[JR HIGH – YEAR 1 – STUDENT]

BOOK 1

PRINCIPLES OF MATHEMATICS

BIBLICAL WORLDVIEW CURRICULUM

[Katherine A. Loop]

[JR HIGH – YEAR 1 – STUDENT]

BOOK 1

PRINCIPLES OF MATHEMATICS

BIBLICAL WORLDVIEW CURRICULUM

[Katherine A. Loop]

First printing: April 2015

Master Books®, P.O. Box 726, Green Forest, AR 72638
Master Books® is a division of the New Leaf Publishing Group, Inc.

ISBN: 978-0-89051-875-5
Library of Congress Number: 2015937807

Cover by Diana Bogardus

Unless otherwise noted, Scripture quotations are from the King James Version of the Bible.

Some sections are heavily adapted with permission from Katherine Loop, *Revealing Arithmetic: Math Concepts from a Biblical Worldview* (Fairfax, VA: Christian Perspective, 2010).

Please consider requesting that a copy of this volume be purchased by your local library system.

Printed in the United States of America

Please visit our website for other great titles:
www.masterbooks.com

For information regarding author interviews,
please contact the publicity department at (870) 438-5288.

Master Books®
A Division of New Leaf Publishing Group
www.masterbooks.com

Acknowledgments

This curriculum was a major part of my life for several years, but I don't believe you would be holding it in your hands today if it were not for some very special people. I'd like to acknowledge and thank

my mom (Cris Loop) for being my right-hand helper throughout this project, solving hundreds of math problems and painstakingly working with me to make the material understandable.

my dad (Bob Loop) for his overall support (and specifically for all his input into the statistics chapter).

my brother (Brian Loop) for putting up with math questions at all hours of the day and helping me think through concepts and flow.

my cousin Hannah and my aunt Marie Ferreira for testing the initial draft, giving me a weekly deadline and providing invaluable feedback.

Joy Dubbs for being my technical sounding board on so many concepts, for reading drafts for me, and for never tiring of a math question.

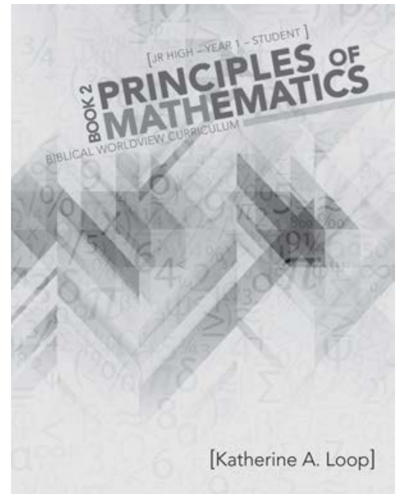
Abbey Ryan for reading the material, cheering me onward, and reminding me why I was writing this.

the people at Master Books for believing in the project and moving it into publication.

everyone else who has prayed for and supported me during this project in one way or another.

Above all, I'd like to thank the Lord, without whom all our labors are in vain.

Once you're done with Book 1 . . . you'll be ready for Book 2!



Get ready to continue exploring principles of mathematics! Now that you know the core principles of arithmetic and geometry, you're ready to move on to learning even more skills that will allow you to explore more aspects of God's creation.

In Book 2, we'll focus on the core principles of algebra, coordinate graphing, probability, statistics, functions, and other important areas of mathematics. The topics may sound intimidating, but you'll discover that they are simply useful techniques that serve a wide range of practical used. As we do in this book, we'll continue to discover that all of math boils down to a way of describing God's creation and a useful tool we can use to serve God, all while worshiping Him!

About the Author

Katherine Loop is a homeschool graduate from Northern Virginia. Understanding the biblical worldview in math made a tremendous difference in her life and started her on a journey of researching and sharing on the topic. For over a decade now, she's been researching, writing, and speaking on math, along with other topics. Her previous books on math and a biblical worldview have been used by various Christian colleges, homeschool groups, and individuals.

Website and Blog: www.ChristianPerspective.net
(Check out the free e-newsletter.)

Facebook: https://www.facebook.com/katherinealoop
(Join the conversation!)

YouTube: https://www.youtube.com/user/mathisnotneutral

About This Curriculum

What Is This Curriculum?

This is Year 1 of a two-year math course designed to give students a firm mathematical foundation, both academically and spiritually. Not only does the curriculum build mathematical thinking and problem-solving skills, it also shows students how a biblical worldview affects our approach to math's various concepts. Students learn to see math, not as an academic exercise, but as a way of exploring and describing consistencies God created and sustains. The worldview is not just an addition to the curriculum, but the starting point. Science, history, and real life are integrated throughout.

How Does a Biblical Worldview Apply to Math… and Why Does It Matter?

Please see lessons 1.1–1.3 and 2.7 for a brief introduction to how a biblical worldview applies to math and why it matters.

Who Is This Curriculum For?

This curriculum is aimed at **grades 6-8**, fitting into most math approaches the **year or two years prior to starting high school algebra**. If following traditional grade levels, Year 1 should be completed in grade 6 or 7, and Year 2 in grade 7 or 8.

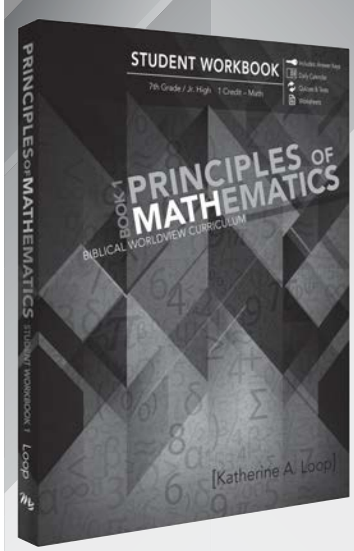
The curriculum also works well for **high school students** looking to firm up math's foundational concepts and grasp how a biblical worldview applies to math. High school students may want to follow the alternate accelerated schedule in the *Student Workbook* and complete each year of the program in a semester, or use the material alongside a high school course.

Where Do I Go Upon Completion?

Upon completion of Year 1, students will be ready to move on to Year 2 (coming Spring 2016). Upon completion of both years, **students should be prepared to begin or return to any high school algebra course.**

Are There Any Prerequisites?

Year 1: Students should have a **basic knowledge of arithmetic** (basic arithmetic will be reviewed, but at a fast pace and while teaching problem-solving skills and a biblical worldview of math) and **sufficient mental development** to think through the concepts and examples given. Typically, anyone in 6th grade or higher should be prepared to begin.



Student Workbook

Year 2: It is strongly recommended that students complete Year 1 before beginning Year 2 (coming Spring 2016), as math builds on itself.

What Are the Curriculum's Components?

The curriculum consists of the ***Student Text*** and the ***Student Workbook***. The *Student Text* contains the lessons, and the *Student Workbook* contains all the worksheets, quizzes, and tests, along with an Answer Key and suggested schedule.

How Do I Use This Curriculum?

General Structure — This curriculum is designed to be self-taught, so students should be able to read the material and complete assignments on their own, with a parent or teacher available for questions. This student book is divided into chapters and then into lessons. The number system used to label the lessons expresses this order. The first lesson is labeled as 1.1 because it is Chapter 1, Lesson 1.

Worksheets, Quizzes, and Tests — The accompanying *Student Workbook* includes worksheets, quizzes, and tests to go along with the material in this book, along with a suggested schedule and answer key.

Answer Key — A complete answer key is located in the *Student Workbook*.

