

math

MAMMOTH

Grade 7-B Worktext

Ratios and proportions

Percent

Geometry

Pythagorean theorem

Probability

Statistics



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Foreword

Math Mammoth Grade 7-A and Grade 7-B worktexts comprise a complete math curriculum for seventh grade mathematics studies. This is a pre-algebra course after which students can continue to an Algebra 1 curriculum.

The curriculum meets and actually exceeds the Common Core Standards (CCS) for grade 7. The two major areas where it exceeds those standards are linear equations (chapter 5) and the Pythagorean Theorem (chapter 9). Linear equations are covered in more depth than the CCS requires, and the Pythagorean Theorem belongs to grade 8 in the CCS. You can access a document detailing the alignment information either on the Math Mammoth website or in the download version of this curriculum.

Part B starts out with a study of ratios and proportions (chapter 6). Students study unit rates, proportions, proportional relationships and graphing, scaling geometric figures, floor plans, and maps

The next chapter provides thorough lessons on the concept of percent. Students learn to solve a wide variety of problems involving percentages, including percentage of change, percentages of comparison, and simple interest problems.

Geometry is our focus in chapter 8. Students draw geometric figures using a protractor and a ruler, and they also learn some basic geometric constructions. The other themes of this chapter are various angle relationships, area and the perimeter of a circle, conversions between units of area and of volume, surface area, volume, and cross-sections when solids are sliced with a plane.

Chapter 9 covers square roots, the Pythagorean Theorem, and its applications. I have included the Pythagorean Theorem in order to make the curriculum work as a pre-algebra course, and you can omit the entire chapter 9 if you are following the Common Core Standards for grade 7.

Chapter 10 is an introduction to probability. Besides learning the basic idea behind probability as the ratio of favorable events to all possible events, students compare experimental probabilities to theoretical ones in probability simulations and even design some on their own.

Lastly, in chapter 11, the curriculum covers statistical concepts. The major areas of study are random sampling and learning to compare two populations using some basic statistical measures and graphs.

Part 7-A covers an introduction to basic concepts of algebra, integers, one-step equations, fractions and decimals, and linear equations.

I wish you success in teaching math!

Maria Miller, the author

Chapter 6: Ratios and Proportions

Introduction

Chapter 6 reviews the concept, which has already been presented in previous grades, of the ratio of two quantities. From this concept, we develop the related concepts of a rate (so much of one thing per so much of another thing) and a proportion (an equation of ratios). We also study how tables of equivalent ratios can help to solve problems with rates, and how cross-multiplying can help to solve problems with proportions.

The lesson *Unit Rates* defines the concept of the unit rate, shows how to calculate one, and gives practice at doing so, including practice with complex fractions. We also consider rates as two quantities that vary, graph the corresponding equation in the coordinate grid, and tie in the concept of unit rate with the concept of slope.

The concept of direct variation is introduced in the lesson *Proportional Relationships*. Writing and graphing equations gives a visual understanding of proportionality. In two following lessons on proportions, students also practice solving rate problems in different ways, using the various methods they have learned throughout the chapter.

The lessons *Scaling Figures, Floor Plans, and Maps* give useful applications and more practice to master the concepts of proportions.

Before the *Chapter Review* there is also an optional lesson, *Significant Digits*, that deals with the concept of the accuracy of a measurement and how it limits the accuracy of the solution. It is optional because significant digits is not a standard topic for seventh grade, yet the concept in it is quite important, especially in science.

The Lessons in Chapter 6

	page	span
Ratios and Rates	12	3 pages
Solving Problems Using Equivalent Rates	15	2 pages
Solving Proportions: Cross Multiplying	17	6 pages
Why Cross-Multiplying Works	23	1 page
Unit Rates	24	5 pages
Proportional Relationships	29	6 pages
Graphing Proportional Relationships—More Practice	35	2 pages
More on Proportions	37	4 pages
Scaling Figures	41	6 pages
Floor Plans	47	4 pages
Maps	51	6 pages
Significant Digits	57	2 pages
Chapter 6 Mixed Review	59	3 pages
Chapter 6 Review	62	5 pages

Helpful Resources on the Internet

Language of Algebra - Video Lessons by Maria

A set of free videos that teach the topics in this book - by the author herself.

<http://www.mathmammoth.com/videos/prealgebra/pre-algebra-videos.php#algebra>

Equivalent ratios

Ratio Pairs Matching Game

Match cards representing equivalent ratios.

Easy: <http://nrich.maths.org/4824> Challenge: <http://nrich.maths.org/4821>

Equivalent Ratios Workout

10 online practice problems.

<http://www.math.com/school/subject1/practice/S1U2L1/S1U2L1Pract.html>

Ratio Stadium

A multi-player online racing game for matching equivalent ratios. The student with the fastest rate of correct answers will win the race.

<http://www.arcademicskillbuilders.com/games/ratio-stadium/>

All About Ratios - Quiz

An interactive five-question quiz about equivalent ratios presented with pictures

<http://math.rice.edu/~lanius/proportions/quiz1.html>

Ratios and rates

Rate lesson from BrainingCamp

A comprehensive interactive lesson on the concepts of ratio, rate, and constant speed (for 6th and 7th grades). Includes an animated lesson, a virtual manipulative, and questions and problems to solve.

<http://www.brainiaccamp.com/content/rates/>

Ratios Activity from BBC Bitesize

An animated and narrated tutorial about ratios and scale models with some quiz questions along the way.

<http://www.bbc.co.uk/education/guides/znnycdm/activity>

Practice with Ratios

An online quiz from Regents Exam Prep Center that includes both simple and challenging questions and word problems concerning ratios.

<http://www.regentsprep.org/Regents/math/ALGEBRA/AO3/pracRatio.htm>

Exploring Rate, Ratio and Proportion (Video Interactive)

The video portion of this resource illustrates how these math concepts play a role in photography. The interactive component allows students to explore ratio equivalencies by enlarging and reducing images to compare an original ratio and a target ratio.

<http://www.learnalberta.ca/content/mejhm/index.html?l=0&ID1=AB.MATH.JR.NUMB&ID2=AB.MATH.JR.NUMB.RATE>

If the link above doesn't work, use this link:

<http://www.learnalberta.ca/content/mejhm/index.html?l=0&ID1=AB.MATH.JR.NUMB>

First choose Rate/Ratio/Proportion, and then *Exploring Rate, Ratio, and Proportion*.

Self-Check Quiz from Glencoe

A five-question multiple-choice quiz about comparing with ratios and rates. By reloading the page you will get different questions.

<http://www.glencoe.com/sec/math/studytools/cgi-bin/msgQuiz.php4?isbn=1-57039-855-0&chapter=8&lesson=1>

Free Ride

An interactive activity about bicycle gear ratios. Choose the front and back gears, which determines the gear ratio. Then choose a route, pedal forward, and make sure you land exactly on the five flags.

<http://illuminations.nctm.org/ActivityDetail.aspx?ID=178>

Ratio Quiz from BBC Skillswise

A multiple-choice quiz about the concept of ratio. You can take the quiz online or download it as a PDF or doc file.

<http://www.bbc.co.uk/skillswise/quiz/ma19rati-e1and2-quiz>

Practice with Ratios

A set of 11 word problems that involve ratios. You can reveal the answer and the full solution at your will.

<http://www.regentsprep.org/regents/math/algebra/ao3/pracRatio.htm>

Ratio Quiz from Syvum

A 10-question online quiz about ratios and problem solving.

http://www.syvum.com/cgi/online/mult.cgi/gmat/math_review/arithmetric_5.tdf?0

Three-Term Ratios

Practice the equivalency of ratios by filling in the missing numbers in three-term ratios (for example, $2:7:5 = _ : 105 : _$) where the three numbers represent the amounts of red, blue, or green colors in nine different photographs. After finishing the activity you get to assemble a picture puzzle from the nine photographs.

http://www.learnalberta.ca/content/mejhm/index.html?l=0&ID1=AB.MATH.JR.NUMB&ID2=AB.MATH.JR.NUMB.RATE&lesson=html/object_interactives/3_term_ratio/use_it.html

<http://www.learnalberta.ca/content/mejhm/index.html?l=0&ID1=AB.MATH.JR.NUMB>

If the link above doesn't work, use this link:

<http://www.learnalberta.ca/content/mejhm/index.html?l=0&ID1=AB.MATH.JR.NUMB>

First choose Rate/Ratio/Proportion, and then either *3-Term Ratios*.

Proportions

Ratios and Proportions

A tutorial with interactive practice exercises about ratios and proportions.

<https://www.wisc-online.com/learn/formal-science/mathematics/gem2004/ratios-and-proportions>

Practice with Proportions

An online quiz from Regents Exam Prep Center

<http://www.regentsprep.org/Regents/math/ALGEBRA/AO3/pracProp.htm>

Challenge Board

Choose questions from the challenge board about rates, ratios, and proportions.

<http://www.quia.com/cb/158527.html>

<http://www.quia.com/cb/101022.html>

Rags to Riches - Proportions

Solve proportions and advanced towards more and more difficult questions.

<http://www.quia.com/rr/35025.html>

Dirt Bike Proportions

A racing game where you need to find the unknown in a simple proportion. This game would actually work equally well for practicing equivalent fractions because the proportions are quite simple.

<http://www.arcademicskillbuilders.com/games/dirt-bike-proportions/dirt-bike-proportions.html>

Ratio and Proportion Game From BBC Skillswise

Write the simplified ratio of red to black marbles. Answer simple questions about ratios and marbles.

<http://www.bbc.co.uk/skillswise/numbers/wholenumbers/ratioandproportion/ratio/game.shtml>

Proportions Battleship Game

Solve proportions whenever you get a 'hit' on enemy ships in the classic battleship game.

<http://www.quia.com/ba/83811.html>

How Much Is a Million?

This is a lesson plan for a hands-on activity where students count grains of rice in a cup, weigh that amount of rice, and then build a proportion to figure out the weight of 1 million grains of rice.

<http://illuminations.nctm.org/Lesson.aspx?id=2674>

Scale drawings and maps

Ratio and Scale

An online unit about scale models, scale factors, and maps with interactive exercises and animations.

<http://www.absorblearning.com/mathematics/demo/units/KCA024.html>

Self-Check Quiz from Glencoe

A five-question multiple-choice quiz about scale drawings. By reloading the page you will get different questions.

<http://www.glencoe.com/sec/math/studytools/cgi-bin/msgQuiz.php4?isbn=0-02-833050-1&chapter=8&lesson=3>

Maps

A tutorial with worked out examples and interactive exercises about how to calculate distances on the map or in real life based on the map's scale.

http://www.cimt.plymouth.ac.uk/projects/mepres/book7/bk7i19/bk7_19i3.htm

Scale Drawings and Models - Word Problems

Solve word problems involving scale drawings and models. You can choose between multiple-choice, match answers, or flashcard style questions.

<https://flashcards.engage.com/scaledrawingsandmodels>

Use Proportions To Solve Problems Involving Scale Drawings

A set of word problems. You can choose how they are presented: as flashcards, as a quiz where you match questions and answers, as a multiple choice quiz, or a true/false quiz. You can also play a game (Jewels) .

<http://www.cram.com/flashcards/use-proportions-to-solve-problems-involving-scale-drawings-3453121>

Scale Drawings - Problem Solving and Constructing Scale Drawings Using Various Scales

A comprehensive lesson with several worked out examples concerning scale drawings.

<http://www.ck12.org/user:c2ZveDJA3N3ZWdvLm9yZw../book/Oswego-City-School-District---Grade-7-Common-Core/section/12.0/>

Constructing Scale Drawings

Practice making scale drawings on an interactive grid. The system includes hints and the ability to check answers.

<https://www.khanacademy.org/math/cc-seventh-grade-math/cc-7th-geometry/cc-7th-scale-drawings/e/constructing-scale-drawings>

Interpreting Scale Drawings

Solve word problems involving scale drawings in an online practice environment. The system includes hints and the ability to check answers.

<https://www.khanacademy.org/math/cc-seventh-grade-math/cc-7th-geometry/cc-7th-scale-drawings/e/interpreting-scale-drawings>

Significant digits

Sig Fig Rules

Drag Sig J. Fig to cover each significant digit in the given number.

<http://www.sigfig.dreamhosters.com/>

Practice on Significant Figures

A multiple-choice quiz that also reminds you of the rules for significant digits.

