

A *Activities for Learning, Inc.*

RIGHTSTART™ MATHEMATICS

by Joan A. Cotter, Ph.D.

LEVEL A LESSONS
FOR HOME EDUCATORS

Special thanks to Sharalyn Colvin, who converted *RightStart™ Mathematics: Grade K Lessons* into *RightStart™ Mathematics: Level A For Home Educators*.

Note: Rather than use the designation, K-4, to indicate a grade, levels are used. Level A is kindergarten, Level B is first grade, and so forth.

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Home Educators

RIGHTSTART™ MATHEMATICS

by Joan A. Cotter, Ph.D.

The following are items needed to complete the RightStart™ Mathematics Level A Lessons:

STATUS	ITEM	CODE
REQUIRED	<i>Level A Lessons</i>	T-A
REQUIRED	<i>Level A worksheets</i>	W-A
RECOMMENDED	<i>Math Card Games</i> book	M
REQUIRED; either abacus	Classic AL Abacus - 8-1/2" x 9-1/2" hardwood frame & beads	A-CL
	Standard AL Abacus - 7-1/2" x 9-1/2" plastic frame & beads	A-ST
REQUIRED	Place Value Cards	P
REQUIRED	Abacus Tiles	AT
REQUIRED	Cards, Six Special Decks needed for Games	C
RECOMMENDED	Basic Drawing Board Geometry Set	DS
REQUIRED	Geoboards	R3
REQUIRED	Colored Tiles, apx 200 in set	RH2
RECOMMENDED	Casio Calculator SL-450	R4
REQUIRED	4" Geared Clock	R12
RECOMMENDED	Math Balance (Invicta)	R7
REQUIRED	Tally Sticks, apx 50 count	RH1
REQUIRED	Yellow is the Sun CD	CD Sun
REQUIRED	Geometry Reflector	RF
RECOMMENDED	Wood Geometry Solids, 12 in set	R14
RECOMMENDED	Plastic Coins	R5

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RightStart™ MATHEMATICS: OBJECTIVES FOR LEVEL A

Name _____

Year _____

Teacher _____

Numeration

- Can count out 31 objects and arrange in groups of tens
- Can recognize quantities 1-100 and represent them on abacus
- Knows even numbers to 20
- Knows odd numbers to 19
- Can count by twos to 30
- Can count by fives to 100
- Can count by tens to 100

1ST QTR	2ND QTR	3RD QTR	4TH QTR
N/A			
N/A			
N/A			
N/A	N/A		

Money

- Knows name and value of penny, nickel, and dime

N/A			
-----	--	--	--

Place Value

- Knows 10 ones is 1 ten
- Knows 10 tens is 1 hundred
- Knows, for example, 37 as 3-tens 7

N/A			
N/A	N/A		
N/A	N/A		

Addition

- Understands addition as combining parts to form whole
- Can partition numbers 3-10 into parts
- Knows number combinations equal to 10
- Knows number combinations up to 10

N/A	N/A		
N/A	N/A		

Subtraction

- Understands subtraction as missing addend
- Understands subtraction as separating

N/A	N/A	N/A	
N/A	N/A	N/A	

Problem Solving

- Can solve addition problems
- Can solve missing addend problems
- Can solve basic subtraction problems

N/A			
N/A	N/A		
N/A	N/A	N/A	

Geometry

- Knows mathematical names of triangle, rectangle, and circle
- Knows parallel and perpendicular lines
- Can continue a pattern on the geoboard

N/A			

Time

- Knows days of the week
- Knows months of the year
- Can tell time to the hour
- Can tell time to the half hour

N/A			
N/A			
N/A	N/A	N/A	
N/A	N/A	N/A	

Measurement

- Can determine length with nonstandard measure

N/A	N/A	N/A	
-----	-----	-----	--

Fractions

- Can divide into halves and fourths
- Knows unit fractions up to 1/16

N/A	N/A	N/A	
N/A	N/A	N/A	

How This Program Was Developed

We have been hearing for years that Japanese students do better than U.S. students in math in Japan. The Asian students are ahead by the middle of first grade. And the gap widens every year thereafter.

Many explanations have been given, including less diversity and a longer school year. Japanese students attend school 240 days a year.

A third explanation given is that the Asian public values and supports education more than we do. A first grade teacher has the same status as a university professor. If a student falls behind, the family, not the school, helps the child or hires a tutor. Students often attend after-school classes.

A fourth explanation involves the philosophy of learning. Asians and Europeans believe anyone can learn mathematics or even play the violin. It is not a matter of talent, but of good teaching and hard work.

Although these explanations are valid, I decided to take a careful look at how mathematics is taught in Japanese first grades. Japan has a national curriculum, so there is little variation among teachers.

I found some important differences. One of these is the way the Asians name their numbers. In English we count ten, eleven, twelve, thirteen, and so on, which doesn't give the child a clue about tens and ones. But in Asian languages, one counts by saying ten-1, ten-2, ten-3 for the teens, and 2-ten 1, 2-ten 2, and 2-ten 3 for the twenties.

Still another difference is their criteria for manipulatives. Americans think the more the better. Asians prefer very few, but insist that they be imaginable, that is, visualizable. That is one reason they do not use colored rods. You can imagine the one and the three, but try imagining a brown eight—the quantity eight, not the color. It can't be done without grouping.

Another important difference is the emphasis on non-counting strategies for computation. Japanese children are discouraged from counting; rather they are taught to see quantities in groups of fives and tens.

For example, when an American child wants to know $9 + 4$, most likely the child will start with 9 and count up 4. In contrast, the Asian child will think that if he takes 1 from the 4 and puts it with the 9, then he will have 10 and 3, or 13.

Unfortunately, very few American first-graders at the end of the year even know that $10 + 3$ is 13.