Schedule — A suggested schedule for completing the material in 1 year, along with an accelerated 1-semester schedule, is located in the *Student Workbook.*

General Notes to Students

Review — If at any point you hit a concept that does not make sense, *back up and review the preceding concepts.*

Showing Your Work — Except for mental arithmetic problems, you should show your work on all word problems — this means you should write down enough steps of what you did that someone can see how you solved the problem (what you added, subtracted, etc.). Unless otherwise specified, it does not matter how you show your work (it doesn't have to be as in-depth as the answer key) — the important thing is that you can see how you obtained your answer. While showing your work may seem like busy work on simple problems, forming the habit of organizing your steps on paper from the beginning will greatly help you when you come to in-depth problems involving numerous steps.

Units of Measurement — If a unit is given in the problem (miles, feet, etc.), you should always include a unit in your answer.

Fractions — From 5.3 on, fractional answers should be denoted in simplest terms, unless otherwise specified. This includes rewriting mixed numbers as improper fractions. If a question is asked using only fractions, your answer should be listed as a fraction.

Decimals — From 7.4 on, decimal answers should be rounded to the hundredth digit unless otherwise specified.

[CONTENTS]

Preface

Growing up, I never pictured myself writing a math curriculum. While I was good at math, I never grasped its importance nor understood how to apply some of the more advanced concepts outside of a textbook. It was a subject of rules to be memorized, applied, and forgotten. After all, would I ever really use most of it in life?

Nor did I see how math could be viewed from a biblical worldview. I was blessed to grow up in a Christian homeschool family and to see the worldview battle in my other subjects, but not in math. I subconsciously delegated math to some sort of "neutral" category.

Obviously, something changed. During my senior year, my mom had me read a book by James D. Nickel titled *Mathematics: Is God Silent?* As the book delved in depth into the history and philosophy of math, I realized that math wasn't neutral at all—and how I saw all of math began to transform.

Excited, I desperately wanted to share what I'd learned with everyone I could. My heart ached for the many young people growing up without seeing God in math or really understanding how it served as a tool to describe His creation. Yet there simply weren't many resources available on the topic.

The need prompted me to begin speaking and writing on math. I left library after library with stacks of math books. I read and read and read. The more I learned, the more amazed I grew. Around two years later, I wrote my first book, *Beyond Numbers: A Practical Guide to Teaching Math Biblically*. After much more research, I wrote my second book, *Revealing Arithmetic: Math Concepts from a Biblical Worldview*.

Both these books gave parents tools and information on how to teach math from a biblical worldview. *Revealing Arithmetic* added details and specific ideas for arithmetic concepts. Still, though, everyone kept asking for an actual curriculum—something I insisted I'd never write. The mere thought was overwhelming.

It eventually became clear that writing a curriculum was exactly what I was supposed to do. After unsuccessfully trying my hardest to get out of the task, I succumbed and began writing.

The journey proved tougher than I imagined, but also more amazing as I got to watch the Lord provide despite (or rather through) many obstacles, including a concussion that took more than a year and a half to recover from. I discovered over and over again that it's truly only by God's enabling that we can do anything. He's the One who causes our brains to work and who sustains the universe in a consistent fashion, making math possible. God's provisions also included many precious people who helped, as you'll see from the Acknowledgments.

It's my earnest prayer that as you use this curriculum and study math from a biblical worldview, you'll be reminded of God's greatness and encouraged that you can trust Him completely.

By His grace,

Katherine Loop

[CHAPTER 1]

Introduction and Place Value

1.1 Math Misconceptions

Math — what does the word bring to mind? Numbers in a textbook? Lists of multiplication and division facts? Problems to solve?

That about sums up the typical view of math, doesn't it? Yet while math does have numbers, facts, and problems, there's much more to math than typically presented.

But before we look at what math is, let's start by examining what it is not. Specifically, let's take a look at three common — but dangerous — misconceptions about math.

Misconception 1: Math Is Neutral

Most math books approach math as a neutral subject. And at first glance, math certainly appears neutral. Neutral means "not engaged on either side; not aligned with a political or ideological grouping."[1] Christians and atheists all can agree that "1 + 1 = 2." This makes math neutral, right?

To answer this, consider a tree. A tree seems pretty neutral too, doesn't it? People of all religions can see a tree, touch a tree, smell a tree, and study a tree, agreeing on a tree's basic features. But this does not mean a tree is neutral. A tree's very existence begs for an explanation. Where did trees originate? Why does a tree have intricate parts that all work and grow together?

Our underlying perspective regarding a tree is determined by what we would call a **worldview**. In *Understanding the Times*, David Noebel (founder of Summit Ministries) defines a worldview this way: "A worldview is like a pair of glasses — it is something through which you view everything. And the fact is, everyone has a worldview, a way he or she looks at the world."[2] In other words, a worldview is a set of truths (or falsehoods we believe to be true) through which we interpret life.

Those with a biblical worldview — those looking at life in light of what the Bible teaches — would see a tree as part of God's originally perfect but now fallen creation, while those with a naturalistic worldview might say a tree evolved from a cosmic bang. When we look at the essential questions of a tree — where it came from, how we should use it, etc. — we see a tree is not really neutral.

In a similar way, math facts may seem neutral. People of all religions can use math and agree that "1 + 1 = 2." But this does not mean math is neutral. Where did math originate? Why does math work the way it does? Why are we able to use math?

Just as it does in the case of a tree, the Bible gives us a framework from which we can answer these questions and build our understanding of math. As we'll discover, only the biblical explanation for math's very existence makes sense out of math and transforms math from a dry list of facts to an exciting exploration.

The point here is simply that math cannot be neutral. The Bible teaches Jesus is Lord of all — the Creator and Sustainer of *all* things (Colossians 1:16–17). He doesn't exempt math from that. Math cannot be separated into a "neutral" box.

Misconception 2: A Biblical Math Curriculum Is the Same as Any Other, with a Bible Verse or Problem Thrown In Now and Then

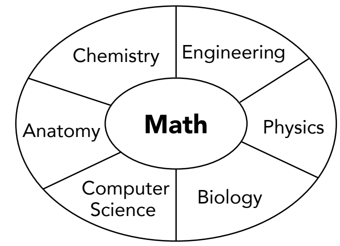
If you're wondering if we're just going to add a Bible verse to the top of the page, mention God dividing the Red Sea when we discuss division, and have you solve Bible-based word problems, let me assure that this is *not* what this curriculum aims to do. Although thinking about how God divided the Red Sea might be helpful in turning our eyes to the Lord, it does nothing for helping us understand how to view *division itself* in light of biblical principles. In this course, we're aiming to let the Bible's principles transform our view of *math itself.*

Misconception 3: Math Is a Textbook Exercise

Quite often, math comes across as a textbook exercise. We memorize this and solve that. There are so many seemingly arbitrary rules to follow that it's easy to scratch your head and wonder who invented this complex system in the first place.

If your view of math is confined to rules and problems — or even if you know there's more to math but are not sure why it feels so dry — there is good news! Math is *not* a mere textbook exercise. Math helps us observe the design throughout creation, design instruments, draw, build boats, operate a business, work with computers, cook, sew, and more. In this course, we'll incorporate history, science, and real-life applications as we go, exploring math both inside *and* outside of a textbook.

1.2 What Is Math?



If you were to try to work in nearly any field of science — be it chemistry, engineering, or anatomy — you would need to study and use math. Why? *Because math is the tool scientists use to explore creation.*

Not only do scientists use math, but artists, pilots, musicians, business managers, clerks, sailors, and homemakers do too. All occupations use math to one extent or another!