<http://www.chemistrywithmsdana.org/wp-content/uploads/2012/07/SigFig.html>

Significant digits quiz

A 10-question multiple-choice quiz about significant digits.

http://www.quia.com/quiz/114241.html?AP_rand=1260486279

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Unit Rates

Remember that a rate is a ratio where the two terms have different units, such as 2 kg/\$0.45 and 600 km/5 h.

In a **unit rate**, the second term of the rate is one (of some unit).

For example, 55 mi/1 hr and \$4.95/1 lb are unit rates. The number “1” is nearly always omitted so those rates are usually written as 55 mi/hr and \$4.95/lb.

To convert a rate into an equivalent unit rate simply divide the numbers in the rate.

Example 1. Mark can ride his bike 35 km in $1\frac{1}{2}$ hours. What is the unit rate?

To find the unit rate, we use the principles of division by fractions to divide 35 km by $1\frac{1}{2}$ h. The units “km” and “hours” are divided, too, and become “km per hour” or “km/hour.”

$$\begin{aligned}\frac{35 \text{ km}}{1\frac{1}{2} \text{ h}} &= 35 \div \frac{3}{2} \text{ km/h} = 35 \times \frac{2}{3} \text{ km/h} \\ &= \frac{70}{3} \text{ km/h} = 23\frac{1}{3} \text{ km/h}.\end{aligned}$$

We could also use decimal division:
 $35 \text{ km} \div 1.5 \text{ h} = 23.333\dots \text{ km/h}.$

So the unit rate is $23\frac{1}{3}$ km per hour.

Example 2. A snail can slide through the mud 5 cm in 20 minutes. What is the unit rate?

Here, it is actually not clear whether we should give the unit rate as cm/min or cm/hr. Let’s do both.

(1) To get the unit rate in cm/min, we simply divide 5 cm \div 20 min. We get the fraction $5/20$. We also divide the units to get “cm/min.” So we get

$$5 \text{ cm} \div 20 \text{ min} = 5/20 \text{ cm/min} = 1/4 \text{ cm/min}$$

$$\begin{aligned}\text{Or use decimals: } 5 \text{ cm}/20 \text{ min} &= 25/100 \text{ cm/min} \\ &= 0.25 \text{ cm/min}.\end{aligned}$$

(2) For centimeters per hour, we multiply both terms of the rate by 3 to get an equivalent rate of 15 cm in 60 minutes, which is 15 cm in 1 hour.

1. Find the unit rate.

a. \$125 for 5 packages

b. \$6 for 30 envelopes

c. \$1.37 for $\frac{1}{2}$ hour

d. $2\frac{1}{2}$ inches per 4 minutes

e. 24 m^2 per $\frac{3}{4}$ gallon

2. A person is walking $\frac{1}{2}$ mile every $\frac{1}{4}$ hour. Choose the correct fraction for the unit rate and simplify it.

$$\frac{\frac{1}{4}}{\frac{1}{2}} \text{ miles per hour} \quad \text{or} \quad \frac{\frac{1}{2}}{\frac{1}{4}} \text{ miles per hour}$$

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Chapter 7: Percent

Introduction

In this chapter we review the concept of percent as “per hundred” or as hundredth parts and how to convert between fractions, decimals, and percents. *Solving Basic Percentage Problems* is another lesson intended for review of sixth grade topics, focusing on finding a known percentage of a number (such as 21% of 56) or finding a percentage when you know the part and the total.

We take a little different perspective of these concepts in the lesson *Percent Equations*. Students write simple equations for situations where a price increases or decreases (discounts). This lesson also explains what a percent proportion is. Personally, I prefer *not* to use percent proportion but to write the percentage as a decimal and then write an equation. I feel that approach adapts better to solving complex problems than using percent proportion.

Here is a quick example to show the difference between the two methods. Let’s say an item is discounted by 22% and it now costs \$28. Then, the new price is 78% of the original. If we let p be the price of the item before the discount, we can write the percent proportion $\$28/p = 78/100$ and solve for p . If, we write the percentage 78% as the decimal 0.78, we get the equation $0.78p = \$28$. Personally, I consider percent proportion to be an optional topic, and the reason I have included it here is to make this curriculum fully meet the Common Core Standards for seventh grade.

The lesson *Circle Graphs* provides students a break from new concepts and allows them to apply the concept of percent in a somewhat familiar context. Next, we delve into the percentage of change. Students sometimes view the percentage of change as a totally different concept as compared to other percentage topics, but it is not that at all. To calculate the percentage of change, we still use the fundamental idea of *percentage = part/total*, only this time, the “part” is how much the quantity in question changes (the difference) and the “total” is the original quantity.

Tying in with percentage of change, students also learn to compare values using percentages, such as how many percent more or less one thing is than another. Once again, this is not really a new concept but is based on the familiar formula *percentage = part/total*. The percentage difference (or relative difference) is the fraction (*actual difference*)/(*reference value*).

Simple Interest is a lesson on the important topic of interest, using as a context both loans and savings accounts. Students learn to use the formula $I = prt$ in a great variety of problems and situations.

The text concludes with a review lesson of all of the concepts taught in the chapter.

The Lessons in Chapter 7

	page	span
Review: Percent	72	3 pages
Solving Basic Percentage Problems	75	3 pages
Percent Equations	78	5 pages
Circle Graphs	83	2 pages
Percentage of Change	85	3 pages
Percentage of Change: Applications	88	4 pages
Comparing Values Using Percentages	92	4 pages
Simple Interest	96	6 pages
Chapter 7 Mixed Review	102	3 pages
Chapter 7 Review	105	2 pages

Helpful Resources on the Internet

Percent videos by Maria

Videos on percent-related topics that match the lessons in this chapter.

<http://www.mathmammoth.com/videos/percent.php>

Percentages, fractions, and decimals

Virtual Manipulative: Percentages

An interactive tool where you fill in any two of the three boxes (whole, part, and percent), and it will calculate the missing part and show the result visually in two ways.

http://nlvm.usu.edu/en/nav/frames_asid_160_g_2_t_1.html

Mission: Magnetite

Hacker tries to drop magnetite on Motherboard. To unlock a code to stop him, match up percentages, fractions, and images showing fractional parts in five different sets of items.

<http://pbskids.org/cyberchase/media/games/percent/index.html>

Fractions and Percent Matching Game

A simple matching game: match fractions and percentages.

http://www.mathplayground.com/matching_fraction_percent.html

Fraction/Decimal/Percent Jeopardy

Answer the questions correctly, changing between fractions, decimals, and percentages.

<http://www.quia.com/cb/34887.html>

Flower Power

Grow flowers and harvest them to make money in this addictive order-'em-up game. Practice ordering decimals, fractions, and percentages. The game starts with ordering decimals (daisies), and proceeds into fractions (tulips or roses).

<https://www.mangahigh.com/en/games/flowerpower>

Matching Fractions, Decimals, and Percentages

A simple matching memory game.

<http://nrich.maths.org/1249>

Sophie's Dominoes

Place dominoes that contain either numbers or a percentage of a number (such as 15% of 300) in the correct order in the work area.

http://www.bsquaredfutures.com/pluginfile.php/212/mod_resource/content/1/doms.swf

Percent Goodies: Fraction-Decimal-Percent Conversions

Practice conversions between fractions, decimals and percents. There are three levels of difficulty and instant scoring for each. Note that fractions must be written in lowest terms.

<http://www.mathgoodies.com/games/conversions/>

Percentage of a number

Percent worksheets

Create an unlimited number of free customizable percent worksheets to print.

<http://www.homeschoolmath.net/worksheets/percent-decimal.php>

<http://www.homeschoolmath.net/worksheets/percent-of-number.php>

<http://www.homeschoolmath.net/worksheets/percentages-words.php>

Penguin Waiter

A simple game where you calculate the correct tip to leave the waiter (levels “easy” and “medium”), the percentage that the given tip is (level “hard”), or the original bill (level “Super Brain”).

<http://www.funbrain.com/penguin/>

Percent Jeopardy

An interactive jeopardy game where the questions have to do with a percentage of a quantity.

<http://www.quia.com/cb/42534.html>

Math At the Mall

Practice percentages while shopping at a virtual mall. Find the percentage of discount and the sales price, calculate the interest earned at the bank, compare health memberships at the gym and figure out how much to tip your waiter at the Happy Hamburger.

<http://www.mathplayground.com/mathatthemall2.html>

Matching Percentage of a Number

Match cards that ask for a percentage of a number (such as 75% of 40) with the values. The game is fairly easy and can be completed using mental math.

<http://www.sheppardsoftware.com/mathgames/percentage/MatchingPercentNumber.htm>

Discount Doors

Calculate the price after the discount.

http://www.bsquaredfutures.com/pluginfile.php/214/mod_resource/content/1/doors.swf

The Percentage Game

This is a printable board game for 2-3 players that practices questions such as “20 percent of ___ is 18” or “___ is 40 percent of 45”.

<http://nzmaths.co.nz/resource/percentage-game>

A Conceptual Model for Solving Percent Problems

A lesson plan that uses a 10 x 10 grid to explain the basic concept of percent and to solve various types of percentage problems. The lesson includes seven different word problems to solve. Please note their solutions are included on the same page.

<http://illuminations.nctm.org/LessonDetail.aspx?id=L249>

Percentages of Something

This is a simple illustration to help students understand the meaning of percent. Choose a way to illustrate percentages and then drag percentages into the blackboard.

<http://www.bbc.co.uk/skillswise/game/ma16perc-game-percentages-of-something>

Percent of change

Percent of Change Matching

Match five flashcards with given increases or decreases (such as “25 is decreased to 18”) with five percentages of increase/decrease.

<https://www.studystack.com/matching-182854>

Percent Shopping

Choose toys to purchase. In level 1, you find the sale price when the original price and percent discount are known. In level 2, you find the percent discount (percent of change) when the original price and the sale price are known.

http://www.mathplayground.com/percent_shopping.html

Rags to Riches: Percent Increase or Decrease

Answer simple questions about percent increase or decrease and see if you can win the grand prize in the game.

<http://www.quia.com/rr/230204.html>

Percentage Change 1

A self-marking quiz with 10 questions about percentage change. The link below goes to level 1 quiz, and at the bottom of that page you will find links to level 2, 3, 4, 5 and 6 quizzes.

http://www.transum.org/software/SW/Starter_of_the_day/Students/PercentageChange.asp

Percentage increase and decrease 4 in a line

The web page provides a game board to print. Players take turns picking a number from the left column, and increase or decrease it by a percentage from the right column. They cover the answer on the grid with a counter. The first player to get four counters in a line wins.

<https://www.tes.co.uk/teaching-resource/percentage-increase-and-decrease-4-in-a-line-6256320>

Treasure Hunt - Percentage Increase and Decrease

The clues of this treasure hunt are printable percentage increase/decrease questions.

<https://www.tes.co.uk/teaching-resource/treasure-hunt--percentage-increase-and-decrease-6113809>

Percent Change Practice

The page provides interactive flash cards with simple questions about percentage of change. Three difficulty levels.

<http://www.thegreatmartinicompany.com/percent-percentage/percent-change.html>

Percentage Increase and Decrease

Multiple-choice questions about percentage of change to be solved without a calculator (mental math).

https://www.mangahigh.com/en/maths_games/number/percentages/percentage_increase_and_decrease_-_no_calculator

Percent of Change Jeopardy

This is an online jeopardy game that provides you the game board, questions for percent increase, percent decrease, sales tax, discounts, and markups, the answers, and a scoreboard where you can enter the teams' points. However, it doesn't have a place to enter answers and requires someone to supervise the play and the teams' answers.

<https://www.superteachertools.net/jeopardyx/jeopardy-review-game.php?gamefile=1396523998>

Interest

Math At the Mall

Practice percentages while shopping at a virtual mall. Find the percent discount and the sales price, calculate the interest earned at the bank, compare health memberships at the gym and figure out how much to tip your waiter at the Happy Hamburger.

<http://www.mathplayground.com/mathatthemall2.html>

Compound interest

A simple introduction to compound interest with many examples.

<http://www.mathsisfun.com/money/compound-interest.html>

Quiz: Simple Interest

A multiple-choice quiz with five questions that ask for the interest earned, final balance, interest rate, or the principal.

