EXAMPLE | Which of the numbers below is divisible by 28?

$10{,}540 = 2^2 \cdot 5 \cdot 17 \cdot 31$ $\quad 12{,}936 = 2^3 \cdot 3 \cdot 7^2 \cdot 11$ $\quad 14{,}406 = 2 \cdot 3 \cdot 7^4$

The prime factorization of 28 is $2^2 \cdot 7$. So, a number that is divisible by 28 must have at least two 2's and one 7 in its prime factorization.

$10{,}540 = 2^2 \cdot 5 \cdot 17 \cdot 31$:
Since there is no 7 in the prime factorization of 10,540, it is not divisible by 28.

$12{,}936 = 2^3 \cdot 3 \cdot 7^2 \cdot 11$:
Since the prime factorization of 12,936 has at least two 2's and one 7, it is divisible by 28.
$12{,}936 = 2^3 \cdot 3 \cdot 7^2 \cdot 11 = (2 \cdot 2 \cdot 7) \cdot 2 \cdot 3 \cdot 7 \cdot 11 = (28) \cdot 2 \cdot 3 \cdot 7 \cdot 11$.

$14{,}406 = 2 \cdot 3 \cdot 7^4$:
Since there is only one 2 in the prime factorization of 14,406, it is not divisible by 28.

So, the only number above that is divisible by 28 is **12,936**.

PRACTICE | Answer each question below.

17. Circle every number below that is divisible by 54.

$675 = 3^3 \cdot 5^2$ $\quad 882 = 2 \cdot 3^2 \cdot 7^2$ $\quad 1{,}782 = 2 \cdot 3^4 \cdot 11$ $\quad 2{,}160 = 2^4 \cdot 3^3 \cdot 5$

18. Circle every number below that is divisible by 308.

$3{,}360 = 2^5 \cdot 3 \cdot 5 \cdot 7$ $\quad 4{,}312 = 2^3 \cdot 7^2 \cdot 11$ $\quad 6{,}468 = 2^2 \cdot 3 \cdot 7^2 \cdot 11$ $\quad 9{,}702 = 2 \cdot 3^2 \cdot 7^2 \cdot 11$

19. Circle every number below that is a factor of $4{,}095 = 3^2 \cdot 5 \cdot 7 \cdot 13$.

27 91 105 225 315

20. Circle every number below that is a factor of $5{,}472 = 2^5 \cdot 3^2 \cdot 19$.

32 48 108 119 171

EXAMPLE | The prime factorization of 11,625 is $3 \cdot 5^3 \cdot 31$. What is $11{,}625 \div 155$?

The prime factorization of 155 is $5 \cdot 31$.

We can use prime factorization to write 11,625 as the product of 155 and another integer:

$$\begin{aligned} 11{,}625 &= 3 \cdot 5^3 \cdot 31 \\ &= 3 \cdot 5 \cdot 5 \cdot 5 \cdot 31 \\ &= (5 \cdot 31) \cdot (3 \cdot 5 \cdot 5) \\ &= 155 \cdot 75. \end{aligned}$$

Since $11{,}625 = 155 \cdot \boxed{75}$, we have $11{,}625 \div 155 = \boxed{\mathbf{75}}$.

PRACTICE | Answer each question below.

21. The prime factorization of 7,425 is $3^3 \cdot 5^2 \cdot 11$. What is $7{,}425 \div 75$? **21.** ________

22. The prime factorization of 49,392 is $2^4 \cdot 3^2 \cdot 7^3$. What is $49{,}392 \div 196$? **22.** ________

23. The prime factorization of 3,780 is $2^2 \cdot 3^3 \cdot 5 \cdot 7$. What number can be multiplied by 135 to get 3,780? **23.** ________

24. ★ Ivan divides 504 by its largest odd factor. What is the result? **24.** ________

25. ★ The prime factorization of 64,800 is $2^5 \cdot 3^4 \cdot 5^2$. What is the smallest integer quotient Myrtle can get if she divides 64,800 by a power of 6? **25.** ________

EXAMPLE | Which of the following are perfect squares?

$2{,}401 = 7^4 \qquad 21{,}296 = 2^4 \cdot 11^3 \qquad 28{,}900 = 2^2 \cdot 5^2 \cdot 17^2$

A perfect square is the product of an integer and itself. So, we try to split the prime factorizations above into two identical groups of prime factors.

We can do this for 2,401 and for 28,900:

$$\begin{aligned} 2{,}401 &= 7^4 \\ &= 7 \cdot 7 \cdot 7 \cdot 7 \\ &= (7 \cdot 7) \cdot (7 \cdot 7) \\ &= 49 \cdot 49 \\ &= 49^2. \end{aligned} \qquad \begin{aligned} 28{,}900 &= 2^2 \cdot 5^2 \cdot 17^2 \\ &= 2 \cdot 2 \cdot 5 \cdot 5 \cdot 17 \cdot 17 \\ &= (2 \cdot 5 \cdot 17) \cdot (2 \cdot 5 \cdot 17) \\ &= (170) \cdot (170) \\ &= 170^2. \end{aligned}$$

However, $$\begin{aligned} 21{,}296 &= 2^4 \cdot 11^3 \\ &= 2 \cdot 2 \cdot 2 \cdot 2 \cdot 11 \cdot 11 \cdot 11 \\ &= (2 \cdot 2 \cdot 11) \cdot (2 \cdot 2 \cdot 11) \cdot 11. \end{aligned}$$

There is no way to group the last 11 so that we have two identical groups of factors.

So, only **2,401** and **28,900** are perfect squares.

PRACTICE | Answer each question below.

26. Write the prime factorization of each perfect square below.

81 = ______________________ 121 = ______________________

1,600 = ______________________ 3,600 = ______________________

27. Write each prime factorization below as a perfect square.

Ex: $2^4 \cdot 5^2 =$ $\underline{20^2}$ $3^2 \cdot 11^2 =$ _____ $2^8 =$ _____ $2^4 \cdot 3^2 \cdot 7^2 =$ _____

28. Circle every number below that is a perfect square.

$7{,}776 = 2^5 \cdot 3^5 \qquad 3{,}136 = 2^6 \cdot 7^2 \qquad 81{,}796 = 2^2 \cdot 11^2 \cdot 13^2 \qquad 444{,}771 = 3^4 \cdot 17^2 \cdot 19$

PRACTICE | Answer each question below.

29. Circle each number below that is a perfect square when x and y are different prime numbers.

$x^5 \cdot y^5$ x^{81} $x^2 \cdot y^3$ y^{12} $x \cdot y$ $x^4 \cdot y^{10}$

30. Grogg says that if a number is a perfect square, then its prime factorization includes only even exponents. Lizzie says that if the prime factorization of a number includes only even exponents, then the number is a perfect square. Who is correct: Grogg, Lizzie, or both? Explain.

31. Is 9^3 a perfect square? If so, write 9^3 as a perfect square. If not, explain why not.

32. What is the smallest positive integer n for which $180n$ is a perfect square?

32. __________

33. The prime factorization of 6,174 is $2 \cdot 3^2 \cdot 7^3$. What is the smallest positive integer that can be multiplied by 6,174 to make a perfect square?

33. __________

34. What is the largest perfect square factor of $23{,}520 = 2^5 \cdot 3 \cdot 5 \cdot 7^2$?

34. __________