Math also shows up in everyday life. Every time you go shopping, you use math — math helps you know how much an item costs (price tags use numbers!), find the unit price of an item, estimate your total, etc. You use math in the kitchen when you measure ingredients. You use math to count the number of silverware to put on the table, to balance a checkbook and track your finances, to understand loans and car payments, to figure out how many bags of bark you need to landscape a flowerbed or how many square feet of carpet to cover a room — the list of math's everyday uses goes on and on.

Math is clearly more than intellectual rules and techniques found in a textbook. Which brings us to the question: what *is* math?

When we **start with the Bible** — God's revealed Word to man — as our source of truth, it makes sense out of every area of life, including math. It gives us a framework for answering not only *what* math is, but also *where* it came from and *why* it works. Take a look at just a few truths with me.

- **All things were created and are sustained by the eternal, triune God of the Bible.**

> *In the beginning God created the heaven and the earth. And the earth was without form, and void; and darkness was upon the face of the deep. And the Spirit of God moved upon the face of the waters (Genesis 1:1–2).*
>
> *In the beginning was the Word, and the Word was with God, and the Word was God. The same was in the beginning with God. All things were made by him; and without him was not any thing made that was made. . . . And the Word was made flesh, and dwelt among us (and we beheld his glory, the glory as of the only begotten of the Father,) full of grace and truth (John 1:1–3, 14).*
>
> *Jesus answered them. . . . "I and my Father are one" (John 10:25, 30).*
>
> *For by him [Jesus] were all things created, that are in heaven, and that are in earth, visible and invisible, whether they be thrones, or dominions, or principalities, or powers: all things were created by him, and for him: And he is before all things, and by him all things consist (Colossians 1:16–17).*

[referring to Jesus] . . . upholding all things by the word of his power . . . (Hebrews 1:3).

- **God is a consistent God who never changes — and who has appointed the ordinances of heaven and earth.**

For I am the LORD, I change not; therefore ye sons of Jacob are not consumed (Malachi 3:6).

Thus saith the LORD; If my covenant be not with day and night, and if I have not appointed the ordinances of heaven and earth; Then will I cast away the seed of Jacob and David my servant, so that I will not take any of his seed to be rulers over the seed of Abraham, Isaac, and Jacob: for I will cause their captivity to return, and have mercy on them (Jeremiah 33:25–26).

- **God created man in His own image and gave him the task of subduing the earth.**

So God created man in his own image, in the image of God created he him; male and female created he them. And God blessed them, and God said unto them, Be fruitful, and multiply, and replenish the earth, and subdue it: and have dominion over the fish of the sea, and over the fowl of the air, and over every living thing that moveth upon the earth (Genesis 1:27–28).

Let's look at how these truths apply to math. A never-changing God is holding *all* things together and has appointed the ordinances — or the decrees — by which heaven and earth operate. God created and sustains a consistent universe. God also created man in His image, capable of subduing and ruling over the earth.

We already established that math is the tool scientists use to describe creation. In other words, math is a way of describing the consistencies God created and sustains! Man is able to use math to, in a very limited way, think "God's thoughts after Him" (Johannes Kepler) because God made us in His image and gave us the ability to subdue the earth.

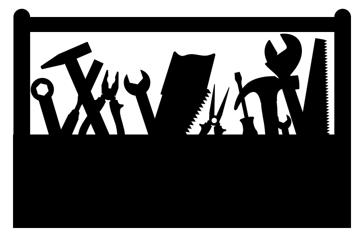
The Bible teaches that God created all things — and math is no exception. The symbols and techniques we think of as math describe on paper the ordinances by which God governs all things. Men might develop different symbols (people have used many different numerals and techniques throughout history, as we'll see throughout this course), but men have no control over the principles. No matter what symbols we use to describe it, one plus one consistently equals two because God both decided it would and, day in and day out, keeps this ordinance in place!

Math, in essence, is **a way of describing the consistent way this universe operates.** It is the language, so to speak, we use to express the quantities and consistencies around us — quantities and consistencies God created and sustains.

Math works outside a textbook *because* God is faithful to uphold all things. Math facts never change *because* God never changes. We can rely on math *because* we can rely on God. Math is complex and complicated *because* God created a complex universe and it takes a lot of different rules and methods to even begin to describe it! Math applies universally *because* God's rule is universal — He's present everywhere. Math helps us see the incredible wisdom and care displayed throughout creation — an order, wisdom, and care that is there *because* we have a wise and caring Creator! At the same time, math reveals the effects of sin that mar God's original design — effects that are there *because* of man's sin, but which remind us of the mercy found in Jesus.

> *Mathematics transfigures the fortuitous concourse of atoms into the tracery of the finger of God. — Herbert Westren Turnbull*[3]

Do you catch how this understanding could revolutionize our view of math? Math doesn't have to be a dry subject of mere numbers and techniques. Numbers and techniques are tools to describe God's creation and help us with the real-life tasks God's given us to do. As Walter W. Sawyer points out, mathematics is like a chest of tools.



> *Mathematics is like a chest of tools. — Walter W. Sawyer*[4]

I love that imagery. Picture a chest of tools for a moment. Some tools — such as a screwdriver — are easy to use and apply in thousands of situations. Other tools — such as a router — take more time and dedication to master and serve a more limited, although just as necessary, purpose.

In a similar fashion, some math concepts — such as addition — are fairly easy to grasp and frequent in their applications. Others — such as some aspects of algebra or calculus — require more dedication to grasp. These higher-level concepts, while they might not come in handy as often as addition, have *very* powerful applications.

From basic to advanced, *all* of math is a tool that points us to God and can be used to complete the tasks He gives us to do!

Ready to Begin?

Some of you reading this course probably dislike math or find it an incredibly challenging task. Others of you may love it and be gifted in it.

Whatever your current view of math, I invite you to take a journey with me. While we'll be seeking to approach concepts from a biblical worldview throughout the course, these first two chapters will be extra-intensive in that department, as we want to lay a firm foundation upon which to build the rest. So please bear with the extra amount of reading.

My prayer is that you'll acquire a deeper appreciation for God's greatness and faithfulness and be encouraged in your walk with Him as you delve into the world of mathematics.

1.3 The Spiritual Battle in Math

The Bible gives us a solid foundation for why math works. Math is a tremendous testimony to God's faithfulness and power. Yet math has been sadly twisted.

Let's take a deeper look at the spiritual battle within math, at how men try to explain math apart from God, and at how ultimately only a biblical worldview makes sense out of math.

Naturalism in Math

Consider the following quote:

> *One cannot escape the feeling that these mathematical formulae have an independent existence and an intelligence of their own, that they are wiser than we are, wiser even than their discoverers, that we get more out of them than was originally put into them.*
> *— Heinrich Hertz (German physicist)*[5]

Notice that Mr. Hertz is claiming that mathematical formulae themselves are wise and have an independent existence. Rather than acknowledging God, he is giving *math itself* the credit for math's amazing ability to work. This is a very naturalistic view of math — an attempt to explain math from only natural causes, apart from God.

Let's think about this claim for a moment. Can math itself explain its own existence? Remember, math goes hand in hand with creation. Things don't just "happen." We live in a universe consistent enough that we can describe gravity using math and call it a law. If the universe were run by random processes, why would we see such order, design, and consistency?