<http://www.cliffsnotes.com/math/algebra/algebra-ii/word-problems/quiz-simple-interest>

Simple Interest

Another quiz where you need to find the principal, the amount of time, interest earned, or the final amount in an account earning interest. Four out of nine questions in this quiz have to do with terminology and the rest are math problems.

<http://www.proprofs.com/quiz-school/story.php?title=simple-interest>

Simple Interest Rates Battleship

Play a traditional battleship game while you practice calculating the amount of interest when the principal, rate, and time are given.

<http://www.quia.com/ba/108800.html>

Calculating simple interest

This page includes several video tutorials plus a short three-question quiz on simple interest.

<https://www.sophia.org/concepts/calculating-simple-interest>

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Percentage of Change

Percent(age of) change is a way to describe how much a price or some other quantity is increasing or decreasing (changing). Let's look at how to calculate the percentage a quantity is changing.

Example 1. A phone used to cost \$50. Now it has been discounted to \$45. What percentage was the discount?

Since this problem is asking for the *percentage*, we will use our basic formula $\frac{\text{part}}{\text{total}} = \text{percentage}$.

Because the change is relative to the *original* price, that original price becomes the "total" in our equation. The "part" is the actual amount by which the quantity changes, in this case \$5. So we get

$$\text{percentage} = \frac{\text{part}}{\text{total}} = \frac{\$5}{\$50} = 1/10 = 10\%$$

Essentially, we wrote **what fraction the \$5 discount is of the original \$50 price** and converted that fraction into a percentage.

In summary: To calculate the percent change, use the same basic formula that defines a percentage: *part/total*. Since the change is relative to the original price, the original price is the "total," and the change in price is the "part."

$$\text{percentage of change} = \frac{\text{part}}{\text{total}} = \frac{\text{difference}}{\text{original}}$$

1. Write an equation and calculate the percentage of change.

a. A toy construction set costs \$12. It is discounted and costs only \$8 now. What percentage is the discount?

$$\frac{\text{difference}}{\text{original}} =$$

b. A sewing kit costs \$20. It is discounted and costs only \$16 now. What percentage is the discount?

c. A bouquet of flowers used to cost \$15, but now it costs \$20. What is the percentage of increase?

d. The price of a stove was \$160. The price has increased, and now it costs \$200. What is the percentage of increase?

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Chapter 8: Geometry

Introduction

The main topics we study in this chapter are:

- various angle relationships
- drawing geometric figures, including basic geometric constructions
- π and the area of a circle
- slicing 3-dimensional solids
- surface area and volume.

In the first lesson of the chapter, we examine various angle relationships: angles that are formed when several rays originate from the same starting point, vertical angles (formed when two lines intersect), and corresponding angles (formed when a line intersects two parallel lines). Then, the lesson *Angles in a Triangle* presents and proves the well-known result that the angles in a triangle sum to 180 degrees. With this knowledge, students are now able to solve various problems that involve unknown angles.

Next, students practice drawing geometric figures. Basic geometric constructions are done just like in ancient times: with only a compass and straightedge (a ruler without measurement units). These constructions help students to think about the main defining features of a figure. Personally I have always enjoyed geometric constructions because they are like little puzzles to solve.

Students also draw figures using a normal ruler and compass in the lesson *Drawing Problems*. They especially determine whether the given information defines a unique figure (triangle or a parallelogram).

Then we turn our attention to π . Students first learn the definition of π as a ratio of a circle's circumference to its diameter in the lesson *Circumference of a Circle*. Then they learn and practice how to calculate the area of a circle in a wide variety of word problems and applications. We also briefly study the proof for the formula for the area of a circle. I feel it is important that students encounter justifications for mathematical formulas and procedures and even read some proofs before high school. We don't want students to think that mathematics is only a bag of magic tricks or formulas to memorize that seemingly came out of nowhere. Proofs and logical thinking are foundations to mathematics and school mathematics should not be left without them.

After this, we slice three-dimensional solids with a plane, and learn that the result is always a two-dimensional shape. Students see that in a concrete way by slicing cubes and pyramids made of modeling clay. Some internet links (provided in the lesson) will also help students to visualize what happens when a solid is cut with a plane.

In this chapter, students also solve a variety of problems concerning surface area and volume and practice converting between various units of area and volume. While these topics tend to involve lots of calculations and less possibilities for hands-on activities, they are very important in real life.

The Lessons in Chapter 8

	page	span
Angle Relationships	114	5 pages
Angles in a Triangle	119	5 pages
Basic Geometric Constructions	124	6 pages
More Constructions	130	5 pages
Drawing Problems	135	7 pages

Circumference of a Circle	142	3 pages
Area of a Circle	145	3 pages
Proving the Formula for the Area of a Circle	148	2 pages
Area and Perimeter Problems	150	5 pages
Surface Area	155	4 pages
Conversions Between Customary Units of Area	159	3 pages
Conversions Between Metric Units of Area	162	3 pages
Slicing Three-Dimensional Shapes	165	7 pages
Volume of Prisms and Cylinders	172	4 pages
Chapter 8 Mixed Review	176	3 pages
Chapter 8 Review	179	8 pages

Helpful Resources on the Internet

Angle relationships

Math Warehouse - Angles

The following links contain examples, diagrams, practice problems, and interactive demonstrations concerning various types of angles.

<http://www.mathwarehouse.com/geometry/angle/complementary-angles.php>

<http://www.mathwarehouse.com/geometry/angle/supplementary-angles.php>

<http://www.mathwarehouse.com/geometry/angle/interactive-supplementary-angles.php>

<http://www.mathwarehouse.com/geometry/angle/vertical-angles.php>

<http://www.mathwarehouse.com/geometry/angle/interactive-vertical-angles.php>

<http://www.mathwarehouse.com/geometry/triangles/#interiorAngle>

Quiz

A 10-question quiz with simple questions about vertical angles, corresponding angles, and angles in a triangle.

<http://www.thatquiz.org/tq-C/?-j7-l8-p1ug>

Working with Angles

Online lessons with explanations and interactive self-check questions from Absorb Mathematics course. The lessons cover measuring angles, the types of angles (acute/obtuse), angles on a straight line, complementary & supplementary angles, and corresponding & alternate angles. Written by Kadie Armstrong, a mathematician.

<http://www.absorblearning.com/mathematics/demo/units/KCA003.html>

Complementary Angles

Includes a clear explanation, an interactive "Play with it" work area to explore supplementary angles, and 10 self-check interactive questions.

<http://www.mathsisfun.com/geometry/complementary-angles.html>

Supplementary Angles

Includes a clear explanation, an interactive "Play with it" work area to explore supplementary angles, and 10 self-check interactive questions.

<http://www.mathsisfun.com/geometry/supplementary-angles.html>

Angles Around a Point

A short lesson showing that angles around a point will always add up to 360 degrees, followed by 10 self-check questions.

<http://www.mathsisfun.com/angle360.html>

Solving for unknown angles from Khan Academy

Use your knowledge of supplementary and complementary angles to solve questions of varying difficulty. Some questions involve writing and solving an equation.

<https://www.khanacademy.org/math/cc-seventh-grade-math/cc-7th-geometry/cc-7th-angles/e/solving-for-unknown-angles>

Interior Angles

A nice explanation about the interior angles of various polygons, starting with the triangle and going to the regular n -gon.

<http://www.coolmath4kids.com/interior.html>

Geometry Bridge

In this interactive activity, you build five bridges in order to let the bus cross them safely and to arrive in the amusement park. You need to classify angles, classify triangles, calculate angles in a triangle, and calculate sides of triangles based on the Pythagorean Theorem in order to build the bridges. Free registration required.

<http://www.sharemylesson.com/teaching-resource/geometry-bridge-50018309/>

Constructions

Geometric Construction

These lessons cover constructions for perpendicular lines, an equilateral triangle, angle bisection, parallel lines, and copying an angle. They include explanations, interactive animations, and self-check questions.

<http://www.absorblearning.com/mathematics/demo/units/KCA006.html>

Animated Geometric Constructions

Simple animations that show how to do basic geometric constructions.

<http://www.mathsisfun.com/geometry/constructions.html>

Geometric Constructions Illustrated with Interactive Java Applets

Interactive java applets and instructions for doing the basic geometric constructions online.

<http://www.personal.psu.edu/dpl14/java/geometry/>

GRACE - Graphical Ruler and Compass Editor

An interactive online tool that allows a dynamic creation and modification of compass & ruler constructions. Constructions may be built from one of five geometric primitives (Line, Line Segment, Ray, Circle, Perpendicular Bisector, and Intersection), and from other constructions; thus you can build constructions by composing them from more basic constructions.

<http://www.cs.rice.edu/~jwarren/grace/>

Circle

Interactive Area of a circle

Explore and discover the relationship between the area, radius, and graph of a circle. Just click and drag the points.

<http://www.mathwarehouse.com/geometry/circle/interactive-area.php>

Circle tool from Illuminations

How do the area and circumference of a circle compare to its radius and diameter? This activity allows you to investigate these relationships in the Intro and Investigation sections and then hone your skills in the Problems section.

<http://illuminations.nctm.org/Activity.aspx?id=3547>

Area of a Circle, Formula & Illustrated Lesson

The interactive tool shows you the area of the circle as the radius increases. The page also includes a short quiz.

<http://www.mathwarehouse.com/geometry/circle/area-of-circle.php>

Circle Tool

An applet that allows you to investigate how the area and circumference of a circle compare to its radius and diameter in the Intro and Investigation sections and then hone your skills in the Problems section. You can drag the radius to various lengths, and then click the “Add to Table” button to record the data in the table. You can also examine the ratios of any two measures, and make a graph of the data
<http://illuminations.nctm.org/ActivityDetail.aspx?ID=116>

Area of Circle

An interactive calculator that calculates the circumference, radius, diameter, and area of a circle when any one of those is entered.
<http://www.mathsisfun.com/geometry/circle-area.html>

Area of Circles

An interactive applet that allows students to explore the formula for the area of a circle by cutting it into sectors and rearranging the sectors to form a figure close to a parallelogram. By increasing the number of sectors, students can see that the figure gets closer and closer to a perfect parallelogram.
<http://www.geogebra.org/student/m279>

The Area of a Circle as a Limit

An animation that illustrates how we can find the area of a circle by drawing triangles into it. The area of the circle is then the limit of the sum of the areas of the interior triangles as the number of triangles goes to infinity.
<http://www.learnerstv.com/animation/animation.php?ani=96&cat=physics>

Area of Circle

An interactive activity where you cut a circle into wedges in order to determine its area.
http://www.learner.org/courses/learningmath/measurement/session7/part_b/index.html

Area & Perimeter Practice

A 10-question quiz that will let you practice finding the area and circumference of a circle.
<http://www.thatquiz.org/tq-4/?-j201g-la-p1ug>

Area and circumference of a circle - Test from BBC Bitesize

A 10-question multiple-choice quiz about the area and circumference of a circle where the questions increase with difficulty.
<http://www.bbc.co.uk/bitesize/quiz/q90581037>

Radius, Diameter, and Circumference

A simple lesson with an interactive quiz about the radius, diameter, and circumference of a circle.
<http://www.mathgoodies.com/lessons/vol2/circumference.html>

Area and Circumference of a Circle

A 15-question multiple-choice quiz.
<http://www.proprofs.com/quiz-school/story.php?title=area-circumference--circle>

Mangahigh.com - Shape

Questions on the area of a circle, including the area of a semi-circle and simple compound shapes.
https://www.mangahigh.com/en/maths_games/shape/circles_and_cylinders/area_of_a_circle

Amazing History of Pi

A short and simple introduction to the history of pi.
<http://ualr.edu/lasmoller/pi.html>

Approximating Pi

How did Archimedes find the approximate value of pi? This interactive tool illustrates Archimedes' basic approach with inscribed or circumscribed polygons.
<http://www.pbs.org/wgbh/nova/archimedes/pi.html>

Pi Day Activities & Links

Two pi-related activities to do at home, plus a lot of fun links about pi. Originally meant for Pi Day (celebrated 3/14 each year).

http://www.exploratorium.edu/pi/pi_activities/index.html

5 Trillion Digits of Pi

As of 2014, the world record for computing digits of pi was over 13 trillion digits. This will keep changing, of course.

<http://www.numberworld.org/digits/Pi/>

A Rolling Circle Illustrating Pi

This is a short animation where a circle with diameter 1 rolls on a number line one complete roll. Of course having rolled once around its circumference, it now lands at pi.