I decided to conduct research using some of these ideas in two similar first grade classrooms. The control group studied math in the traditional work-book-based manner. The other class used the lesson plans I developed. The children used that special number naming for three months.

They also used a special abacus I designed, based on fives and tens. I asked 5-year-old Stan how much is $11 + 6$. Then I asked him how he knew. He replied, "I have the abacus in my mind."

The children were working with thousands by the sixth week. They figured out how to add four-place numbers on paper after learning how to do it on the abacus.

Every child in the experimental class, including those enrolled in special education classes, could add numbers like $9 + 4$, by changing it to $10 + 3$.

I asked the children to explain what the 6 and 2 mean in the number 26. Ninety-three percent of the children in the experimental group explained it correctly while only 50% of third graders did so in another study.

I gave the children some base ten rods (none of them had seen them before) that looked like ones and tens and asked them to make 48. Then I asked them to subtract 14. The children in the control group counted 14 ones, while the experimental class removed 1 ten and 4 ones. This indicated that they saw 14 as 1 ten and 4 ones and not as 14 ones. This view of numbers is vital to understanding algorithms, or procedures, for doing arithmetic.

I asked the experimental class to mentally add $64 + 20$, which only 52% of nine-year-olds on the 1986 National test did correctly; 56% of those in the experimental class could do it.

Since children often confuse columns when taught traditionally, I wrote $2304 + 86 =$ horizontally and asked them to find the sum any way they liked. Fifty-six percent did so correctly, including one child who did it in his head.

This following year I revised the lesson plans and both first grade classes used these methods. I am delighted to report that on a national standardized test, both classes scored at the 98th percentile.

Some General Thoughts on Teaching Mathematics

1. Only five percent of mathematics should be learned by rote; 95 percent should be understood.
2. Teaching with understanding depends upon building on what the child already knows. Teaching by rote does not care.
3. The role of the teacher is to encourage thinking by asking questions, not giving answers. Once you give an answer, thinking usually stops.
4. It is easier to understand a new model after you have made one yourself. For example, a child needs to construct graphs before attempting to read ready-made graphs.
5. Good manipulatives cause confusion at first. If the new manipulative makes perfect sense at first sight, it wasn't needed. Trying to understand and relating it to previous knowledge is what leads to greater learning, according to Richard Behr and others.
6. Lauren Resnick says, "Good mathematics learners expect to be able to make sense out of rules they are taught, and they apply some energy and time to the task of making sense. By contrast, those less adept in mathematics try to memorize and apply the rules that are taught, but do not attempt to relate these rules to what they know about mathematics at a more intuitive level."
7. According to Arthur Baroody, "Teaching mathematics is essentially a process of translating mathematics into a form children can comprehend, providing experiences that enable children to discover relationships and construct meanings, and creating opportunities to develop and exercise mathematical reasoning."
8. Mindy Holte puts learning the facts in proper perspective when she says, "In our concern about the memorization of math facts or solving problems, we must not forget that the root of mathematical study is the creation of mental pictures in the imagination and manipulating those images and relationships using the power of reason and logic."
9. The only students who like flash cards are those who don't need them.
10. Mathematics is not a solitary pursuit. According to Richard Skemp, solitary math on paper is like reading music, rather than listening to it; "Mathematics, like music, needs to be expressed in physical actions and human interactions before its symbols can evoke the silent patterns of mathematical ideas (like musical notes), simultaneous relationships (like harmonies) and expositions or proofs (like melodies)."
11. "More than most other school subjects, mathematics offers special opportunities for children to learn the power of thought as distinct from the power of authority. This is a very important lesson to learn, an essential step in the emergence of independent thinking." (A quote from *Everybody Counts*)

12. Putting thoughts into words helps the learning process.
13. The difference between a novice and an expert is that an expert catches errors much more quickly. A violinist adjusts pitch so quickly that the audience does not hear it.
14. Europeans and Asians believe learning occurs not because of ability, but primarily because of effort. In the ability model of learning, errors are a sign of failure. In the effort model, errors are natural. In Japanese classrooms, the teachers discuss errors with the whole class.
15. For teaching vocabulary, be sure either the word or the concept is known. For example, if a child is familiar with six-sided figures, we can give him the word, hexagon. Or, if he has heard the word, multiply, we can tell him what it means. It is difficult to learn a new concept and the term simultaneously.
16. Introduce new concepts globally before details. This lets the children know where they are headed.
17. Informal mathematics should precede paper and pencil work. Long before a child learns how to add fractions with unlike denominators, she should be able to add one half and one fourth mentally.
18. Some pairs of concepts are easier to remember if one of them is thought of as dominant. Then the non-dominant concept is simply the other one. For example, if even is dominant over odd; an odd number is one that is not even.
19. Worksheets should also make the child think. Therefore, they should not be a large collection of similar exercises, but should present a variety.
20. In Japan students spend more time on fewer problems. Teachers do not concern themselves with attention spans as is done in the U.S.
21. In Japan the goal of the math lesson is that the student has understood a concept, not necessarily has done something (a worksheet).
22. The calendar should show the entire month, so the children can plan ahead. The days passed can be crossed out or the current day circled.
23. A real mathematical problem is one in which the procedures to find the answer or answers are not obvious. It is like a puzzle, needing trial and error. Emphasize the satisfaction of solving problems and like puzzles, of not giving away the solution to others.
24. Keep math time enjoyable. A person who dislikes math will avoid it. We store our emotional state along with what we've learned. A child under stress stops learning. If a lesson is too hard, end it and play a game. Try again another day.

RightStart™ Mathematics

There are 13 major characteristics that make this research-based program effective.