Besides this, viewing math as a self-existent truth still doesn't answer the fundamental question of how we *know* it's true in the first place. Is the foundation for truth our experience? Do we know that one plus one equals two because we experience that it does, and therefore assume that it always will?

Our experience in itself is not a solid foundation for truth. For one, we can never experience everything, so therefore could never truly know anything for sure! Math is so useful because it helps us solve problems we have not experienced. It allows us to calculate the force needed to get a brand-new rocket into the air — and to predict how a bridge will hold weight before we build it. Much of math deals with things that we can never actually experience, but which help us solve a variety of real-life problems. In order to use math, we *have* to assume it works consistently in areas we have never — and never can — experience.

Humanism in Math

Now consider this quote:

> *The German mathematician Leopold Kronecker (1823–91) once said, "God made the integers, all else is the work of man." First causes, this comment suggests, are divine, while the complexities, minutiae, and refinements of mathematics are a human creation. For Kronecker's contemporary Dedekind, however, the integers too were the "free creations of the human mind." . . . For him, as for many modern mathematicians and theorists, mathematics stood as an independent and secular discipline. — Denis Guedj*[6]

Who did Kronecker and Dedekind give credit for math? *Man*. Both these men viewed math as the product of the human mind. Rather than giving God the credit for math's ability to work, they gave it to man. This is a humanistic view of math — a view that focuses on *man* and *his* achievements, ignoring his Creator.

Let's think about the problem with basing truth on human reasoning. Time and again, math concepts men think up using mathematical reasoning end up applying in creation. Why is this? Why do men's thoughts line up with reality? Why do we find such an orderly, mathematical world all around us?

Albert Einstein expressed the problem this way — and admits there's something miraculous in the world that can't be explained by reasoning alone.

> *Even if the axioms of the theory are posited by man, the success of such a procedure supposes in the objective world a high degree of order which we are in no way entitled to expect a priori [based on man's reasoning]. Therein lies the "miracle" which becomes more and more evident as our knowledge develops. . . . And here is the weak point of positivists and of professional atheists, who feel happy because they think that they have not only pre-empted the world of the divine, but also of the miraculous.*[7]

Also, why are there universal laws of logic we rely on to be true? Why can't one person decide that 1 plus 1 will equal 2 and another that it will equal 3 and they both be right? This sort of thinking, if applied consistently, would completely make math, as well as logic itself, meaningless and useless!

The Battle Defined

The spiritual battle over math is the same as the battle we find in other areas of life. Will we recognize our *dependency* on God, or claim *independence* from Him?

Our view of any area of life — including math — is either going to stem from a dependent perspective on life (one that recognizes our dependency on God and His Word) or an independent one. When we get down to the fundamental level, there is no such thing as neutrality. Even a tree is not neutral — as we saw in the first lesson, the tree was either created by God or got here some other way.

While it is true that man developed math symbols and techniques, it makes no sense why those symbols and techniques mean anything in real life if they truly are the "free creations of the human mind" as Dedekind stated.

For more information different worldviews on math and how the biblical worldview makes sense of math, see James D. Nickel, *Mathematics: Is God Silent?* rev. ed. (Vallecito, CA: Ross House Books, 2001). For more information on how logic itself can't be explained apart from God, see Dr. Jason Lisle, *The Ultimate Proof of Creation: Resolving the Origins Debate* (Green Forest, AR: Master books, 2009).

Likewise, math is either dependent on God or it is not. If God does not receive the glory for math's ability to work, that glory goes somewhere else. As R.J. Rushdoony points out:

> *. . . mathematics is not the means of denying the idea of God's pre-established world in order to play god and create our own cosmos, but rather is a means whereby we can think God's thoughts after Him. It is a means towards furthering our knowledge of God's creation and towards establishing our dominion over it under God. The issue in mathematics today is root and branch a religious one.*[8]

The Bible is clear: we are to trust and worship God; He gives us our every breath, He controls each aspect of life, and He determines truth — apart from Him we are nothing. If man ignores this truth, he does so to his own demise.

> *For the wrath of God is revealed from heaven against all ungodliness and unrighteousness of men, who hold the truth in unrighteousness; Because that which may be known of God is manifest in them; for God hath shewed it unto them. For the invisible things of him from the creation of the world are clearly seen, being understood by the things that are made, even his eternal power and Godhead; so that they are without excuse: Because that, when they knew God, they glorified him not as God, neither were thankful; but became vain in their imaginations, and their foolish heart was darkened. Professing themselves to be wise, they became fools, And changed the glory of the uncorruptible God into an image made like to corruptible man, and to birds, and fourfooted beasts, and creeping things (Romans 1:18–23).*

The Depth of the Battle

The battle over math is much more than a theological squabble over numbers. It ultimately affects our entire approach to truth.

If we look at math as something spiritually neutral — a self-existent or man-made fact — then math becomes an independent source of truth. We find ourselves viewing math as the ultimate standard rather than God's Word.

Millions of people have embraced the lie of evolution because they believe it has been scientifically proven to be true. At the root of their belief is the false notion that deductive reasoning or mathematical principles are the ultimate standard ruling the universe.

Yet, apart from God, it does not even make sense why we can reason or why the universe is consistent! Science and math would be impossible in a universe without God. The very tool skeptics try to use to disprove God cannot be explained apart from God. Even honest unbelievers acknowledge their inability to explain math in their worldview. Most simply ignore the *why*.

> *In this article I shall not attempt any deep philosophical discussion of the reasons why mathematics supplies so much power to physics. . . . The vast majority of working scientists, myself included, find comfort in the words of the French mathematician Henri Lebesgue: "In my opinion a mathematician, in so far as he is a mathematician, need not preoccupy himself with philosophy — an opinion, moreover, which has been expressed by many philosophers."*
> *— Freeman J. Dyson*[9]

But when we look at math from a biblical perspective, we understand that math is not a source of truth; it is a description of the consistencies of God. God is the source of truth. We can only rely on math to work because we can trust God. Thus, as we study math in this course, we will not approach it as a means to determine truth or as the source of truth, but rather as a tool to help us understand the trustworthy principles our trustworthy God created and sustains.

Math and the Gospel

Although we might try our hardest, we cannot change math. We can change the symbols or names we use in math, but we cannot change what the names and symbols represent — 1 of something plus 1 of something else will consistently equal 2. Math is not relative. Why?

Because God is God and we are not! He, not us, decides how things will be. He set and keeps certain principles in place, and if we want a math that will actually work, we *have* to conform to those principles.

Math reminds us that God decides truth, not us. We need to be careful in every area that we take head to the truth He has revealed to us in His Word, the Bible, and that we don't try to change those truths to fit what we think or want. For example, the Bible tells us that salvation is by faith in Jesus, and not by works or any other way (Ephesians 2:8; James 14:6), and that hell is real (Revelation 21:8).

It's tempting to try to change this truth, thinking there must be some good in ourselves or that God would not really send people to hell (especially those whom we love and think are nice), but God's truth is not open to negotiation. He's God, not us. If we want salvation, we have to conform to what *God* says will save us.

Over and over again, the Bible, God's Word, urges us to trust in God's way of salvation — Jesus. Only He could pay the penalty for sin. Only by believing upon Him — admitting our own helplessness — can we be saved. Just as God is faithful to hold this universe together consistently, He will be faithful to everything else He says in His Word. You can rely on what God says.

If you've not responded to God's gift of salvation, today is the day to do so! He will keep His Word — both to save and to punish.

If you're not sure if you have trusted God's way of salvation, don't delay in making sure. If you are sure, then take tremendous comfort in the knowledge that God is faithful and will complete what He began in you.