<http://i.imgur.com/dsCw0.gif>

Area and perimeter

Math Playground: Party Designer

You need to design areas for the party, such as a crafts table, a food table, a seesaw, and so on, so that they have the given perimeters and areas.

<http://www.mathplayground.com/PartyDesigner/PartyDesigner.html>

Free worksheets for the area of triangles, quadrilaterals, and polygons

Generate printable and customizable worksheets to practice finding the area of triangles, parallelograms, trapezoids, or polygons in the coordinate grid. Options include choosing either the first or all quadrants, scaling, image size, workspace, and border.

http://www.homeschoolmath.net/worksheets/area_triangles_polygons.php

BBC Bitesize - Area

Brief revision (review) “bites,” including a few interactive questions, about the area of triangles, parallelograms, and compound shapes.

<http://www.bbc.co.uk/bitesize/ks3/maths/measures/area/revision/4/>

Geometry Area/Perimeter Quiz from ThatQuiz.org

An online quiz that asks either the area or the perimeter of rectangles, triangles, parallelograms, and trapezoids.

You can also modify the quiz parameters to your liking, for example to omit a shape, or instead of solving for the area, solve for an unknown side when perimeter/area is given.

<http://www.thatquiz.org/tq-4/?-j1i00f-lc-p0>

Area Tool

Use this tool to determine how the length of the base and the height of a figure can be used to determine its area. Can you find the similarities and differences between the area formulas for trapezoids, parallelograms, and triangles?

<http://illuminations.nctm.org/Activity.aspx?id=3567>

Geometry - Math Warehouse

Detailed lessons about angles, triangles, quadrilaterals, circles, similar triangles, parallelograms, polygons, and trapezoids.

<http://www.mathwarehouse.com/geometry/>

Cross-sections of solids

Cross sections

An interactive activity that allows you to slice a cube with a plane and find its different cross sections.

http://www.learner.org/courses/learningmath/geometry/session9/part_c/index.html

Cross-Section Flyer from Shodor

Examine cross-sections of a cone, cylinder, pyramid, prism, and a double-cone. You can rotate and move the cutting plane, plus decide the number of faces for the pyramid and prism. An excellent tool!

<http://www.shodor.org/interactivate/activities/CrossSectionFlyer/>

Shapes – 3D Geometry Learning

An app for iPad and iPhone

Explore various properties of 3D shapes, such as edges, vertices, and faces. You can rotate the solids and unfold them into their nets. Choose from 27 different solids.

<https://itunes.apple.com/au/app/solids-elementary-hd/id501650786?mt=8>

Surface area and volume

2-D and 3-D Shapes

Learn about different solids: rotate them and see their nets.

http://www.bgfl.org/bgfl/custom/resources_ftp/client_ftp/ks2/maths/3d

Geometric Solids

Manipulate (rotate) various geometric solids by dragging with the mouse and see their nets. Count the number of faces, edges, and vertices.

<http://illuminations.nctm.org/Activity.aspx?id=3521>

Surface area practice

A 10- question quiz where you can practice finding the surface area of prisms, pyramids, and cylinders. Note: you need to input the surface area of cylinders as a multiple of $\pi \cdot \text{cm}$. For example, a cylinder with height 7 cm and bottom radius of 2 cm has the surface area of $(4 \text{ cm} \cdot \pi \cdot 7 \text{ cm}) + (2 \cdot (2 \text{ cm})^2 \cdot \pi) = 28\pi \text{ cm} + 8\pi \text{ cm} = 36\pi \text{ cm}$.

<http://www.thatquiz.org/tq-4/?-j824a0-l6-p1ug>

Interactivate: Surface Area and Volume

Explore or calculate the surface area and volume of rectangular and triangular prisms. You can change the base, height, and depth interactively.

<http://www.shodor.org/interactivate/activities/SurfaceAreaAndVolume>

Interactives - Surface area - Cylinders

A lesson that includes an interactive portion where you calculate the surface area of a cylinder step-by-step.

http://www.learner.org/interactives/geometry/area_surface2.html

Volume of a Cylinder

Discussion about the volume of a cylinder, including a regular calculator and an interactive calculator where you can drag the orange dot to resize the cylinder, and the volume is calculated as you drag. By allowing oblique cylinders and then dragging the top orange dot, you can clearly see that the volume of an oblique cylinder is equal to the volume of a right cylinder.

<http://www.mathopenref.com/cylindervolume.html>

Surface Area and Volume of 3-D Shapes

This page contains three worked out examples about the volume and surface area of cylinders and prisms, followed by over a dozen exercises and word problems. You can self-check your answers by clicking the "Check" buttons.

http://www.cimt.plymouth.ac.uk/projects/mepres/book9/bk9i9/bk9_9i4.html

Volume of a cylinder

Interactive and guided questions about the volume of a cylinder. Students also find the height or the radius of a cylinder when given the volume. Some problems leave the answer in terms of pi.

https://www.mangahigh.com/en/math_games/shape/circles_and_cylinders/volume_of_a_cylinder

Geometry Volume Quiz

A 10-question quiz where you can practice finding the volume of prisms and cylinders. Note: you need to input the volume of cylinders as a multiple of π · cm. For example, a cylinder with height 7 cm and bottom radius of 2 cm has the volume of $(2 \text{ cm})^2 \cdot \pi \cdot 7 \text{ cm} = 28\pi \text{ cm}$.

<http://www.thatquiz.org/tq-4/?-j28g0-la-p1ug>

MathGuide's Volume of Cylinders Quizmaster

Interactive questions where you calculate the area of the bottom and the volume of a cylinder, given its radius and height.

<http://www.mathguide.com/cgi-bin/quizmasters/CylindersV.cgi>

Just for Fun

Online Kaleidoscope

Create your own kaleidoscope pattern with this interactive tool.

http://www.zefrank.com/dtoy_vs_byokal/

Interactive Tangram Puzzle

Place the tangram pieces so they form the given shape.

http://nlvm.usu.edu/en/nav/frames_asid_112_g_2_t_1.html

Interactivate! Tessellate

An online, interactive tool for creating your own tessellations. Choose a shape, then edit its corners or edges. The program automatically changes the shape so that it will tessellate (tile) the plane. Then push the tessellate button to see your creation!

<http://www.shodor.org/interactivate/activities/Tessellate>

National Library of Virtual Manipulatives for Interactive Mathematics: Geometry

A collection of interactive geometry activities: Congruent triangles, fractals, geoboard, golden rectangle, ladybug leaf, ladybug mazes, platonic solids, tangrams, tessellations, transformations and more.

http://nlvm.usu.edu/en/nav/category_g_3_t_3.html

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Area of a Circle

The area of a circle is given by this formula: $A = \pi r^2$, where r is the radius of the circle.

Read the formula as: “Area equals pi r squared.” It means that you first multiply the radius by itself and then multiply the result by π .

You can remember the formula by thinking, “Pie are square.” Of course pies are usually round, not square! It is bad English, as well, but the purpose of this silly mnemonic is just to help you remember the formula.

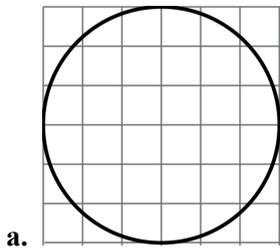
Example 1. The radius of a circle measures 8 cm. What is its area?

We use the formula: $A = \pi r^2 = \pi \cdot 8 \text{ cm} \cdot 8 \text{ cm} \approx 3.14 \cdot 64 \text{ cm}^2 = 200.96 \text{ cm}^2$, or about 200 cm^2 .

Remember to always give your answer for an area in square units, be it square inches, square centimeters, square meters, square feet, *etc.* If no measuring unit is given, use “square units.”

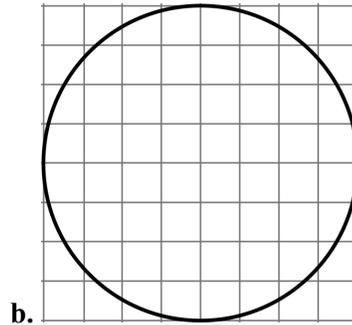
You can use a calculator for all the problems in this lesson.

1. Estimate the area of the circles by counting squares and parts of squares. After that, calculate the area to the nearest hundredth of a square unit.



Estimation: _____ square units

Calculation: _____ square units



Estimation: _____ square units

Calculation: _____ square units

2. Find the areas of these circles.

a. A circle with a radius of 7.0 cm.

Round the answer to the nearest ten square centimeters (to 2 significant digits).

Area =

b. A circle with a radius of $10 \frac{1}{4}$ in.

Round the answer to the nearest square inch (to 3 significant digits).

Area =

c. A circle with a *diameter* of 75.0 cm.

Round the answer to the nearest ten square centimeters (to 3 significant digits).

Area =

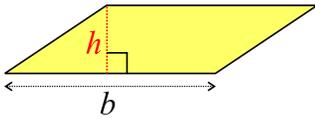
d. A circle with a radius of 17 ft 4 in.

Round the answer to the nearest thousand square inches (to 3 significant digits).

Area =

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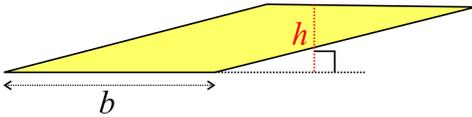
Area and Perimeter Problems



The area of a parallelogram is

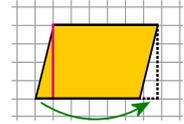
$$A = bh$$

where b is the base and h is the altitude (height).



The **altitude** of a parallelogram is a perpendicular line segment from the base, or the extension of the base, to the top. Thus, the altitude might not be inside the parallelogram.

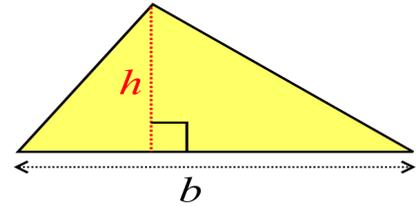
Recall that from any parallelogram we can cut off a triangular piece and move it to the other side to make it a rectangle. That is why the formula for the area of a parallelogram is so similar to the formula for the area of a rectangle.



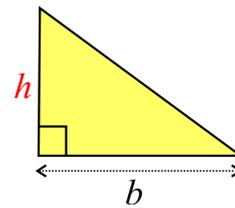
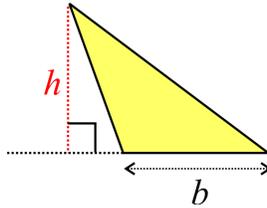
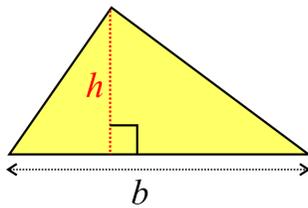
Since any triangle is a half of its corresponding parallelogram, the area of a triangle is half the area of that parallelogram:

$$A = \frac{bh}{2}$$

where b is the base and h is the altitude of the triangle.



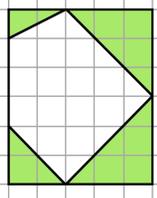
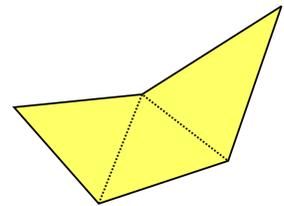
The **altitude** of a triangle is a line from one vertex to the opposite side that is perpendicular to that side. It can:



- (1) fall inside the triangle; (2) fall outside the triangle; (3) be one of the sides of a right triangle.

To calculate the area of a polygon:

- (1) Divide it into rectangles, triangles, and other simple shapes.
- (2) Calculate the area of each part separately.
- (3) Add the areas of the parts.



Sometimes we can use another strategy to find the area of a polygon.

- (1) Draw a rectangle around the polygon.
- (2) Calculate the areas of the triangles and quadrilaterals that are outside the polygon but inside the rectangle.
- (3) Subtract those areas from the area of the entire rectangle.

You may use a calculator for all the problems in this lesson.

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Chapter 9: The Pythagorean Theorem

Introduction

This is a relatively short chapter focusing on the Pythagorean Theorem and its applications. The Pythagorean Theorem is actually not part of the Common Core Standards for seventh grade. The Common Core places it in eighth grade. However, I have included it in this curriculum because it is a traditional topic in pre-algebra. That way, Math Mammoth Grade 7 works as a full pre-algebra curriculum while fully meeting (and exceeding) the Common Core Standards for grade 7. If you are following the Common Core Standards strictly, you can safely omit this entire chapter, because your student(s) will encounter these topics in eighth grade.