1. Refers to quantities of up to 5 as a group; discourages counting individually.
2. Uses fingers and tally sticks to show quantities up to 10; teaches quantities 6 to 10 as 5 plus a quantity, for example $6 = 5 + 1$.
3. Avoids counting procedures for finding sums and remainders. Teaches five- and ten-based strategies for the facts that are both visual and visualizable.
4. Employs games, not flash cards, for practice.
5. Once quantities 1 to 10 are known, proceeds to 10 as a unit. Uses the “math way” of naming numbers for several months; for example, “1 ten-1” (or “ten-1”) for eleven, “1-ten 2” for twelve, “2-ten” for twenty, and “2-ten 5” for twenty-five.
6. Uses expanded notation (overlapping) place-value cards for recording tens and ones; the ones card is placed on the zero of the tens card. Encourages a child to read numbers starting at the left and not backward by starting at the ones column.
7. Proceeds rapidly to hundreds and thousands using manipulatives and place-value cards. Provides opportunities for trading between ones and tens, tens and hundreds, and hundreds and thousands with manipulatives.
8. Only after the above work, about the fourth month of first grade, introduces the traditional English names for quantities 20 to 99 and then 11 to 19.
9. Teaches mental computation. Investigates informal solutions, often through story problems, before learning procedures.
10. Teaches four-digit addition on the abacus, letting the child discover the paper and pencil algorithm. This occurs in Level B. Four-digit subtraction is mastered in Level C.
11. Introduces fractions with a linear visual model.
12. Approaches geometry through drawing boards and tools.
13. Teaches short division (where only the answer is written down) for single-digit divisors, before long division. Both are taught in Level E.

Some Pointers

Kindergarten. Most of the kindergarten lesson plans have two distinct topics, which can be taught on alternate days.

Transition Lessons. These lessons are designed for children starting Levels C, D, or E (grades 2, 3, or 4) who have not been doing RightStart™ Mathematics previously. The lessons need to be studied before the regular manual, except where noted. The manual tells which lessons to use for the particular grade.

Counting. Counting needs to be discouraged because it is slow and inaccurate. It also interferes with understanding quantity and learning place-value.

Warm-up. The warm-up time is the time for quick review, memory work, and sometimes an introduction to the day's topics. The drawing board makes an ideal slate for quick responses.

Place value. In order to understand addition algorithms, place-value knowledge is essential. From the very beginning, the children are helped to see quantities grouped in fives and tens. Children can understand place value in first grade and even in kindergarten when it is approached as it is in this program.

Worksheets. The worksheets are designed to give the children a chance to think about and to practice the day's lesson. Some lessons, especially in the early grades, have no worksheet.

Games. Games, not worksheets or flash cards, are used for practice. They can be played as many times as necessary until memorization takes place. Games are as important to math as books are to reading.

Some games are incorporated in this manual. Extra games, found in the book, *Math Card Games*, are suggested in the Review and Practice lessons in Levels C to E. There are games for the child needing extra help, as well as for the advanced child.

Teaching. Establish with the children some indication when you want a quick response and when you want a more thoughtful response. Research shows that the quiet time for thoughtful response should be about three seconds. Avoid talking during this quiet time; resist the temptation to rephrase the question. This quiet time gives the slower child time to think. It also gives a quicker child time to think more deeply.

Encourage the child to develop perseverance. Avoid giving answers too quickly. Children tend to stop thinking once they hear the answer.

Help the children realize that it is their responsibility to ask questions when they do not understand. Do not settle for "I don't get it."

Number of lessons. It is not necessary that each lesson be done in one day. Sometimes two days may be more appropriate. However, do complete each manual before going to the next one.

Visualization. The ability to imagine or visualize is an important skill to develop in mathematics and other subjects as well. Often you are called upon to suggest to the children that they imagine a particular topic.

Questions. I really want to hear how this program is working. Please let me know any improvements and suggestions that you may have.

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Level A Lessons for Home Educators

Each lesson is designed with enough activities for two days.

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Lesson 1

Quantities 1 to 3

- OBJECTIVES**
1. To recognize quantities 1 to 3 without counting
 2. To learn finger sets and tally marks for quantities 1 to 3
- MATERIALS**
- 4 tally sticks (craft sticks, also called Popsicle sticks)
 - Various collections of 1 to 3 interesting objects
 - Two trays or two sheets of construction paper
- WARM-UP**
- Teach the nursery rhyme, “One, Two, Buckle my Shoe.” This rhyme will be used later for teaching other skills.

One, Two, Buckle my Shoe

*One, two,
Buckle my shoe.*

*Three, four,
Shut the door.*

*Five, six,
Pick up sticks.*

*Seven, eight,
Lay them straight.*

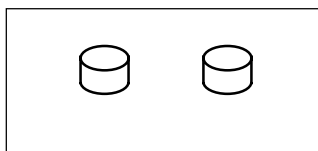
*Nine, ten,
A big, fat hen.*

- NOTES**
- Recognizing quantities. We now know that babies 5 months old can recognize quantities up to 3 and they’re not counting. Many babies at 1 year of age can recognize 4 objects. When very young children count small collections, they are tagging, or naming each item and are not aware of the quantity. In other words, if you ask them after they have counted 3 objects, “Where are the 3 objects,” they will point to the last one, not the whole collection. What happens when you point to an object in counting is that the child concentrates on the single object and loses the concept of the whole. Therefore, do not count fewer than 5 objects.

Finger sets. Finger sets, or the use of fingers, to show the quantity gives the child a concrete feel of numbers. Many parents have started this by encouraging their children to show their ages with fingers. You will be building on this idea. However, we want the children to use their left hands for showing numbers 1 to 5 to match the abacus and to reinforce the left to right orientation. It may be easier when showing fingers if the thumb touches the remaining fingers kept down.