For more information on God's plan of salvation, please see www.biblicalperspective.net.

Keeping Perspective

The battle we face in math is ultimately a battle to remember our complete dependency on God. Even our ability to count comes from Him! Each math concept works only *because* of His faithfulness. Apart from Him, we truly can do *nothing*.

Ever since the Garden of Eden, Satan has been trying to distort the truth and get men to trust themselves instead of God. He has done this very thing in math — turning what should be a testimony to God into a testimony to man and math.

We can all see that math works. Someone or something has to be responsible for math's ability to work. If we're not giving the glory for math to God, then we're ending up giving it to man or to math. If math is not encouraging us that we can depend on our faithful, all-powerful God, then it is subtly telling us we can live independently from Him and determine our own truth.

Yes, indeed, there is a spiritual battle in math — and it's the same battle we face in every area of life.

1.4 Numbers, Place Value, and Comparisons

Now that we've seen the overall foundation the Bible gives us and explored a little about the spiritual battle in math, let's begin applying what we've discussed to specific aspects of math. In order to build our understanding of math from the foundation up, we'll be exploring simple review concepts for these first few chapters. As we do, though, we'll be learning important principles that apply to more advanced concepts.

An Overview of Mathematical Symbols and Terms

Math is filled with symbols and terms. Just as it is helpful if we use the same words to refer to objects (a book, sink, couch, etc.), it's helpful to use standardized symbols and terms in math.

Before we jump into looking at specific symbols and terms, though, let's take a minute to look at the big picture. Much of math is a naming process — a way of describing quantities and consistencies God created. So let's take a look at the first "naming" process the Bible describes: Adam naming the animals.

> *And out of the ground the LORD God formed every beast of the field, and every fowl of the air; and brought them unto Adam to see what he would call them: and whatsoever Adam called every living creature, that was the name thereof (Genesis 2:19).*

In naming the animals, Adam

1. observed God's creation (the animals) and
2. assigned names to describe the different animals.

In describing quantities, we

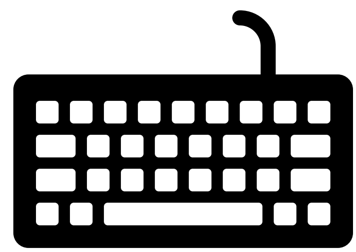
1. observe God's creation (the quantities around us) and
2. assign names (or symbols) to different quantities.

So what can we learn from Adam naming the animals? Well, notice that God brought the animals to Adam for naming — Adam was in God's presence while observing and naming. While sin separated man from his Creator, through Jesus we can again **know God and worship Him while using math to describe His creation.** This holds true not just for basic math, but for *every* area of math we'll explore. In fact, the Bible urges us to do "whatsoever" we do "as to the Lord"!

> *And whatsoever ye do, do it heartily, as to the Lord, and not unto men; (Colossians 3:23).*

Number Systems: Beyond Quantities

Number systems prove useful in other ways besides recording quantities too. For example, house numbers and telephone numbers don't record quantities — instead, they give us a way of "naming" homes and telephone lines. As another example, numbers and math are used in cryptography ("the art of writing or solving codes")[11] to help code messages. And before you picture coding as only wartime messages across enemy lines, did you realize that computers use a code to translate the letters or symbols you type on a keyboard? There's a number assigned to every letter or symbol that can be typed on a keyboard!

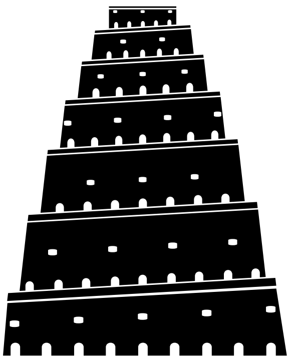


Reviewing Numerals and Place Value

Undoubtedly, you already know how to count (use words — like "one" — to describe quantities) and write numerals (use symbols — like "1" — to describe quantities). Below is just a quick review.

"Zero" is the name we mainly use in English to describe having nothing (you may also sometimes hear other names, such as "nought," "oh," or "nil," used to mean nothing). "One" is the name for a single unit — a single pen, dollar, CD, etc. "Two" is the name for a group of 2 units of anything.

Rather than words, we commonly use symbols. It's a lot easier to write "1" than to spell out "one" all the time! At the same time, though, it would be impossible to have a different symbol for *every* number. Instead, we use a concept known as place value.



Different Words for Quantities — The Tower of Babel

Different cultures use different words to describe quantities. For example, a single quantity is called *one* in English, *uno* in Spanish, *eine* in German, and *один* in Russian. Once again, the Bible tells us why.

Genesis 11:1–9 tell us about an event that changed the world — the Tower of Babel. Prior to the Tower of Babel, "the whole earth was of one language, and of one speech" (Genesis 11:1). Thus, men would have used the same words to describe quantities.

At the Tower of Babel, men misused the ability God had given them to communicate and sought to unite against God and make a name for themselves. The project stopped abruptly when God came down and confused their languages. The Tower of Babel accounts for the many different language systems we find, including the different words used to describe numbers.

In place value, the place, or location, of a number determines its value. So we use the symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 to represent up to nine. Once we reach ten, we move to the next "place" over, using the same digits, but knowing that each one represents a set, or group, of ten. So "10" represents 1 set of ten, and 0 sets of one, "20," represents 2 sets of ten, and 0 sets of one, and "21" represents 2 sets of ten, and 1 set of one.

This place-value concept can be extended as far as necessary to represent numbers. Once we have ten tens, we move on to hundreds, then thousands, then ten thousands, then hundred thousands, and so forth.

Hundred trillion	Ten trillion	One trillion	Hundred billion	Ten billion	One billion	Hundred million	Ten million	One million	Hundred thousand	Ten thousand	One thousand	Hundred	Ten	One

Using our place-value system, we would represent the approximate world population in 2011 as 6,946,043,989 (or six billion, nine hundred forty-six million, forty-three thousand, nine hundred eighty-nine).[10]

Hundred trillion	Ten trillion	One trillion	Hundred billion	Ten billion	One billion	Hundred million	Ten million	One million	Hundred thousand	Ten thousand	One thousand	Hundred	Ten	One
					6	9	4	6	0	4	3	9	8	9

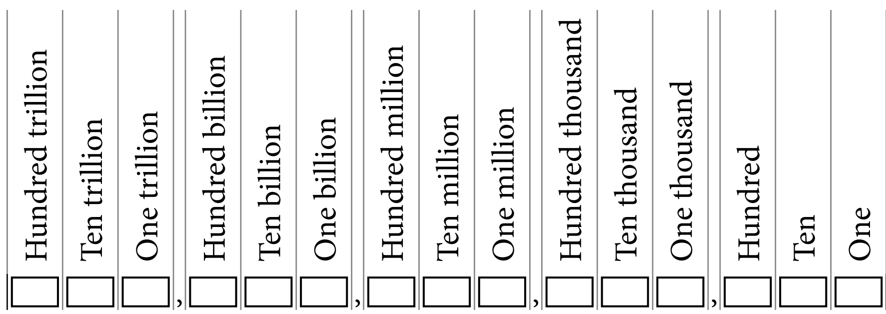
Notice how the commas every three places help our eyes count the places and determine the value.

4444444 4,444,444

In other countries, decimal points (4.444.444) or other separators are used instead of commas. (An important thing to keep in mind if ordering something online from another country!) Spaces (4 444 444) are also a recognized way of separating the places.

