with the GCF.

Factor out the GCF ; $(2b - a) = (2b - a) \cdot 1$

Factor out the GCF again.

$$\begin{aligned}
 2. \quad 2ab + bc - 2bc + 4ab &= (\mathbf{2ab + 4ab}) + (\mathbf{bc - 2bc}) \\
 &= 6ab - bc \\
 &= \mathbf{b(6a - c)}
 \end{aligned}$$

Rearrange and group terms with the same pattern.

Combine like terms.

Factor out the GCF.

$$\begin{aligned}
 3. \quad x^3 - xy^2 - x^2y + y^3 &= (\mathbf{x^3 - x^2y}) - (\mathbf{xy^2 - y^3}) \\
 &= \mathbf{x^2(x - y) - y^2(x - y)} \\
 &= (x - y)(x^2 - y^2) \\
 &= (x - y)(x + y)(x - y) \\
 &= \mathbf{(x - y)^2(x + y)}
 \end{aligned}$$

Group

Factor out the GCF.

$$a^2 - b^2 = (a + b)(a - b)$$

Keep factoring until cannot factor any further.

Tip: Recognize factoring patterns, such as $2b - a$, $x - y$, ...

$$\begin{aligned}
 4. \quad 32x^3y - 2xy^3 &= 2xy(16x^2 - y^2) \\
 &= 2xy[(4x)^2 - y^2] \\
 &= \mathbf{2xy(4x + y)(4x - y)}
 \end{aligned}$$

Factor out the GCF.

$$a^2 - b^2 = (a + b)(a - b)$$

$(4x + y)$ and $(4x - y)$ cannot be factored further.