- ACTIVITIES**
- Left hand.*** Ask the child to raise her *left* hand. Also ask her to point to her left foot, left eye, or other left body parts.

Quantity of 2. Place 2 objects on the tray or paper within sight of the child. Ask her how many she sees. [2] Ask her if she needed to count. [no] Demonstrate 2 with your fingers and ask the child to show the number with her left hand. (Left hands are used for numbers less than 5 to correspond to the abacus and tally marks and to



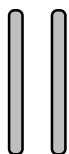
Two objects set on a tray.



2 with fingers on the left hand.

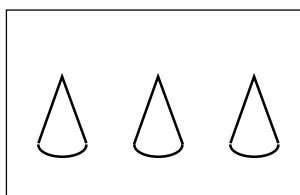
correlate with reading from left to right.) Use your right hand if you are facing her. It does not matter which fingers she uses as long as she uses her left hand.

Place the tally sticks within reach of the child. Tell her that we will call them *tally sticks* because we will use them to make tally marks. Ask her to take 3 tally sticks. Next ask her to lay out the same number of tally sticks as objects on the tray. [2] Show her how to lay the tally sticks vertically as shown.



Making 2 with tally sticks.

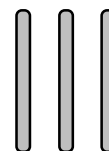
Quantity of 3. Place 3 different objects in a line on a second tray. Ask, How many do you see? [3] Ask her to show the number with her fingers and with the tally sticks. Did you need to count? [no]



3 objects on tray.



3 on fingers.



3 on tally sticks.

Rearrange the objects and again ask how many and to show it. [3] Ask, Did you need to count? [no]

Quantity of 1. On the third tray place one object (choose an object larger than those used previously). Ask the child how many [1] and to show it with her fingers and tally sticks.

Finding given quantities. Now ask the child to look around the room and name 1 of something. [for example, the table] Repeat for finding 2 and 3 objects.

Disagreeing. Place 3 objects on one tray and 2 objects on a second tray. Ask the child if she agrees that there are 3 objects on both trays. It is very important that the child feels comfortable disagreeing with you during math time. After a short discussion about the number of objects on the trays, add one more object to the tray with 2 objects and ask her if she agrees now that they both have 3.

Changing quantities. Start with 3 objects on a tray; ask the child to show the number with her fingers. Now ask her to watch. Remove an object and ask her to show the new number. [2] Repeat adding or removing objects, but do not exceed 3 on the tray.

Lesson 2

Quantities 1 to 4 & Sorting

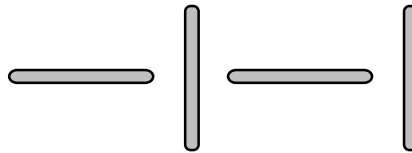
- OBJECTIVES**
1. To recognize and continue a simple pattern
 2. To review finger sets for quantities 1 to 3
 3. To recognize 4 objects and to represent 4 with finger sets and tally sticks
 4. To determine likenesses and differences through sorting

- MATERIALS**
- Tally sticks (see Lesson 1)
 - Various collections of 1 of 4 interesting objects
 - Two trays or sheets of construction paper
 - Various containers of interesting objects for sorting, such as colored tiles, crayons, beads, geometric figures, seeds, beans, washers, bolts, pictures of flowers, and pictures of birds. Items can be sorted by color, size, or other attributes.

- WARM-UP** Continue teaching the rhyme, "One, Two, Buckle my Shoe." See Lesson 1.

- NOTE** Sorting is an activity with many applications. For example, in science, items can be sorted whether alive or inanimate; in geography, by continent; in music, by composer or period; in everyday life, clean or dirty.

- ACTIVITIES** ***Continuing the pattern.*** Take a group of tally sticks or a container of crayons and lay one out horizontally. Place the next one vertically, the third one horizontally and the fourth one vertically. Ask the child what direction the fifth stick or crayon should go. Ask the child to continue to lay out the pattern. Ask him what the pattern is. [horizontal, vertical, or similar words]



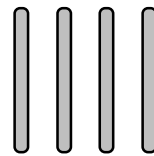
Continuing the pattern.

Review. Show 1 to 3 objects and ask the child to show the number with his fingers on his left hand and with tally sticks. Stressing the number words, ask him to build 3 with the tally sticks. Then ask him to change it to 1, to 3, to 1, and to 2.

Quantities 1 to 4. Show 3 objects on the tray and ask the child how many he sees. [3] Add 1 more and ask him how many he sees now. [4] Ask him to show it with his fingers. See the figures below.



Showing 4 with fingers.



Making 4 with tally sticks.

Ask him to take 4 tally sticks and to lay them out as shown above. Then ask him to build 3 with the tally sticks. Then ask him to change it to 4, to 1, to 2 and so on.

Now tell him that you will play a different game. Ask him to start with 4 tally sticks. Then ask him to remove 2 sticks and say how many he has. [2] Ask him to add 1 and say how many. [3] Ask him to add another 1 and name the number. [4] Repeat for other quantities up to 4.

Do the same type of activity with objects on a tray.

NOTE Ordinal counting is familiar to most children. It has an additional value in beginning mathematics because of the sounds “thir” and “fif,” which we need in English to pronounce thirteen, thirty, one-third, as well as fifteen, fifty, and one-fifth.