Reading Numbers

Notice how when reading numbers, we recycle terms. We start with ones (our basic units), tens, and hundreds. Then we have thousands (our new unit), followed by *ten* thousands and *hundred* thousands. We repeat this for millions, billions, etc.



Notice also that in writing, we use commas every three digits, thereby separating the "thousands," "millions," etc.

To read a number, we read the number from left to right. If a digit has a zero, we don't read that place, as there's nothing to "report" there (as in the 0 in the hundred's place in 123,456,567,087).

123,456,567,087 would be read "one hundred twenty-three billion, four hundred fifty-six million, five hundred sixty-seven thousand, eighty-seven."

Now, I am sure you already know how to read numbers in English, but did you realize that there are variations in how to read numbers? The British often add an "and" (example: "one hundred *and* twenty-one" instead of "one hundred twenty-one"). 1,325 could also be read as "thirteen twenty-five" instead of as "one thousand three hundred twenty-five." This might make sense for dates ("the year thirteen twenty-five") or even house numbers ("I live at thirteen twenty-five Pleasant Lane"). When reading a street address over the phone, you might even just read each digit by itself, as in "one, three, two, five Pleasant Lane" to avoid confusion. These variations remind us that **names are a tool to help us communicate**, so clearly communicating is the most important thing.

When asked to write the word you would use to read a number in this course, use the traditional American method ("one thousand, three hundred twenty-five" for 1,325).

Reviewing Basic Comparison Terms and Symbols

If a number is larger, or has more, than another number, we say it is **greater than** the other number. If it is smaller/has less, we say it is **less than** the other number. If two quantities are the same, we say they are **equal**. If they are not the same and we do not want to make a specific comparison as to which one is greater, we say they are **not equal**. (Any number that is greater than or less than another number is also not equal to it — it's just a matter of what point we want to make.)

Notice that the "less than" and "greater than" signs are the same, but pointing the opposite directions. You can remember which direction to put the symbol by remembering that the **larger side goes with the larger number.**

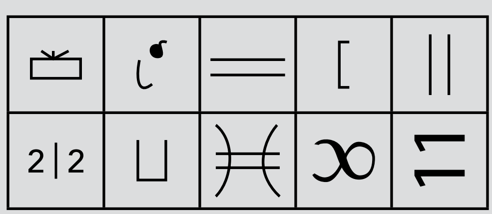
The symbols <, >, =, and ≠ are merely "shortcuts" for describing how numbers compare. They save our fingers from having to write the word out every time. It's a lot easier to write < than "less than." It also makes equations easier to read.

● ● ● ● ● 5	5 is less than or does not equal 6. < or ≠	● ● ● ● ● ● 6	● ● ● ● ● 5	5 equals 5 =	● ● ● ● ● 5
● ● ● ● ● ● 6	6 is greater than or does not equal 5 > or ≠	● ● ● ● ● 5	● ● ● ● ● ● 6	6 equals 6 =	● ● ● ● ● ● 6

Would it surprise you to learn that >, <, =, and ≠ are algebraic symbols? Any time we use a non-numerical symbol in math, we are actually using algebra. So >, <, =, and ≠ are actually part of algebra! Algebra is nothing to fear — it's just a way of using symbols to describe God's creation. In the case of >, <, =, and ≠ , we're using symbols to describe how numbers compare.

Different Ways to Compare Numbers

Much as symbols for writing numbers have varied, so have symbols for comparing them. While we're used to using the "=" sign to mean "equal to," other symbols have been used throughout history — the box shows just a few. Instead of symbols, many cultures also used words or contractions to describe equality (*pha, equantur, aequales, gleich,* etc.).[12] Once again, history helps us see that the symbols we study in math are just an agreed-upon language system we use today to describe the quantities and consistencies God created and sustains.



Keeping Perspective

We looked today at a few names (one, two, three, etc.) and symbols (1, 2, 3, =, >, <, etc.) used in math. As we continue our study of math, we're going to learn various names and symbols men have adopted to describe different consistencies or operations. Keep in mind that **terms and symbols are like a language** — agreed-upon ways of communicating about the quantities and consistencies around us.

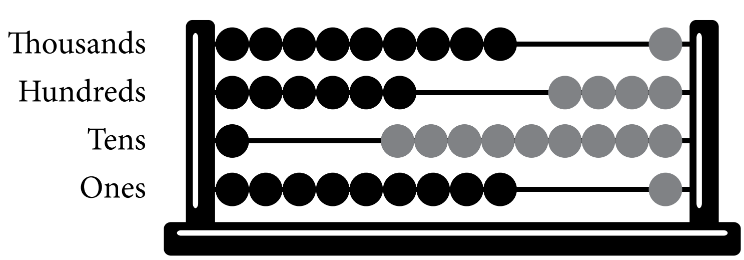
1.5 Different Number Systems

It's all too easy to start viewing the terms, symbols, and methods we learn in math as math itself, thereby subtly thinking of math as a man-made system. A look at history, however, reveals many other approaches to representing quantities. Let's take a look at a few of them and at how they compare with our place-value system.

Place-Value Systems

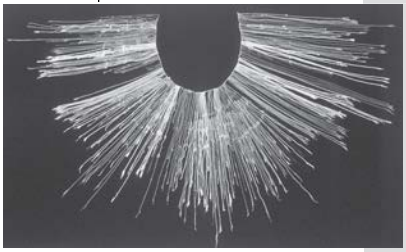
In the last lesson, we reviewed how the number system we're mainly familiar with uses the place, or location, of a digit to determine its value. This is known as a **place-value system.**

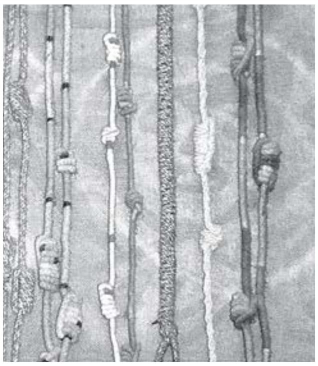
Perhaps place value is easiest to picture using a device used extensively throughout the Middle Ages: an abacus. Each bead on the bottom wire of an abacus represents one; on the next, ten; on the third, one hundred; and on the fourth, one thousand. To represent a quantity on an abacus, we move the appropriate number of beads from each wire to the right. In the abacus shown, the 1 bead to the right on the thousands wire represents 1 thousand, the 4 beads to the right on the hundreds wire represent 4 hundred, the 9 beads to the right on the tens wire represent 9 tens, and the 1 bead on the ones wire represent 1. Altogether, that makes 1,491.



1,491

Just as the place, or line, of a bead changes its value, the place, or location, of a symbol in a place-value system changes its value. The number system commonly used today is called the **Hindu-Arabic decimal system** (or just the "**decimal system**" for short). This system came from the Hindu system, which the Arabs adopted and brought to Europe.





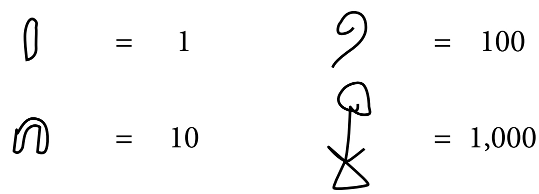
The Quipu — An Intriguing Approach

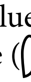
The Incas — an extensive empire in South America spanning more than 15,000 miles — had a fun approach to recording quantities. They tied knots on a device called a quipu (kē pōō).[13] The quipu system was extremely complicated, and only special quipu makers, called quipucamayocs, were able to interpret them. Although we do not know a lot about quipus, we do know they used place value. The location of the knot, along with some other factors, determined its value.