First, students need to become familiar with square roots, so they can solve the equations that result from applying the Pythagorean Theorem. The first lesson of the chapter introduces taking a square root as the opposite operation to squaring a number. The lesson includes both applying a guess-and-check method and using a calculator to find the square root of a number.

Next, students learn how to solve simple equations that include taking a square root. This makes them fully ready to study the Pythagorean Theorem and apply it.

The Pythagorean Theorem is introduced in the lesson by that name. Students learn to verify that a triangle is a right triangle by checking if it fulfills the Pythagorean Theorem. They apply their knowledge about square roots and solving equations to solve for an unknown side in a right triangle when two of the sides are given.

Next, students solve a variety of geometric and real-life problems that require the Pythagorean Theorem. This theorem is extremely important in many practical situations. Students should show their work for these word problems to include the equation that results from applying the Pythagorean Theorem to the problem and its solution.

There are literally hundreds of proofs for the Pythagorean Theorem. In this chapter, we present one easy proof based on geometry (not algebra). As an exercise, students are asked to supply the steps of reasoning to another geometric proof of the theorem, and for those interested the lesson also provides an internet link that has even more proofs of this theorem.

The Lessons in Chapter 9

	page	span
Square Roots	190	4 pages
Equations That Involve Taking a Square Root	194	5 pages
The Pythagorean Theorem	199	5 pages
The Pythagorean Theorem: Applications	204	7 pages
A Proof of the Pythagorean Theorem	211	1 page
Chapter 9 Mixed Review	212	3 pages
Chapter 9 Review	215	3 pages

Helpful Resources on the Internet

Square roots

Squares and Square Roots

A fun lesson about squares and square roots with lots of visuals and little tips. It is followed by 10 interactive multiple-choice questions.

<http://www.mathsisfun.com/square-root.html>

The Roots of Life

Practice finding square roots of perfect squares and help the roots of a tree grow. Easy, medium, hard, and contest levels.

<http://www.hoodamath.com/games/therootsoflife.html>

Square Root Game

Match square roots of perfect squares with the answers. Includes several levels.

<http://www.math-play.com/square-root-game.html>

Pyramid Math

Choose "SQRT" to find square roots of perfect squares. Drag the correct answer to the jar on the left. This game is pretty easy.

<http://www.mathnook.com/math/pyramidmath.html>

Rags to Riches Square Root Practice

Answer multiple-choice questions that increase in difficulty. The questions include finding a square root of perfect squares, determining the two nearest whole numbers to a given square root, and finding square roots of numbers that aren't perfect squares to one decimal digit.

<http://www.quia.com/rr/382994.html>

The Pythagorean Theorem

Video Lessons by Maria

Several of my own videos present word problems that involve the Pythagorean Theorem.

https://www.youtube.com/playlist?list=PLYM4yncims39aK3r_qc4ZsGcqGJgu-xX0

Pythagorean Theorem - Braining Camp

This learning module includes a lesson, an interactive manipulative, multiple-choice questions, real-life problems, and interactive open-response questions.

<https://www.brainingcamp.com/content/pythagorean-theorem/>

Pythagoras' Theorem from Maths Is Fun

A very clear lesson about the Pythagorean Theorem and how to use it, followed by 10 interactive practice questions.

<http://www.mathsisfun.com/pythagoras.html>

Pythagorean Triplets

Move the two orange points in this activity to find Pythagorean Triplets, sets of three whole numbers that fulfill the Pythagorean Theorem.

<http://www.interactive-maths.com/pythagorean-triples-ggb.html>

The Pythagorean Theorem Quiz

A 10-question quiz that asks for the length of the third side of a right triangle when the lengths of two sides are given.

<http://www.thatquiz.org/tq-A/?-j10-la-p1ug>

Sample worksheet from
www.mathmammoth.com

Interactivate: Pythagorean Theorem

Interactive practice problems for calculating the third side of a right triangle when two sides are given.

<http://www.shodor.org/interactivate/activities/PythagoreanExplorer/>

Practice with Pythagorean Theorem

Twelve word problems to solve that all involve the usage of the Pythagorean Theorem. The page also includes a concise solution for each problem (click the “Explanation” drop down menu).

<http://www.regentsprep.org/regents/math/algebra/AT1/PracPyth.htm>

Pythagoras in 3D

Can you find the longest dimension of a box? Includes an interactive illustration for the problem.

<http://www.interactive-maths.com/pythagoras-in-3d-ggb.html>

Proof

Two interactive and animated proofs of the Pythagorean Theorem

The first proof is by dissection: Rearrange the colored pieces from the smaller squares so that they fill the square on the hypotenuse. You can manipulate the pieces yourself or see an animation of it. The second is titled “Chinese” and is also based on rearranging pieces.

http://hotmath.com/learning_activities/interactivities/pythagorean.swf

Pythagorean Theorem and its many proofs

A collection of 111 approaches to prove this theorem. Many proofs are accompanied by interactive Java illustrations.

<http://cut-the-knot.com/pythagoras/>

Annotated Animated Proof of the Pythagorean Theorem

Watch the animation to learn a proof of the Pythagorean Theorem.

<http://www.davis-inc.com/pythagor/proof2.html>

Proving the Pythagorean Theorem

See if you can figure out two more proofs of the Pythagorean theorem. Only the pictures are given to you. Tips and Solutions are available.

http://www.learner.org/courses/learningmath/geometry/session6/part_b/more.html

Many Proofs of the Pythagorean Theorem

A list of animated proofs.

http://www.takayaiwamoto.com/Pythagorean_Theorem/Pythagorean_Theorem.html

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The Pythagorean Theorem: Applications

Example 1. An eight-foot ladder is placed against a wall so that the base of the ladder is 2 ft away from the wall. What is the height of the top of the ladder?

Since the ladder, the wall, and the ground form a right triangle, this problem is easily solved by using the Pythagorean Theorem. Let h be the distance asked. From the Pythagorean Theorem, we get:

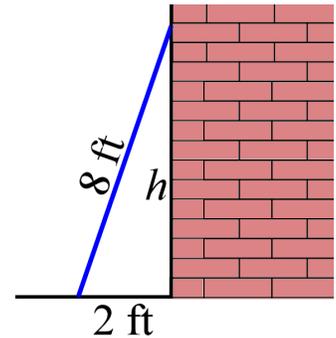
$$2^2 + h^2 = 8^2$$

$$4 + h^2 = 64$$

$$h^2 = 60$$

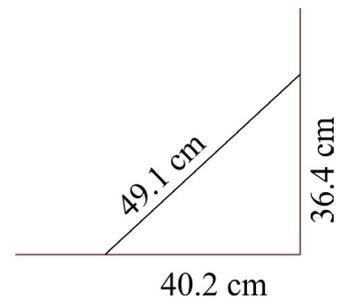
$$h = \sqrt{60}$$

$$h \approx 7.75$$



Our answer, 7.75, is in feet. This means the ladder reaches to about $7 \frac{3}{4}$ ft = 7 ft 9 in. high.

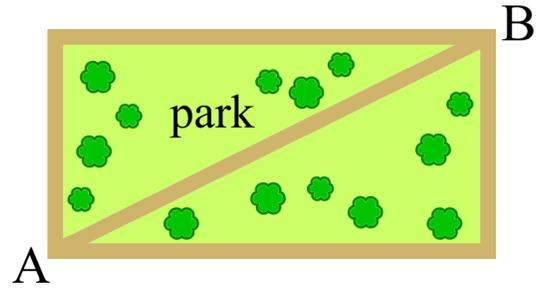
1. Is this corner a right angle?



2. How long is the diagonal of a laptop screen that is 9.0 inches high and 14.4 inches wide?

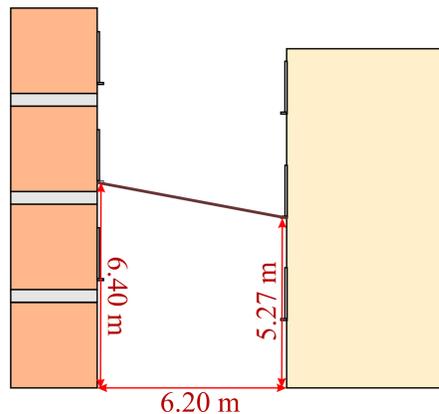
Note: computer screen sizes refer to the length of the screen's *diagonal*. For example, a 15-inch screen means that the diagonal is 15 inches, not the width nor the height.

3. A park is in the shape of a rectangle and measures 48 m by 30 m. How much longer is it to walk from A to B along the diagonal of the park than to walk along the edges of the park?

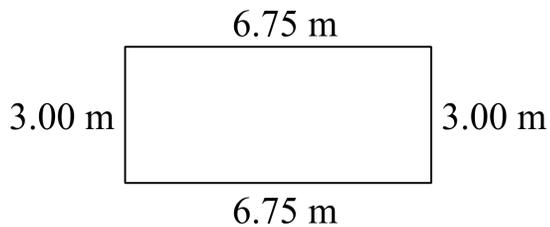


4. The area of a square is 100 m^2 . How long is the diagonal of the square?

5. A clothesline is suspended between two apartment buildings.
Calculate its length, assuming it is straight and doesn't sag any.



6. Construction workers have made a rectangular mold out of wood, and they are getting ready to pour cement into it. How could they make sure that the mold is indeed a rectangle and not a parallelogram?
After all, in a parallelogram the opposite sides are equal, so simply measuring the opposite sides does not guarantee that a shape is a rectangle.



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Chapter 10: Probability

Introduction

Probability is new to Math Mammoth students, as the topic doesn't appear at all in grades 1-6. However, most students have an intuitive understanding of probability based on hearing the terms “probably” and “likely,” listening to weather forecasts, and so on.

In the past, probability wasn't taught until high school—for example, I personally encountered it for the first time in 12th grade. However, since probability is such a useful and easily accessible field of math, it was felt that it should be introduced sooner, so during the 1990s and 2000s it “crept” down the grade levels until many states required probability even in elementary school. The Common Core Standards include probability starting in 7th grade. I feel that is good timing because by 7th grade students have studied fractions, ratios, and proportions, so they have the tools they need to study probability. Moreover, they will need an understanding of the basic concepts of probability in order to understand the statistical concepts that they will study in middle school and high school.

In this chapter we start with the concept of simple (classic) probability, which is defined as the ratio of the number of favorable outcomes to the number of all possible outcomes. Students calculate probabilities that involve common experiments, to include flipping a coin, tossing a pair of dice, picking marbles, and spinning a spinner.

The lesson *Probability Problems from Statistics* introduces probability questions involving the phrase “at least,” which are often solved by finding the probability of the complement event. For example, it might be easier to count the number of students who got at most D+ on a test than to count the number of students who got at least C-.

In the next lesson, *Experimental Probability*, students conduct experiments, record the outcomes, and calculate both the theoretical and experimental probabilities of events, in order to compare the two. They will draw a card from a deck or roll a die hundreds of times. The download version of this curriculum includes spreadsheet files for some of the lengthier probability simulations. You can also access those simulations at the web page http://www.mathmammoth.com/lessons/probability_simulations.php

Next, we study compound events, which combine two or more individual simple events. Tossing a die twice or choosing first a girl then a boy from a group of people are compound events. Students calculate the probabilities of compound events by using the complete sample space (a list of all possible outcomes). They construct the sample space in several ways: by drawing a tree diagram, by making a table, or simply by using logical thinking to list all the possible outcomes.

The last major topic in this chapter is simulations. Students design simulations to find the probabilities of events. For example, we let heads represent “female” and tails represent “male,” so we can toss a coin to simulate the probability of choosing a person of either sex at random. Later in the lesson, students design simulations that use random numbers. They generate those numbers by using either the free tool at <http://www.random.org/integers> or a spreadsheet program on a computer.

In the last lesson of the chapter, *Probabilities of Compound Events*, we learn to calculate the probability of a compound event by *multiplying* the probabilities of the individual events (assuming the outcomes of the individual events are independent of each other). This topic exceeds the Common Core Standards for 7th grade and thus is optional. I have included it here because the idea studied in the lesson is very simple and I feel many students will enjoy it.