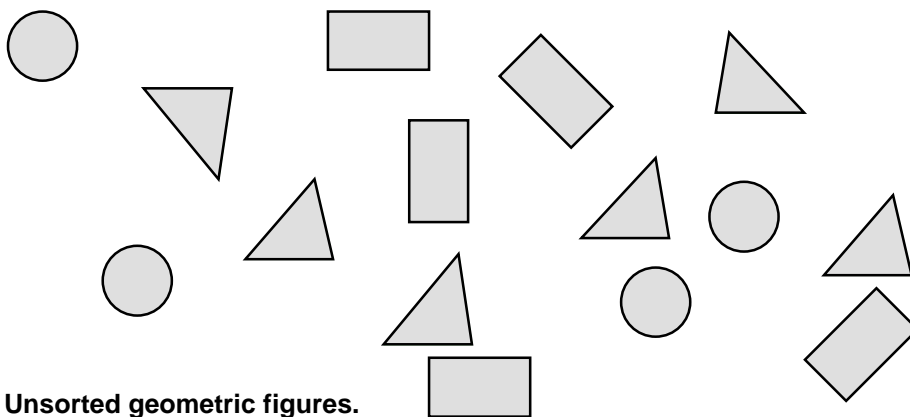
Ordinal counting. Ask the child to place 3 favorite toys in a row. Ask him which toy is first in line. Discuss which end is first. Once the beginning is established, ask what toys are second and third.

Also ask the inverse; in what position is the red block; in what position is the doll.

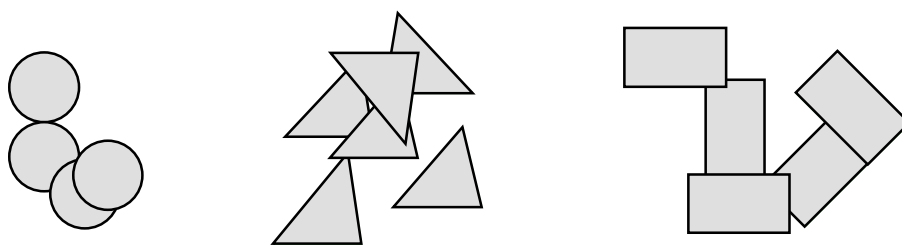
Sorting. Combine the contents of two containers onto a special area. Then let the child proceed to sort the objects and return them to their original containers.

Alternately, where a container has several different types of objects, (for example, different colored tiles) the child may sort them into different piles and then return all the objects to the original container.

Remind the child that he must not choose too large of a sorting job since it must be finished before the end of the lesson time.



Unsorted geometric figures.



Sorted geometric figures.

Lesson 3

Quantities 1 to 5 & Matching

- OBJECTIVES**
1. To learn finger sets and tally marks for quantities 1 to 5
 2. To identify without counting 1 to 5 objects
 3. To learn to detect the number of taps from 1 to 3
- MATERIALS**
- 5 interesting objects and tray
 - Tally sticks
 - Various containers of 6 to 8 identical or similar in some aspect, pairs of interesting objects for matching, such as colored tiles, beads, seeds, beans, washers, bolts, pictures of flowers, birds, or animals, and cards with numerals.
- WARM-UP**
- Continue teaching the rhyme, “One, Two, Buckle my Shoe.” Ask her to say it while clapping only on the numbers.
- NOTE**
- Matching as a life skill can encourage children to look beyond the immediately obvious and to integrate other concepts. Some examples of matching include: a fruit with its seed, pairs of shoes or socks, lower case letters to capital letters, initial sounds with letters, and sports with equipment. It will be used frequently throughout the year for practicing mathematics concepts.
- ACTIVITIES**
- Quantities 1 to 5.** Use objects and review using fingers on the *left* hand to show up to 4. With 4 objects on the tray, ask the child to watch while you add 1 more. Tell her it is 5.

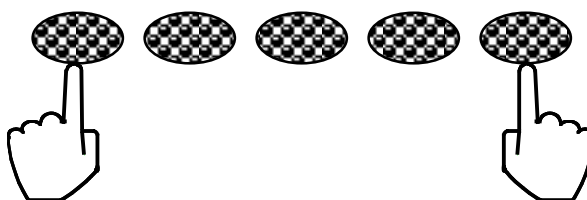
Ask her to show 5 with her fingers. Ask, What is special about 5 on your hand? [whole hand]

Seeing 5 as having a middle. With the 5 objects in a row, explain, Five has something else special about it. It has a *middle*.

Demonstrate how to find a middle. With a row of 5 objects, point to the first object with your left hand and last object with your right hand. Then simultaneously point to the second and fourth objects. The remaining object is the middle. See the figures below.

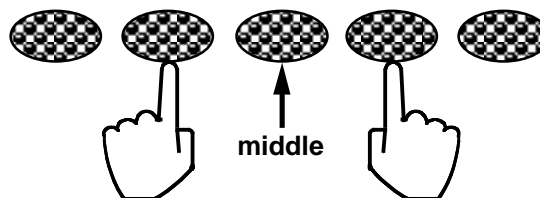


Showing 5 with fingers.



Finding the middle of 5.

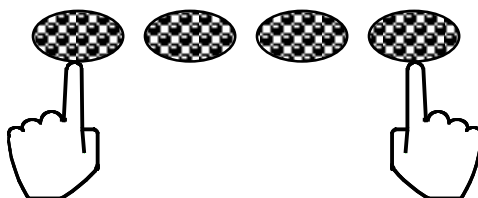
Step 1.



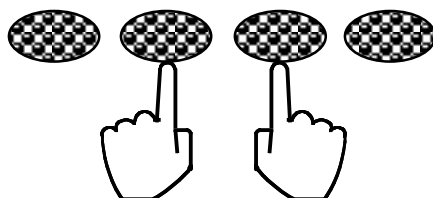
Step 2.

Ask, Does your hand have a middle finger? [yes]

Comparing 5 to 4. Remove 1 object and ask, Do you think 4 has a middle? [no] Repeat the same procedure to see if 4 has a middle. See the figures below. Then ask, Do your fingers have a middle when they show 4? [no]

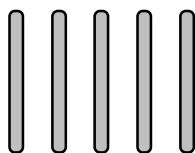


Does 4 have a middle?

