Lesson 17

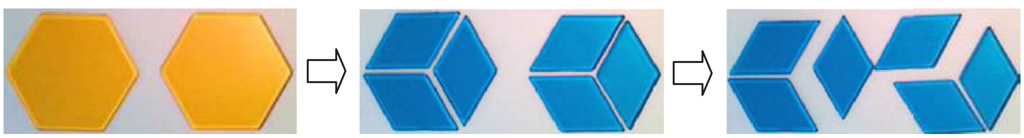
Understanding the Relationship Between $a \div b$ and $\frac{a}{b}$: Part I

Let's consider this problem:

Your Mom would like to share two candy bars of the same size equally among three kids. How much of the candy bar should she give each kid?

One way to solve this problem is to use our fraction blocks, and to represent a whole candy bar by the yellow block. Hence, two candy bars are represented by 2 yellow blocks. Our task now is to break up the 2 yellow blocks into three equal parts.

In order to accomplish this, we will first replace each yellow block by three blue blocks, as shown in the middle image below. Next, we can partition those 6 blue blocks into 3 equal parts of two blue blocks each. Since each blue block represents one-third of a candy bar, we see that each kid should get $\frac{2}{3}$ of a candy bar.



We can use the mathematical notation "$2 \div 3$" to indicate that we need to partition the 2 candy bars into 3 equal parts. Since the size of each of those parts is $\frac{2}{3}$, we have this relationship: $2 \div 3 = \frac{2}{3}$. We read this equation as "Two divided by 3 is two-thirds."

Please use your fraction blocks to answer these sharing problems:

a. **One cake shared equally among 2 kids. Each one gets _____of the cake.**

b. **Two cakes shared equally among 6 kids. Each one gets _____ of a cake.**

c. **Two cakes shared equally among 4 kids. Each one gets _____ of a cake.**

d. $1 \div 2 =$ **e.** $2 \div 6 = \frac{2}{6} =$ **f.** $2 \div 4 =$

What we have learned:

A sharing problem can be expressed using division. If we share 2 cakes among 3 kids, each one gets $\frac{2}{3}$ of a cake. We can express this as $2 \div 3 = \frac{2}{3}$.

Lesson 30

A Visual Model for Fraction Multiplication

In this lesson we show how the symbolic partition of the multiplicand into equal parts can serve as a visual model for fraction multiplication.

Let's consider the product $\frac{3}{4}\times\frac{8}{5}$. We are able to partition $\frac{8}{5}$ into 4 equal parts, $\frac{8}{5}=\frac{2}{5}+\frac{2}{5}+\frac{2}{5}+\frac{2}{5}$. Since one of those 4 equal parts is $\frac{2}{5}$, we have $\frac{1}{4}\times\frac{8}{5}=\frac{2}{5}$.

$$\frac{8}{5}=\overline{\frac{2}{5}}+\frac{2}{5}+\frac{2}{5}+\frac{2}{5}$$

Since 3 of those parts, $\frac{2}{5}+\frac{2}{5}+\frac{2}{5}$, are $\frac{6}{5}$, we have $\frac{3}{4}\times\frac{8}{5}=\frac{6}{5}$.

$$\frac{8}{5}=\overline{\frac{2}{5}+\frac{2}{5}+\frac{2}{5}}+\frac{2}{5}$$

A. Find each product by using the given partition. The first one is done for you.

a. $\frac{1}{3}\times\frac{6}{7}$ $\qquad$ $\frac{6}{7}=\overline{\frac{2}{7}}+\frac{2}{7}+\frac{2}{7}$ $\qquad$ **Answer:** $\frac{1}{3}\times\frac{6}{7}=\frac{2}{7}$

b. $\frac{2}{3}\times\frac{6}{7}$ $\qquad$ $\frac{6}{7}=\frac{2}{7}+\frac{2}{7}+\frac{2}{7}$ $\qquad$ **Answer:** $\frac{2}{3}\times\frac{6}{7}=$

c. $\frac{3}{3}\times\frac{6}{7}$ $\qquad$ $\frac{6}{7}=\frac{2}{7}+\frac{2}{7}+\frac{2}{7}$ $\qquad$ **Answer:** $\frac{3}{3}\times\frac{6}{7}=$