The Lessons in Chapter 10

	page	span
Probability	223	3 pages
Probability Problems from Statistics	226	2 pages
Experimental Probability	228	3 pages
Count the Possibilities	231	6 pages
Using Simulations to Find Probabilities	237	6 pages
Probabilities of Compound Events	243	4 pages
Chapter 10 Mixed Review	247	3 pages
Chapter 10 Review	250	2 pages

Helpful Resources on the Internet

Simple probability

Probability videos by Maria

These video lessons cover topics that have been chosen to complement the lessons in this chapter.

http://www.mathmammoth.com/videos/probability/probability_lessons.php

Probability Fair

Choose the probability that has the best chance when the colorful spinner is spun.

<http://www.mrnussbaum.com/probfair/index.html>

“Data Analysis & Probability Games” from MathWire

A list of board and dice games to help to teach topics appropriate for beginners in probability.

<http://mathwire.com/games/datagames.html>

Probability Game with Coco

A multiple-choice online quiz on simple probability.

<http://www.math-play.com/Probability-Game.html>

Math Goodies Probability Lessons

Tutorials with lots of worked-out examples and online interactive exercises. Topics include independent and dependent events, addition rules, conditional probability, and more.

www.mathgoodies.com/lessons/toc_vol6.html

Cross the Bridge

This is a printable board game based on throwing two dice and the probabilities for the sum of the dice.

<http://www.mathsphere.co.uk/downloads/board-games/board-game-17-crossing-the-river.pdf>

Rocket Launch

A three-stage rocket is about to be launched. In order for a successful launch to occur, all three stages of the rocket must successfully pass their pre-takeoff tests. By default, each stage has a 50% chance of success, however, this can be altered by dragging the bar next to each stage. Observe how many tries it takes until there is a successful launch.

<http://mste.illinois.edu/activity/rocket/>

How could I send the check and not pay the bill?

What is the probability that Tessellation will put each of the three checks into the correct envelopes if she does it randomly? The page includes a hint and a complete solution (click “answer” at the bottom of page).

<http://figurethis.nctm.org/challenges/c69/challenge.htm>

She always wins. It's not fair!

Is this game with two dice fair? We can use basic probability concepts to analyze the situation. The page includes a hint and a complete solution (click “answer” at the bottom of page).

<http://figurethis.nctm.org/challenges/c26/challenge.htm>

Sticks and Stones

Play the game “Sticks and Stones,” gather data from the game play, and then consider the probabilities for the four possible moves in the game.

<http://illuminations.nctm.org/Lesson.aspx?id=1915>

Flippin' Discs

In this interactive activity, you throw two discs. You win if they both show the same color. You can run the game 100 times and see the detailed results. Can you explain why you win approximately half the time? The questions below the activity lead the student to explore the situation with 3, 4, and even 5 discs. The solution is found in a link near the top left of the page.

<http://nrich.maths.org/4304>

At Least One...

The tree diagram and related discussion on this page guides students' thinking to help them answer probability questions like, “What is the probability of getting at least one head by flipping a coin ten times?” A link near the top left of the page leads to the solution.

<http://nrich.maths.org/7286>

Same Number

Imagine you are in a class of thirty students. The teacher asks everyone to secretly write down a whole number between 1 and 225. How likely is it for everyone's numbers to be different? The web page provides an interactive simulation so you can experiment with this problem. The following discussion also leads students to the classic birthday problem. The solution is found in a link near the top left of the page.

<http://nrich.maths.org/7221>

What Does Random Look Like?

This problem challenges our thinking about randomness. Make up a sequence of twenty Hs and Ts that *could* represent a sequence of heads and tails generated by a fair coin. Then use the animation to generate truly random sequences of 20 coin flips. Can you learn how to spot fakes?

<http://nrich.maths.org/7250>

Interactivate: *Fire!!* and *Directable Fire!!*

In these two activities, you first set the probability that a fire will spread from tree to tree in a forest of 100 trees. Then you click the tree where the fire starts and watch it spread. In the Directable Fire activity, you can set the probabilities for each direction to be different. Repeat the activity several times to see that the amount of forest that burns varies (for any set probability of fire spreading).

<http://www.shodor.org/interactivate/activities/Fire/>

<http://www.shodor.org/interactivate/activities/DirectableFire/>

The Monty Hall Problem

Try this interactive version of the famous Monty Hall problem. The page includes a simple explanation of the solution. In the Monty Hall Problem you choose one of three closed doors. Behind one door is a big prize and behind the others something else, such as a goat. After you choose a door, Monty opens one of the doors with no prize, and you have a chance to switch.

www.nytimes.com/2008/04/08/science/08monty.html?_r=1

Virtual tools for experimental probability

Virtual Experimental Probability

Virtually roll one or two dice, spin a spinner, choose a card, or toss a coin virtually many times. The results are recorded so you can compare them to the theoretical probability.

http://staff.argyll.epsb.ca/jreed/math9/strand4/probability_display.htm

Sample worksheet from
www.mathmammoth.com

Coin Flip

This virtual coin toss shows the results numerically and can generate at least 100,000 flips.

<http://www.btwaters.com/probab/flip/coinmainD.html>

Coin Toss Simulation

Another virtual coin toss. This one shows the results both using images of coins and numerically.

<http://syzygy.virtualave.net/multicointoss.htm>

Coin Tossing - National Library of Virtual Manipulatives

This coin-toss tool not only lets you set the number of virtual tosses, but also the probability of heads, which means that you can observe what happens when a coin is weighted.

http://nlvm.usu.edu/en/nav/frames_asid_305_g_3_t_5.html

Theoretical and Experimental Probability Gizmo

Experiment with spinners and compare the experimental probability of a particular outcome to the theoretical probability. Select the number of spinners, the number of sections on a spinner, and a favorable outcome of a spin. Then tally the number of favorable outcomes. The gizmo works for 5 minutes for free. You can also sign up for a free trial account.

<http://www.explorelearning.com/index.cfm?method=cResource.dspView&ResourceID=310>

Dice Roll

Choose the number of virtual dice to roll and how many times you want to roll them. The page shows both the actual results and expected (theoretical) probabilities, and the simulation works for a very large numbers of rolls.

<http://www.btwaters.com/probab/dice/dicemain3D.html>

Interactive: Spinner

You can adjust the number of regions (each is a different color) and choose the number of spins. The results show the frequencies for each color, the experimental probabilities, and the theoretical probabilities.

<http://www.shodor.org/interactivate/activities/BasicSpinner/>

Adjustable Spinner

Create a virtual spinner with variable-sized sectors to compare experimental results to theoretical probabilities. You can choose the sizes of the sectors, the number of sectors, and the number of trials.

<http://www.shodor.org/interactivate/activities/AdjustableSpinner/>

Experimental Probability

Experiment with probability using virtual spinners or dice. The sections on the spinner can be of the same size or of different sizes, and the dice can be regular 6-sided dice or customized.

<http://www.shodor.org/interactivate/activities/ExpProbability/>

Box Model - Random Drawings

First, select some numbers from which the computer draws. You can even select the same number multiple times. Tick the box for “Quick draw” and enter a number of repetitions. Then press “Start” to have the program draw a number randomly from among the numbers you selected. A bar graph shows the relative frequencies for your experiment.

http://nlvm.usu.edu/en/nav/frames_asid_146_g_3_t_5.html

Marbles

Run repeated experiments where you draw 1, 2, or 3 marbles from a set of blue, red, purple, and green marbles. You choose the number of each kind of marble and how many repetitions there are. The results shown include the frequencies of each possible outcome, the experimental probabilities, and the theoretical probabilities. This activity can be used not only to explore probabilities but also to perform simulations.

<http://www.shodor.org/interactivate/activities/Marbles/>

Racing Game with One Die

Explore how experimental probability relates to fair and unfair games with this two-car race. You choose which and how many numbers of the die make each of the cars move. Other options include the number of runs (trials) and the length of the race (in segments). The program calculates the percentage of wins for each car and draws a pie chart.

<http://www.shodor.org/interactivate/activities/RacingGameWithOneDie/>

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Counting the Possibilities

A **sample space** is a list of all possible outcomes of an experiment.

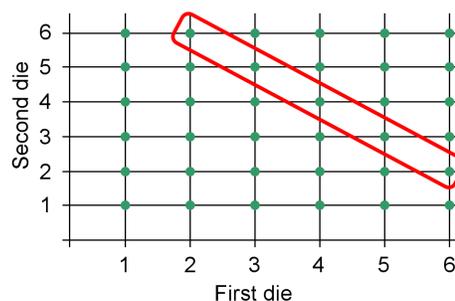
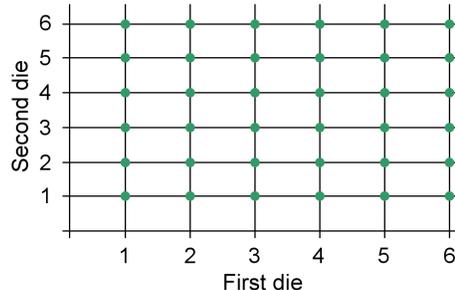
Example 1. We roll two dice. The sample space for this experiment is shown in the grid on the right. Each dot represents one outcome. For example, the point (1, 4) means that the first die shows 1 and the second die shows 4.

Notice that there are a total of $6 \cdot 6 = 36$ possible outcomes.

What is the probability of getting the sum of 8 when rolling two dice? The chart helps answer that question. First we find out and count how many outcomes give you the sum 8:

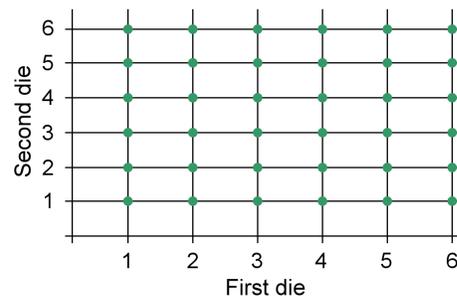
You could roll 2 & 6, 3 & 5, 4 & 4, 5 & 3, or 6 & 2. Those number pairs are circled in the second graphic.

So, there are five favorable outcomes and 36 possible ones. Therefore, the probability of getting 8 as a sum is $5/36$.



1. **a.** How many outcomes are there for rolling the same number on both dice (such as (5, 5))?

b. What is the probability of rolling the same number on both dice?



2. **a.** What is the probability of rolling 5 on the first die and 6 on the second?

b. What is the probability of rolling 5 on one die and 6 on the other?

c. What is the probability of getting the sum 7, when rolling two dice?

d. What is the probability of getting a sum of at least 6, when rolling two dice?

3. You roll a six-sided die two times. Find the probabilities.

a. $P(1; 5)$

c. $P(\text{even}; \text{odd})$

b. $P(2; 5 \text{ or } 6)$

d. $P(6; \text{not } 6)$

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Chapter 11: Statistics

Introduction

Math Mammoth Grade 7 ends with a study of statistics. The chapter begins with a review lesson to remind students how to make a boxplot and a histogram and how to calculate the mean absolute deviation—all topics that were studied in 6th grade.

The first focus area of the chapter is random sampling. Students learn that sampling methods vary and that random sampling is likely to produce an *unbiased sample*—a sample that represents the population well. In the lesson *Using Random Sampling*, students choose several random samples from a population of 100 geometric shapes, and they see first hand that random samples can vary—even a lot. Yet if the sample size is sufficiently large, or if we have several random samples, we can be relatively confident in concluding something about the population itself. Students will also practice making inferences about populations based on several random samples.

The second major topic is comparing two populations, either directly or by using samples from the populations. Students learn to use the overall distributions and the measures of center and variability to compare two sets of data in various ways. While some of the ways in which we compare the data are informal only, all of the concepts presented are fundamental to the use of statistics in various sciences. Students also do a project where they gather data on their own from two populations and compare them.

The Lessons in Chapter 11

	page	span
Review of Data Analysis	256	5 pages
Random Sampling	261	4 pages
Using Random Sampling	265	7 pages
Comparing Two Populations	272	8 pages
Comparing Two Samples	280	6 pages
Chapter 11 Mixed Review	286	4 pages
Chapter 11 Review	290	3 pages

Helpful Resources on the Internet

Statistics videos by Maria

Videos on statistics topics that are helpful for the lessons in this chapter.

http://www.mathmammoth.com/videos/statistics/statistics_lessons.php

Graphs and Plots

Statistics Interactive Activities from Shodor

A set of interactive tools for exploring and creating boxplots, histograms, dot plots, and stem-leaf plots. You can enter your own data or explore the examples.