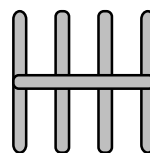


No middle.

5 with tally sticks. Ask the child to lay out 5 tally sticks. Now tell her that we do something to make the 5 special. Demonstrate picking up the last tally stick and laying it across the other four as shown. Be sure the fifth stick covers the other four sticks. The horizontal position is easier for children than the diagonal position.



5 tally sticks.



The special way of showing 5.

Practice. Practice with the child by: (a) showing 1 to 5 objects in various configurations and asking her to show fingers and to say the number, (b) showing fingers and asking her to name the amount, and (c) saying numbers and asking her to show fingers.

Matching. Explain that in matching she is looking for two items that belong together. Demonstrate matching by setting all the items out so they can be seen. Then pick up one item, find its match, and set them aside. Pick up another item and find its match, and so on. The items are returned to their containers at the conclusion of the activity.



Matching a collection of items.

Lesson 4

Quantities 1 to 6 & Making Triangles and Quadrilaterals

- OBJECTIVES**
1. To review *left* and *right*
 2. To learn the quantity 6 as 5 and 1 more
 3. To identify without counting 1 to 6 grouped objects
 4. To learn to make claps or taps for 1 to 3
 5. To construct triangles and quadrilaterals

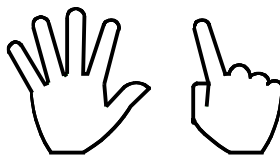
MATERIAL Tally sticks
 5 identical objects and 1 more of a different color
 Tiles or objects of several colors for making 6
 Music for “Yellow is the Sun” (end of manual), optional
 A tool for making audible taps or tones that the child can hear
 4-5 sticks, or pencils, of various lengths. Make the sticks by overlapping two tally sticks and fasten with rubber bands or glue.

WARM-UP Ask the child to clap for the numbers in “One, Two, Buckle my Shoe.” Next ask him to say and clap only the numbers aloud; the remaining words are whispered or said silently.

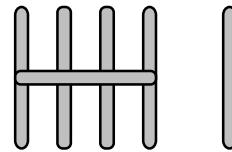
NOTE Seeing quantities between 6 and 9 as 5 plus 1 to 4 makes them visualizable, or imaginable. For example, try to visualize 8 apples without grouping them—impossible. Now think of 5 of the apples as red and 3 as green—very possible. Grouping by 5s also prepares the children to think of ten as a unit.

ACTIVITIES **Quantity 6.** Ask the child to show 5 with his left hand. Ask him to show his left hand, knee, foot, and so forth. Now ask him to show his *right* hand, foot, elbow, and so forth.

Tell him that to show the next number, 6, he will need to also use his right hand. Emphasize that 6 is 5 and 1 more. Ask him how he thinks we are going to show 6. Demonstrate showing the 5 on the left hand and the 1 on the right hand. See the figure below. For practice, ask him to show numbers between 1 and 6.



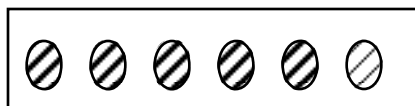
Showing 6 with fingers.



Showing 6 with tally sticks.

Now ask the child to make a 5 with the tally sticks. Remind him that 6 is 5 and 1. Show him how to make the 6 as shown above. For practice, ask him to make various numbers from 1 to 6.

Grouping 6 objects. Review by laying out 1 to 5 identical objects and ask the child to name how many he sees. End with 5 on the tray. Ask him, What would we have if we added 1 more? [6] Add the object of a different color as shown below.



Showing 6 objects using two colors.

Give the child the multicolored objects. Ask him to make quantities 1 to 5 with only one color. Then ask him to make 6, using two colors.

Drawing (optional). Ask the child to draw six of something with 5 objects the same and sixth object different.

Yellow is the Sun. Teach the child the first two lines of the “Yellow is the Sun” song. Ask them to show 6 for the second line.

Yellow is the Sun

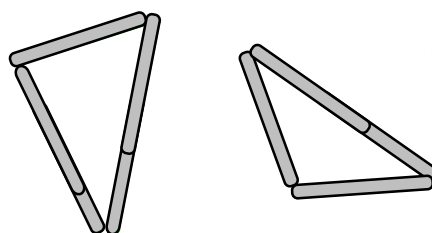
Yellow is the sun.

Six is five and one.

Claps and taps. Ask the child to either clap his hands or tap a hard surface as you say a number from 1 to 3. The clapping or tapping should be done quickly without counting. Then tap once and ask, How many did you hear? [1] Next tap twice in quick succession and ask, How many did you hear? [2] Repeat for 3. Do the tapping in random order and ask the child to show what he heard with his fingers or tally sticks.

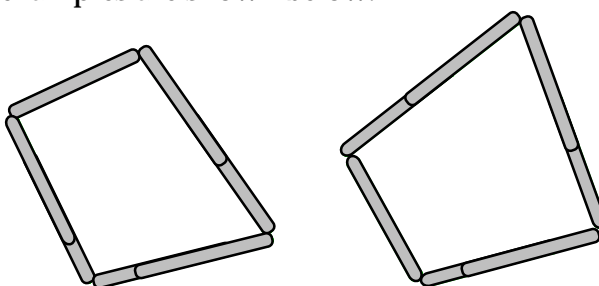
Making triangles and quadrilaterals. Take out the tally sticks, both single and those made longer with rubber bands. If necessary, explain that the longer sticks are necessary for making some shapes and should not be disassembled. See the figure below.

Ask the child to take 3 sticks and make a *triangle*. Explain that a triangle has 3 sides and an inside, which is made by making the ends of the sticks touch. Demonstrate making some: see samples below.



Triangles made with sticks.