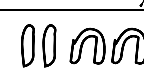
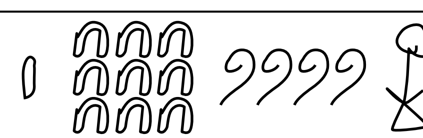
Apparently, the Incas were very successful with this innovative approach. Not only did they operate a huge empire, but the Incas baffled the Spanish conquerors by their ability to record the tiniest details as well as the largest ones on their quipus.[14]

Fixed-Value Systems

A different approach to recording quantities is to *repeat* symbols to represent other numbers. For example, here are some symbols in Egyptian numerals (hieroglyphic style).[15]



The next figure shows two different quantities represented using Egyptian numerals and our decimal place-value system. Notice how when writing twenty-two, the Egyptians repeated their symbol for one and their symbol for ten twice. They put the smaller values on the left and the larger values on the right. Thus the symbol for ten () is to the right of the symbol for one ().

Decimal System a place-value system	**Egyptian System** a fixed-value system
22	
1,491	

We'll refer to number systems that use repeated symbols like this as **fixed-value systems**.

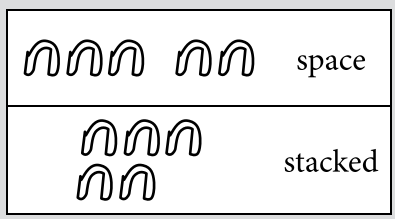
A Deeper Look at the Egyptian System

Notice that in the Egyptian version of 1,491, the symbols representing "ninety" are stacked on top of each other.

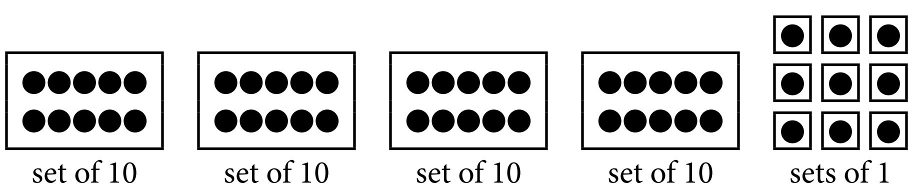
This: Rather than this:

While there were many variations within the Egyptian system over time, in general, when writing more than four of each symbol, the Egyptians **spaced, stacked, or grouped the symbols in sets (groups) of four or less**, with the larger set on top or first.[16] This practice made it easier to count the symbols (and thus to read the number!) at a glance.

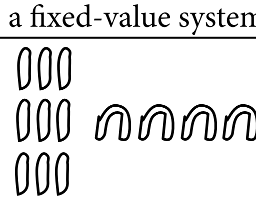
50



Let's compare our decimal place-value system with the Egyptian system. To record forty-nine objects in the Egyptian system, we would repeat the symbol for "one" nine times to show we had nine ones, and then repeat our symbol for "ten" four times to show we had four sets of ten. In the decimal system, we would instead use our symbols for four and nine, putting the 4 in the tens column so it represents four sets of ten and 9 in the ones column, representing nine sets of one.

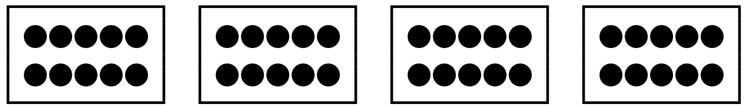


"Forty-nine" = four sets of ten and nine ones

Decimal System a place-value system	**Egyptian System** a fixed-value system
49	

When we compare forty-nine in both systems, we see it takes significantly fewer symbols to represent the number in the decimal system. Place value saves a lot of extra writing!

To represent a number like forty in the decimal system, we would again use a 4, adding a zero (0) to represent that we have no (0) sets of one. Notice the importance of a zero (0) in a place-value system; without it, we would have no way of showing that the 4 represents 4 sets of ten instead of 4 sets of one.



set of 10 set of 10 set of 10 set of 10

"Forty" = four sets of ten and no ones

Decimal System a place-value system	**Egyptian System** a fixed-value system
40	𓎆𓎆𓎆𓎆

Ordered Fixed-Value Systems

Another approach to recording quantities is to again use a limited number of symbols and repeat those symbols, but to add rules regarding their order that change the symbols' meaning. Roman numerals are an example of an ordered fixed-value system.

Take a look at these symbols used for quantities in Roman numerals:

I	1
V	5
X	10
L	50
C	100
D	500
M	1,000

As with the Egyptians, quantities in Roman numerals are represented by repeating symbols, although this time with the larger quantities on the left.

22 is written XXII in Roman numerals.

But unlike in the Egyptian system, the same symbol is generally not repeated more than three times. Instead, it is assumed that whenever a symbol representing a smaller quantity is to the *left* of a symbol representing a larger quantity, one should *subtract* the value of the smaller quantity from the value of larger quantity to get the value the two symbols represent.

Notice that the smaller quantity is to the *left* of the larger—this means to subtract I from V, giving us 5 – 1, or 4.

Notice that the smaller quantity is to the *right* of the larger—this means to add I to V, giving us 5 + 1, or 6.

I	1	XI	11
II	2	XII	12
III	3	XIII	13
IV	4	XIV	14
V	5	XV	15
VI	6	XVI	16
VII	7	XVII	17
VIII	8	XVIII	18
IX	9	XIX	19
X	10	XX	20

There was a time when "four" was written IIII instead of IV. But IV is easier to read, as there are fewer symbols involved.

Now let's take a look at the same number we looked at with the Egyptians: 1,491.

Decimal System a place-value system	**Roman Numeral System** an ordered fixed-value system
1,491	**MCDXCI** M = 1,000 CD = 500 – 100 = 400 XC = 100 – 10 = 90 I = 1 1,000 + 400 + 90 + 1 = 1,491

Notice that Roman numerals would not lend themselves well to quickly adding or subtracting on paper! There is a reason we use the decimal place-value system for most purposes.

Keeping Perspective

While you may use only our current decimal place-value system on a regular basis, being aware of other systems will help you learn to better see our place-value system as just one system to help us describe quantities.

1.6 Binary and Hexadecimal Place-value Systems

Before we move on, we're going to take one more look at the concept of place value, as it's a pretty important concept. While I'm sure you're quite familiar with our current place-value system, did you realize computers use place-value systems based on a value besides ten?

Well, they do! They use what's known as a binary place-value system. Exploring this system, along with the hexadecimal place-value system, is not only cool, but it can also help provide an even firmer grasp of the decimal place-value system. Let's take a look.

Unwrapping Place-Value Systems

The value we choose for each place in the system is called our **base**. You can picture a base like a container — the size of the container determines how much it can hold. In the decimal system, each place, or container, can hold *ten* digits (0, 1, 2, 3, 4, 5, 6, 7, 8, 9); once we reach *ten* of a unit, we move to the next place over, using the same digits, but knowing that each one represents ten of the previous place's value.

We write forty-two as "42" to represent 4 sets of ten (or 40) plus 2.

4	2
0, 1, 2, 3, 4, 5, 6, 7, 8, 9	0, 1, 2, 3, 4, 5, 6, 7, 8, 9
Tens (The digit in this place/container tells us how many sets of ten are part of the number.)	**Ones**

Binary System

Computers actually operate off a base-two place-value system called the **binary system** (*bi* means *two*). In a binary system, instead of allowing *ten* values (0, 1, 2, 3, 4, 5, 6, 7, 8, 9) in each place, we only allow *two* (0, 1). It's as if each place, or container, can only hold the *two* digits: 0 and 1. Once we reach two, we move to the next place over. While in the decimal system, each place is worth ten times the previous place, each place in the binary system is worth two times the previous place.