<http://www.shodor.org/interactivate/activities/BoxPlot/>

<http://www.shodor.org/interactivate/activities/Histogram/>

<http://www.shodor.org/interactivate/activities/PlopIt/>

<http://www.shodor.org/interactivate/activities/StemAndLeafPlotter/>

Sample worksheet from
www.mathmammoth.com

PlotLy

A comprehensive, collaborative data analysis and graphing tool. Bring data in from anywhere, do the math, graph it with interactive plots (scatter, line, area, bar, histogram, heat map, box, and more), and export it.

<http://plot.ly>

How to make a box plot using Plotly

Easy-to-follow instructions.

<https://plot.ly/how-to-make-a-box-plot-online/>

Analyzing and Displaying Data Gizmos from ExploreLearning

Gizmos are interactive online tools with lesson plans that allow you to explore and learn about the topic in a virtual, dynamic environment. This page includes gizmos for box-and-whisker plots, histograms, stem-and-leaf plots, and more. The gizmos work for 5 minutes for free. You can also sign up for a free trial account.

<http://www.explorelearning.com/index.cfm?method=cResource.dspResourcesForCourse&CourseID=231>

Create A Box and Whisker Chart

An online tool for creating a box-and-whisker plot from your own data. Includes lots of options, such as colors, fonts, and more.

<https://www.meta-chart.com/box-whisker>

How to create a BoxPlot (Box-and-Whisker Chart) in Excel

Detailed instructions for how to create a boxplot in different versions of Excel.

<https://support.microsoft.com/en-us/kb/155130>

Make Your Own Stem-and-Leaf Plot

Enter values from your own data, and this web page creates your stem-and-leaf plot for you.

<http://www.mrnussbaum.com/graph/sl.htm>

Stem-and-Leaf Plots Quiz

An online multiple-choice quiz that is created randomly. Refresh the page (or press F5) to get another quiz.

<http://www.glencoe.com/sec/math/studytools/cgi-bin/msgQuiz.php4?isbn=0-07-825200-8&chapter=12&lesson=1&&headerFile=4>

How to Read a Dot Plot

This lesson explains how to answer questions from a dot plot.

<http://www.mathbootcamps.com/how-to-read-a-dotplot/>

Statistical measures

Measures of Center and Quartiles Quiz from ThatQuiz.org

An online quiz about the measures of center and quartiles in boxplots, stem-and-leaf plots, and dot plots.

<http://www.thatquiz.org/tq-5/?-jr0t0-11-p0>

Mean, Median, and Mode

A lesson that explains how to calculate the mean, median, and mode for a set of data given in different ways. It also has interactive exercises.

http://www.cimt.plymouth.ac.uk/projects/mepres/book8/bk8i5/bk8_5i2.htm

GCSE Bitesize Mean, Mode and Median

Lessons with simple explanations and examples.

<http://www.bbc.co.uk/schools/gcsebitesize/maths/statistics/measuresofaveragerev1.shtml>

Measures Activity

Enter your own data and the program will calculate mean, median, mode, range, and some other statistical measures.

<http://www.shodor.org/interactivate/activities/Measures>

Train Race Game

Calculate the median and range of travel times for four different trains, then choose a good train to take based on your results.

<http://www.bbc.co.uk/schools/mathsfile/shockwave/games/train.html>

Measures of Variation - Self-Check Quiz

An online multiple-choice quiz about range, quartiles, and interquartile range that is created randomly. Refresh the page (or press F5) to get another quiz.

<http://www.glencoe.com/sec/math/studytools/cgi-bin/msgQuiz.php4?isbn=0-07-825200-8&chapter=12&lesson=2&&headerFile=4>

Mean Deviation

A simple explanation about what the mean absolute deviation is, how to find it, and what it means.

<http://www.mathsisfun.com/data/mean-deviation.html>

Mean Absolute Deviation

Several videos explaining how to calculate the mean absolute deviation of a data set.

<http://www.onlinemathlearning.com/measures-variability-7sp3.html>

Working with the Mean Absolute Deviation (MAD)

A tutorial and questions where you are asked to create line plots with a specified mean absolute deviation.

http://www.learner.org/courses/learningmath/data/session5/part_e/working.html

Sampling

Random and Biased Sampling

A comprehensive lesson to read that explains about unbiased types of sampling.

<http://www.ck12.org/na/Random-and-Biased-Sampling-in-a-Population---7.SP.1,2-1/lesson/user%3Ac2ZveDJA3N3ZWdVlm9yZw../Random-and-Biased-Sampling-in-a-Population---7.SP.1%252C2/>

How many fish in the pond?

This problem is based on the capture-recapture method scientists use to estimate the size of a population. The page includes a hint and a complete solution.

<http://figurethis.nctm.org/challenges/c52/challenge.htm#hint>

Capture-Recapture

If you are interested in learning more about the capture-recapture method, this web page provides a complete lesson plan and activity where students take a sample of beans to explore this concept.

<http://illuminations.nctm.org/Lesson.aspx?id=2528>

Polling: Neighborhood Gizmo

Conduct a phone poll of citizens in a small neighborhood to determine their response to a yes-or-no question. Use the results to estimate the sentiment of the entire population. Investigate how the error of this estimate becomes smaller as more people are polled. Compare random versus non-random sampling. The gizmo works for 5 minutes for free. You can also sign up for a free trial account.

<http://www.explorelearning.com/index.cfm?method=cResource.dspDetail&ResourceID=507>

Identify a Random Sample

A video lesson about what a sample is and what makes a sample random. On the left of the web page are links to three other videos about representative (unbiased) samples and biased samples.

<https://learnzillion.com/student/lessons/1844>

Valid Claims

Multiple-choice questions to practice figuring out whether we took a random sample and whether we are able to draw valid conclusions from the data.

<https://www.khanacademy.org/math/probability/statistical-studies/statistical-questions/e/valid-claims>

Random Sampling

Multiple-choice questions that test your understanding of the basics of random sampling.

<https://www.khanacademy.org/math/recreational-math/math-warmup/random-sample-warmup/e/random-sample-warmup>

Make inferences about a population by analyzing random samples

A video lesson that teaches how to make inferences about a population based on random samples.

<https://learnzillion.com/student/lessons/1848>

Making Inferences from Random Samples

Multiple-choice questions about what can reasonably be inferred, from a random sample, about an entire population

<https://www.khanacademy.org/quetzalcoat/content-improvement/middle-school-content/e/making-inferences-from-random-samples>

Comparing two populations

Compare Populations Using the Mean

Video lessons that teach how to informally compare two populations using the mean.

<https://learnzillion.com/student/lessons/1445>

Compare Two Populations using the Range and the Interquartile range

A video lesson that teaches you how to informally compare the ranges and the interquartile ranges of two populations.

<https://learnzillion.com/lessons/1453-compare-two-populations-using-range-and-interquartile-range>

Comparing Populations - Khan Academy

Multiple-choice questions to practice comparing centers of distributions in terms of their spread.

<https://www.khanacademy.org/math/cc-seventh-grade-math/cc-7th-probability-statistics/cc-7th-population-sampling/e/comparing-populations>

Displaying Univariate Data Practice

Interactive practice questions about double stem-and-leaf plots.

<http://www.ck12.org/statistics/Displaying-Univariate-Data/asmtpactice/Displaying-Univariate-Data-Practice/>

Double Box-and-Whisker Plots Practice

Interactive practice questions about double box-and-whisker plots.

<http://www.ck12.org/statistics/Double-Box-and-Whisker-Plots/asmtpactice/Double-Box-and-Whisker-Plots-Practice/>

Mean and Median with BoxPlots

This interactive tool allows you to create several box-and-whisker plots. The data sets may contain up to 15 integers, each with a value from 0 to 100.

<http://illuminations.nctm.org/Activity.aspx?id=3576>

Facts & figures

GapMinder

Visualizing human development trends (such as poverty, health, gaps, income on a global scale) via stunning, interactive statistical graphs. This is an interactive, dynamic tool and not just static graphs. Download the software or the reports for free.

<http://www.gapminder.org/data/>

WorldOdometers

World statistics updated in real time. Useful for general educational purposes— for some stunning facts.

<http://www.worldometers.info>

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Comparing Two Populations

In this lesson, we will use measures of center and variability to compare two sets of data.

Example 1. The two dot plots on the right show the ages of two different groups of children. In one group, we have children from ages 5-7, and in the other, children from ages 3-5.

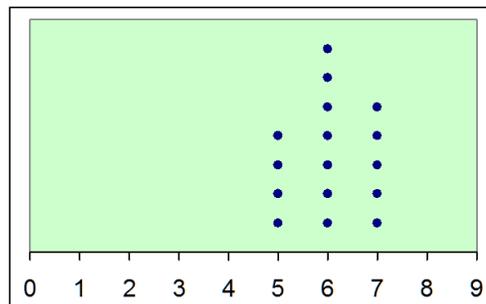
We see from the plots that the centers of the two distributions differ by 2 years. We can also see that both distributions are very clustered (with little spread). The distributions overlap somewhat but not very much: both groups include children that are 5 years old, but otherwise there is no overlap.

Overall, the ages of these two groups are definitely different. In statistical terms, we would say that there is a significant difference in the ages of these children.

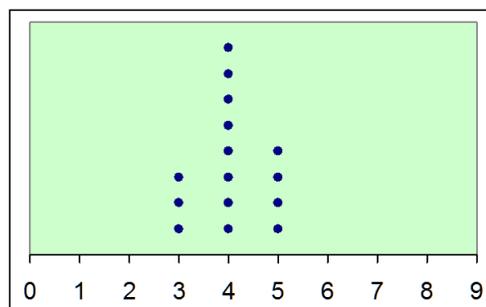
We also have a numerical way to verify that the difference in the means is indeed significant: **we compare the difference in the measures of center to the measure of variability.**

This time, the medians are 6 years and 4 years — a difference of 2 years. We use the interquartile range (IQR) to measure the variability. The IQR of the first group is 1.5 years and of the second, 1 year. Let's use about 1 year in the comparison.

The difference in the medians (2 years) is about twice the measure of variability (about 1 year). This means the difference is quite significant. If the difference had been only a fraction ($1/2$ or less) of the measure of variability, then the difference would not be significant.



median 6 years IQR 1.5 years



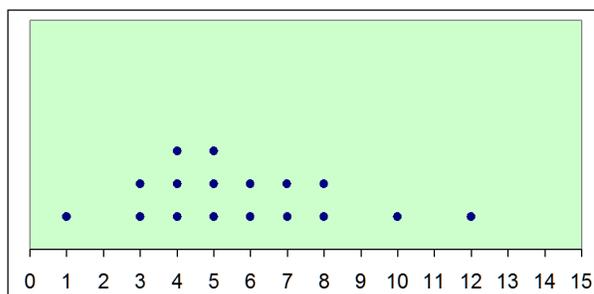
median 4 years IQR 1 year

Example 2. The two dot plots on the right again show the ages of two different groups of children.

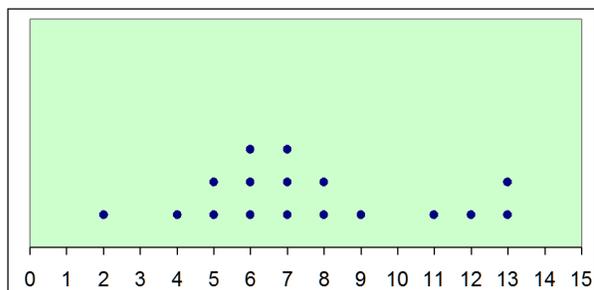
This time, we can see from the plots that there is great variability in the ages in both groups, because the data are very spread out. The data sets also overlap a lot: the first group has children from 1 to 12 years, and the second from 2 to 13 years, which means the overlap is from 2 to 12 years.

The difference in the medians is again 2 years. However, the IQR is much bigger now (3.5 and 4.5 years; we can think of it as being about 4 years). Therefore, the difference in the medians (2 years) is only about $1/2$ of the measure of variability.

This fact, along with the large overlap, helps us to see that the difference in the medians is not very significant. The ages of these groups of children are kind of similar, and not greatly different.



median 5 years IQR 3.5 years



median 7 years IQR 4.5 years

Example 3. The two graphs show science test scores for two classes, 7-A and 7-B. Which class did better, generally speaking?