Next ask the child to take 4 sticks and to make *quadrilaterals*. Some examples are shown below.



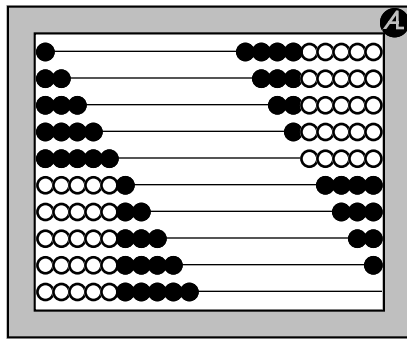
Quadrilaterals made with sticks.

After the child has made some of each, ask, How can you tell a triangle from a quadrilateral? [A triangle has 3 sides while a quadrilateral has 4 sides.] Ask the child, Did you discover anything while making them?

Lesson 9

Memory Games & Perpendicular

- OBJECTIVES**
1. To build the stairs
 2. To learn the meaning of *perpendicular*
 3. To match various representations of quantities 1 to 10 through memory games
- MATERIALS**
- Abacus
Sets of finger, tally stick, and bead pattern cards (see Lesson 8)
2 pencils or sticks of unequal length to show perpendicular lines
- WARM-UP**
- Sing “Yellow is the Sun.” Then sing “Writing Numbers,” (Lesson 8) adding motions: “to the top,” “start at the left,” “start at the right,” and “smack in the middle.”
- Flash various quantities 1 to 7 for 2 seconds, using fingers, tally sticks, or the abacus, asking the child to identify the quantity.
- ACTIVITIES**
- Building the stairs.*** Ask the child to build the stairs: ask her to enter 1 on the first wire, 2 on the second wire, and 3 on the third wire. Ask her to continue to 10.

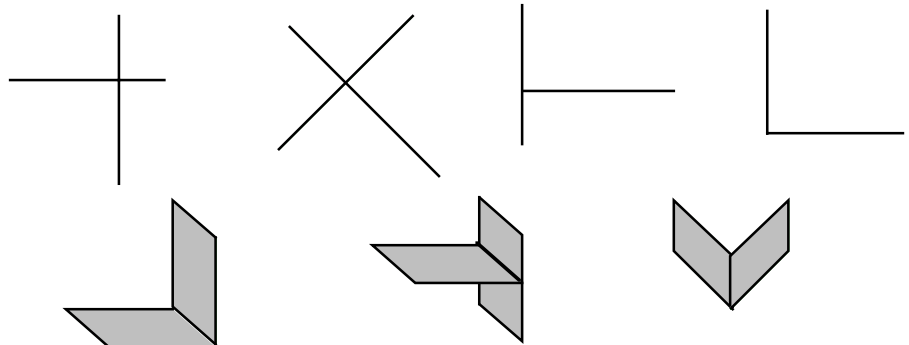


Building the stairs.

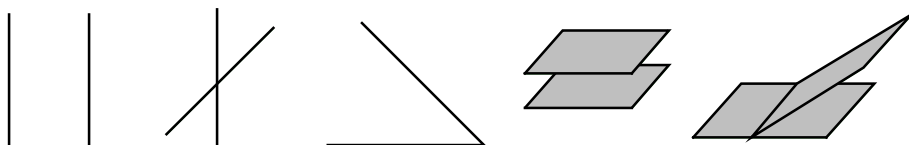
If a child has difficulty building the stairs, try this approach. Ask the child to enter 1, then to copy it onto the next wire and enter another bead. Continue by copying the last quantity to the next wire and adding another bead.

Ask her to name the number of beads from top to bottom. [1, 2, 3, . . . , 10] Also ask her to name the wires: first, second, and so on.

Perpendicular. Using two pencils or straws, demonstrate a variety of *perpendicular* lines, as shown below. Tell the child that these are perpendicular lines. Also show her perpendicular planes and tell them that they are perpendicular planes.

Lines and planes that are *perpendicular*.

Then hold them in non-perpendicular positions and tell the child that they are not perpendicular.



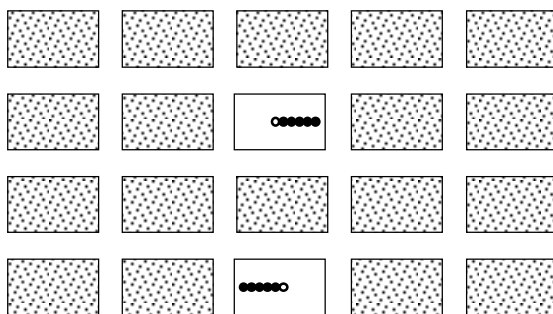
Lines and planes that are *not* perpendicular.

Ask her to find perpendicular lines and planes in the room.

Memory Games. Show the child how to play the Memory (Concentration) Game with you.

MEMORY

Objectives	To learn to quickly recognize the various patterns
Number of players	Two or two teams
Cards	Two sets of Finger, Tally Stick, or Bead Pattern cards from 1 to 10
Layout	Cards face down in rows. It does not matter if some cards are upside down
Object of the game	To collect the most pairs
Play	The first player or team turns over one card for both to see and says <i>aloud</i> the number represented on the card. The player or team then attempts to turn over a second card to match that card. If the cards match, the player or team collects both cards and receives another turn. If the cards do not match, the cards are returned face down in their original places and the other player or team takes a turn. Continue until all cards are matched.



Playing Memory with the Bead cards.

Post-Game Sort	To sort the cards into sets after the last game, the players turn all cards face up. Each player collects a set of cards in order from 1 to 10.
Variations	This game can also be played with different sets of cards; for example, one set of finger pattern cards and one set of tally sticks pattern cards.