In binary, the number "10" represents 1 set of *two* and 0 sets of *one*, or two!

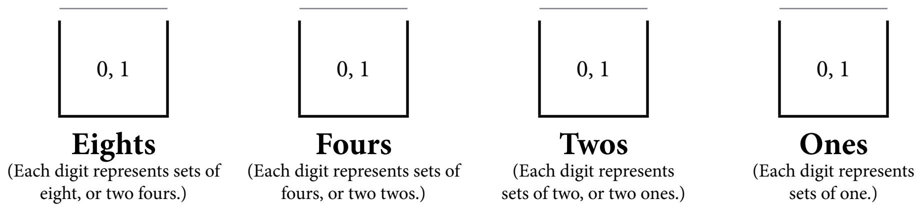
1	0
0, 1	0, 1
Twos (The digit in this place/container tells us how many sets of two are part of the number.)	**Ones**

To make things clearer, take a look at the first four places, or containers, for both systems.

Decimal (base 10)

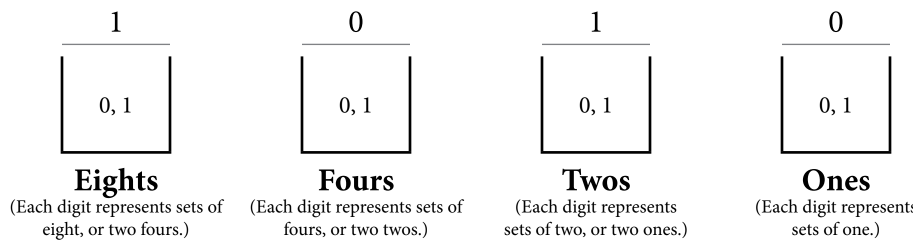
0, 1, 2, 3, 4, 5, 6, 7, 8, 9	0, 1, 2, 3, 4, 5, 6, 7, 8, 9	0, 1, 2, 3, 4, 5, 6, 7, 8, 9	0, 1, 2, 3, 4, 5, 6, 7, 8, 9
Thousands (Each digit represents sets of a thousand, or ten hundreds.)	**Hundreds** (Each digit represents sets of a hundred, or ten tens.)	**Tens** (Each digit represents sets of a ten, or ten ones.)	**Ones** (Each digit represents sets of one.)

Binary (base 2)



Let's take a look at how this plays out with a few numbers.

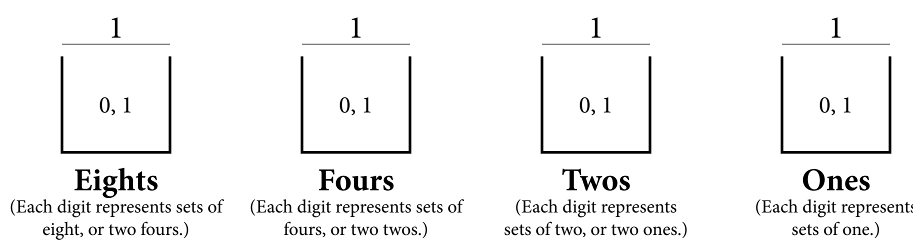
Example: Find the decimal value of the binary number 1010.



1 set of 8 = 1 x 8 = *8*
0 sets of 4 = 0 x 4 = *0*
1 set of 2 = 1 x 2 = *2*
0 sets of 1 = 0 x 1 = *0*
8 + *0* + *2* + *0* = 10

1010 in binary is the same as the decimal number 10.

Example: Find the decimal value of the binary number 1111.

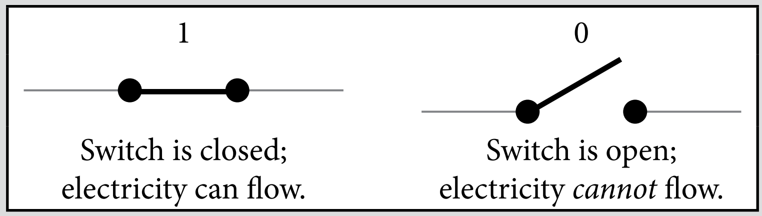


1 set of 8 = 1 x 8 = *8*
1 set of 4 = 1 x 4 = *4*
1 set of 2 = 1 x 2 = *2*
1 set of 1 = 1 x 1 = *1*
8 + *4* + *2* + *1* = 15

1111 in binary is the same as the decimal number 15.

Computer Circuits

Because computer circuits run on electricity, the 0 and 1 used in binary numbers can easily describe the "off" and "on" flows of electricity controlled by an open or closed switch. Whenever there's electricity, the computer interprets it as a 1. When there's no electricity, it interprets it as a 0.



Making Computer Talk More Concise: Hexadecimal Numbers

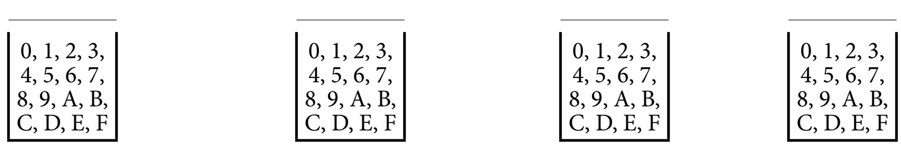
Although binary numbers translate well to electrical pulses, they tend to get long quickly (eight is written 1000), making them difficult for us to read. To help make numbers more readable, computer programs often use hexadecimal numbers (a place-value system based on 16) to represent binary numbers. Because it has a larger base (i.e., a container that can hold more digits), the hexadecimal system can represent very large numbers with fewer digits.

Decimal: 1,200
Binary: 10010110000
Hexadecimal: 4B0

Base 16-Hexadecimal System

16 Symbols: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F

A represents the decimal value of 10.
B represents the decimal value of 11.
C represents the decimal value of 12.
D represents the decimal value of 13.
E represents the decimal value of 14.
F represents the decimal value of 15.



Four thousand ninety-sixes (Each digit represents sets of four thousand ninety-six, or sixteen two hundred fifty-sixes.)

Two hundred fifty-sixes (Each digit represents sets of two hundred fifty-six, or sixteen sixteens.)

Sixteens (Each digit represents sets of sixteen, or sixteen ones.)

Ones (Each digit represents sets of one.)

Example: Find the decimal value of the hexadecimal number 4B0.

4	**B**	**0**
0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F
Two hundred fifty-sixes (Each digit represents sets of two hundred fifty-six, or sixteen sixteens.)	**Sixteens** (Each digit represents sets of sixteen, or sixteen ones.)	**Ones** (Each digit represents sets of one.)

4 sets of 256 = 4 x 256 = *1,024*
11 sets of 16 = 11 x 16 = *176*
0 sets of 1 = 0 x 1 = *0*
1,024 + *176* = 1,200

4B0 in hexadecimal is the same as the decimal number 1,200.

Keeping Perspective

Place-value systems can be based off *any* quantity — and other systems besides the decimal one are in common use today! Each system is a tool to help us describe quantities . . . and each works best in different situations.

While it's not necessary for you to learn the binary or hexadecimal systems (unless you plan to go into computer programming), take some time to explore them a little. Thinking outside the box this way will help you develop your mathematical skills and grow in your ability to use math as a tool.