We can see the answer just by looking at the distributions: The bars in the graph for 7-B are more towards the right than the bars in the graph for 7-A. So, class B did better.

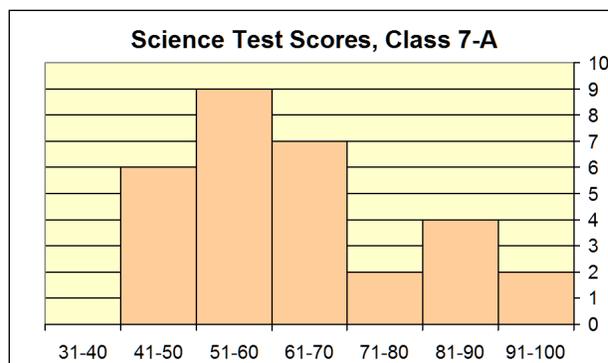
To find out *how much better*, we will compare the means of both data sets. The mean test score for class 7-A was 64.2 points and for class 7-B it was 74.8 points. The difference is 10.6 points.

Is that a significant difference?

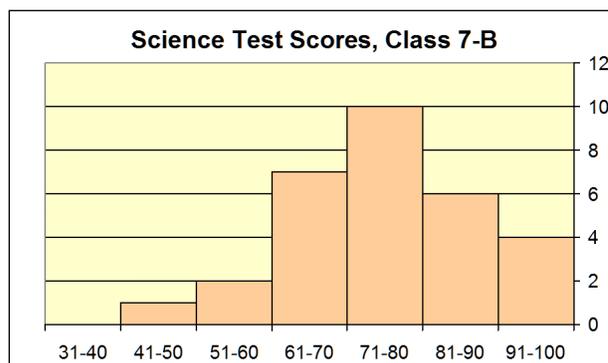
To check that, we compare the difference of 10.6 points to how variable or spread out the distributions are. The more variability there is in the two distributions, the bigger the difference in the means has to be for it to be significant.

We will use mean absolute deviation as a measure of variability. The mean absolute deviation of each data set is close to 11 points. The difference in the two means (10.6 points) is about one time the measure of variability. That tells us that the difference *is* significant.

If the difference in the means had been, say, 0.3 times the measure of variability (only 3 points), then the difference wouldn't have been significant.

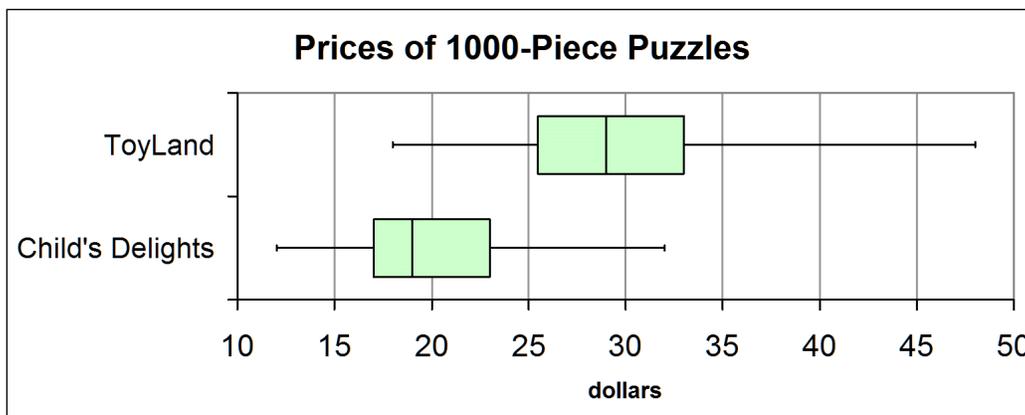


mean 64.2 MAD 11.8



mean 74.8 MAD 10.3

Example 4. The boxplots show the prices of 1000-piece puzzles in two stores, Toyland and Child's Delights.

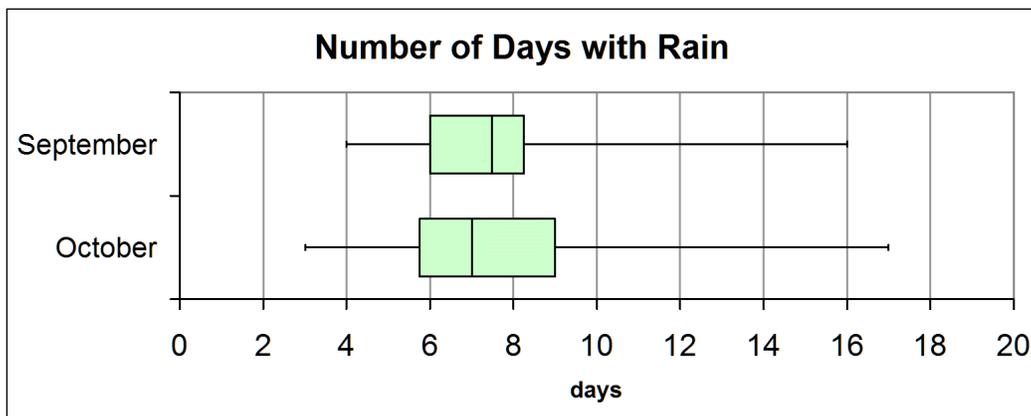


Boxplots make comparing sets of data very easy, since you can immediately see both the centers (median) and the spread (interquartile range) of the data sets from the plot.

We can see from the medians that overall, the puzzles in Child's Delights are cheaper. The prices in Toyland vary more, though, so you can find some cheap puzzles there, as well.

And while the ranges of the prices are quite different, the interquartile ranges (the length of the box) are similar: about 8 dollars for ToyLand and about 6 dollars for Child's Delights. The difference in the medians is about 10 dollars, which is about 1.5 times the interquartile range. That is a significant difference.

1. Jim studied the rain patterns in his hometown. He made boxplots to show the number of days with rain in September and October from data collected over 20 years.



a. Describe the overlap of the two distributions.

b. Based on the medians, which month has more days with rain overall?

Which month has the greater variability in the number of days with rain?

c. Estimate from the plot the medians for October and September and their difference.

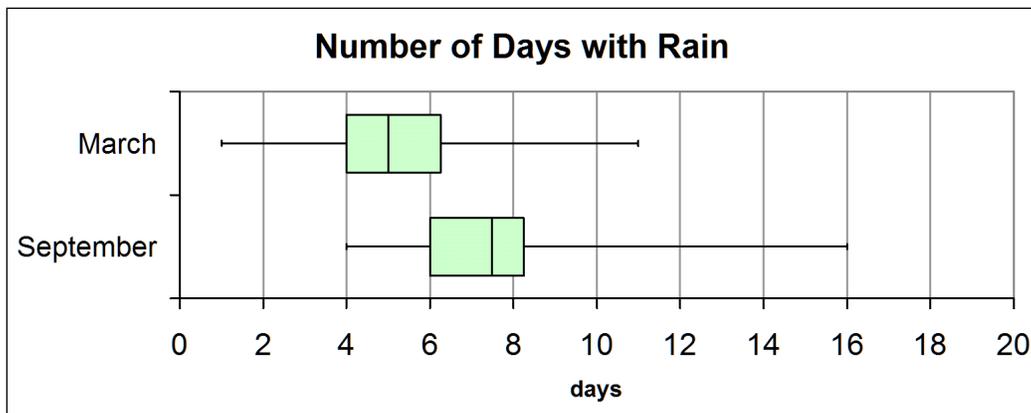
median (September) _____ median (October) _____ difference _____

d. Estimate from the plot the interquartile ranges for October and September.

IQR (September) _____ IQR (October) _____

e. Based on your answers to (c) and (d), is the difference in the medians significant?

2. Jim also made boxplots to compare the number of days with rain in March and September.



a. We can see that these two distributions overlap each other quite a bit, but not completely.

Let's say that a certain month had 4 rainy days.

Which is more likely, that the month was March or September?

b. Let's say that a certain month had 8 rainy days.

Which is more likely, that the month was March or September?

c. Based on the plots, which month has more days with rain overall?

Which month has the greater variability in the number of days with rain?

d. Find the difference in the medians and the interquartile ranges.

e. Compare the difference in the medians to the variability of the data.

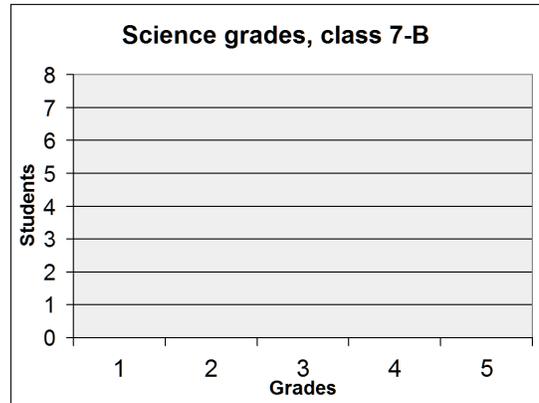
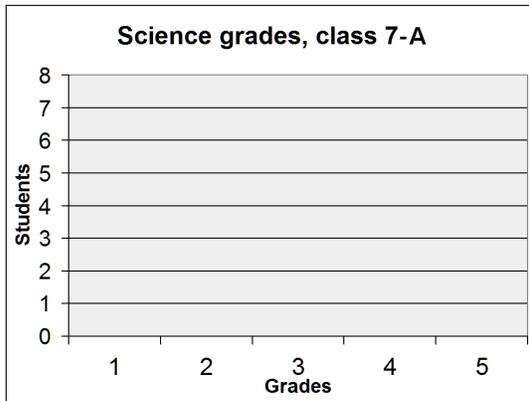
The difference in the medians is about _____ times the IQR.

So, are there significantly more days with rain in September than in March?

3. The following are the science grades of two 7th grade classes, grade 5 being the best possible grade. Make bar graphs from the data.

Class 7-A	
Grades	Students
1	5
2	8
3	7
4	5
5	2

Class 7-B	
Grades	Students
1	3
2	6
3	7
4	7
5	4



a. Compare the bar graphs visually.

Based on the graphs, which class, if either, did better overall?

Based on the graphs, which class, if either, had more variability in the grades?

b. Now, calculate the mean of the grades for each class and the difference in the means.

Class A, mean: _____ Class B, mean: _____ Difference: _____

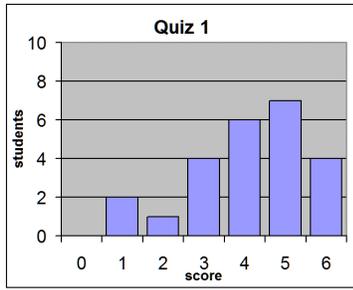


c. The mean absolute deviations of the data are 1.01 (class A) and 1.02 (class B).

The numbers are quite close. This means the variability is similar in both sets of data.

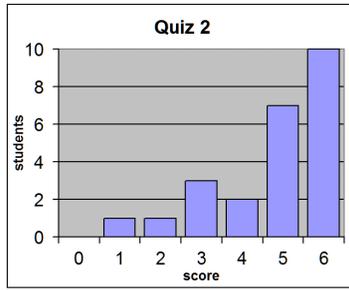
Compare the difference in the mean scores to the variability of the data, and use that to explain whether one of the classes did *significantly* better than the other.

4. Mrs. Ross gave her students several quizzes in the calculus class. The bar graphs for the scores are below.



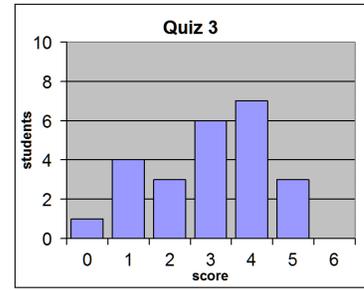
mean _____

MAD 1.14



mean _____

MAD 1.13



mean _____

MAD 1.22

- Look at the graphs. Mrs. Ross felt one of the quizzes turned out too easy (the students didn't!). Which one?
- In which quiz did the students fare the worst?
- The mean scores for the three quizzes were: 2.96, 4.13, and 4.79. Match each mean with the correct graph.
- Compare quiz 2 and quiz 3 now. What is the difference in the means for quiz 2 and quiz 3? _____
This difference is about _____ times the mean absolute deviation of the data (1.13).
Is the difference in the means significant?