Lesson 26

Equations & Overlapping Tens and Ones

- OBJECTIVES**
1. To introduce the term *consecutive*
 2. To introduce the term *equation*
 3. To overlap the 10s and 1s of the place value cards
- MATERIALS** Place value cards 1 to 9 and 10 to 90
Abacus
- WARM-UP** Play the Comes Before Game with basic number cards from 0 to 9.
Ask the child to say the names of the days of the week. Then Play the Comes Before Game with the days of the week.

Play the Name the Pattern (Name the Rule) game with spoken numbers. Tell the child, I am going to name some numbers and you tell me what pattern you hear. Start with 2, 4, 6, 8. [even numbers] Continue with 3-ten, 4-ten, 5-ten. [counting 10s] Say 8, 7, 6, 5. [counting backward] Conclude with 2, 3, 4, 5. [possibly, numbers in order] Give him the word *consecutive*.
- ACTIVITIES** ***More than problem.*** Give the child time to think after asking him, What number can I enter on the top row of the abacus that is more than 5? [6-10] For each answer volunteered by the child, ask him how he could keep track of the answers. [possibly, write them on the drawing board.] After he has his list, have him verify his answers on with the abacus.
- NOTE** An expression such as " $2 + 2 = 4$ " is properly call an *equation*. The term "number sentence," which sometimes is used, makes no sense either grammatically or mathematically and should be avoided. Engineers and scientists don't talk about number sentences. However, before state testing tell the child that the person who wrote the test used the "baby" word *number sentence*.

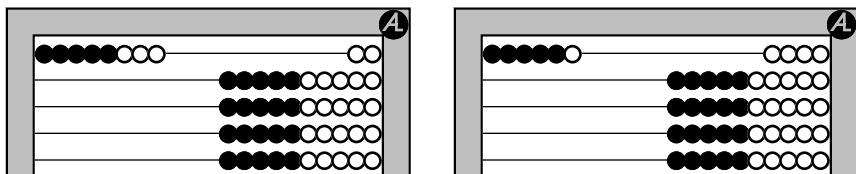
Equations. Remind the child, Last time we talked about 2 plus 2 equals 4. We call that kind of a statement an equation. Ask him if he would like to do some more equations.

Ask him, What is 3 books + 1 book? Tell him to use the abacus to find what it equals. [4 books] Say the entire equation, So 3 plus 1 equals 4.

Repeat for other simple combinations, but for now keep the second addend equal to or smaller than the first. Use 3 forks + 3 forks [3 + 3 = 6], or 5 days plus 2 days. [5 + 2 = 7] Ask him to say the equations.

Comparing quantities. You and the child will work together as partners. Ask him to enter 2 on one abacus. You then say and enter 2-ten on the other abacus. Now ask who has more entered. [you with 2-ten]

Now ask him to enter 5-ten, and you will enter 5. Ask him who now has more entered. [the child with 5-ten]



Comparing $4 + 4$ with $3 + 3$.

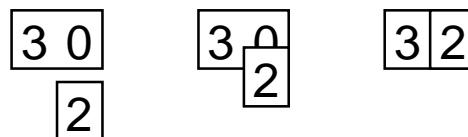
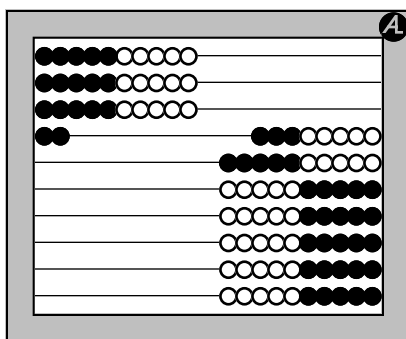
This time ask the child to enter $4 + 4$ [8]. You then say and enter $3 + 3$ [6]. Again ask, Who has more entered? [the child] Ask him, How do you know who had more?

Now ask the child to enter $4 + 1$ [5] while you will say and enter $3 + 2$ [5]. Again ask who has more entered. [No one] Ask the child why that is so. [No one has more; we have the same.]

Naming 10 quantities. Ask the child to enter 6-ten on the abacus, and then to enter 4 more. Ask how much he has entered. [6-ten 4] Repeat for several more combinations, such as 8-ten 7, 9-ten 6, and 4-ten 8.

Find place value cards. Ask the child to spread out the 10s and 1s from the place value cards. Enter 3-ten 2 on the abacus and ask him to read the quantity. [3-ten 2] Then ask him to find place value cards matching the quantities.

Show him how to overlap the cards as shown below.



Overlapping the 2-card on the 0 of the 30 card to show 3-ten 2.

Repeat asking the child to enter a quantity, such as 7-ten 3, 4-ten 8, and 1-ten 9, to find the matching cards, and to overlap them.

Game with tens and ones. Play the Can You Find game, asking for 10s, 1s, and combinations of both; ask the child to overlap when possible. Be careful to keep track of what cards remain.

Yellow is the Sun

Lyrics by Joan A. Cotter

Composed by Rosine Hermodson-Olsen
Arranged by Barbara Ask

The musical score is written for piano in 4/4 time. It consists of four systems of music, each with a treble and bass staff joined by a brace. Chord diagrams are placed above the treble staff at the beginning of each measure. The lyrics are written below the treble staff.

System 1: Measures 1-3. Chords: C, G, C, G. Lyrics: "Yellow is the sun. Six is five and one. Why is the sky so blue?"

System 2: Measures 4-6. Chords: C, F, G, C. Lyrics: "Seven is five and two. Salty is the sea. Eight is five and three."

System 3: Measures 7-9. Chords: Dm, G, C. Lyrics: "Hear the thunder roar. Nine is five and four."

System 4: Measures 10-12. Chords: C, Gmaj7, C. Lyrics: "Ducks will swim and dive. Ten is five and five."

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