B. Find each product by using the given partition.

a. $\frac{1}{3}\times\frac{3}{5}$ $\qquad$ $\frac{3}{5}=\frac{1}{5}+\frac{1}{5}+\frac{1}{5}$ $\qquad$ **Answer:** $\frac{1}{3}\times\frac{3}{5}=$

b. $\frac{2}{3}\times\frac{3}{5}$ $\qquad$ $\frac{3}{5}=\frac{1}{5}+\frac{1}{5}+\frac{1}{5}$ $\qquad$ **Answer:** $\frac{2}{3}\times\frac{3}{5}=$

C. Review Lesson #29. Use your fraction blocks to find these products.

a. $\frac{3}{4}\times\frac{8}{3}=$ $\qquad$ **b.** $\frac{2}{3}\times\frac{3}{2}=$ $\qquad$ **c.** $\frac{3}{4}\times\frac{8}{6}=$

What we have learned:

A partition of a fraction into a sum of equal fractions, such as $\frac{8}{5}=\frac{2}{5}+\frac{2}{5}+\frac{2}{5}+\frac{2}{5}$, enables us to visually see that $\frac{3}{4}\times\frac{8}{5}=\frac{6}{5}$, since 3 of the 4 equal parts sum to $\frac{6}{5}$.

Lesson 29

Concretely Multiplying a Fraction by a Fraction

Let's consider the fraction multiplication problem $\frac{5}{4} \times \frac{8}{6}$. We need to take 5 copies of one-fourth of $\frac{8}{6}$. We can represent the problem concretely. If we let the yellow block be the whole, each green block represents a sixth and 8 green blocks represents $\frac{8}{6}$.

$\frac{8}{6}$ 

To find the value of one-fourth of $\frac{8}{6}$, we will partition the blocks into 4 equal parts, and select one of those parts, as shown below in red. This part consists of 2 green blocks:

$$\frac{1}{4} \times \frac{8}{6} = \frac{2}{6}$$

To get 5-fourths of $\frac{8}{6}$, we need to select 5 copies of 1-fourth of $\frac{8}{6}$. Below we see that we have a total of 10 green blocks, or ten sixths. Hence, $\frac{5}{4} \times \frac{8}{6} = \frac{10}{6}$.

$$\frac{5}{4} \times \frac{8}{6} = \frac{10}{6}$$

A. Review Lesson #23. Find the products below. Example: $\frac{1}{4} \times 20 = 5$.

a. $\frac{1}{2} \times 4 =$ b. $\frac{1}{3} \times 3 =$ c. $\frac{1}{5} \times 10 =$

B. Review Lesson #24. Find the products below. Example: $\frac{3}{4} \times 20 = 3 \times 5 = 15$.

a. $\frac{3}{2} \times 4 =$ b. $\frac{5}{3} \times 3 =$ c. $\frac{4}{5} \times 10 =$

C. Use your fraction blocks to find these products.

a. $\frac{3}{2} \times \frac{4}{3} =$ b. $\frac{5}{3} \times \frac{3}{2} =$ c. $\frac{4}{5} \times \frac{10}{6} =$

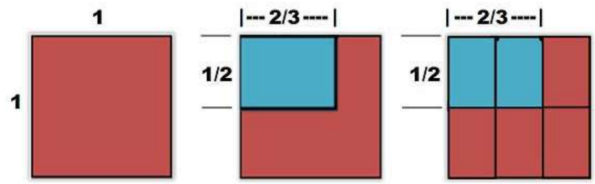
What we have learned:

To find $\frac{3}{4} \times \frac{8}{6}$, we first find one-fourth of $\frac{8}{6}$ and then triple the result.

Lesson 37

Finding the Area of a Rectangle with Fractional Side Lengths: Part I

Let's find the area of a rectangle whose sides are $\frac{1}{2}$ and $\frac{2}{3}$.

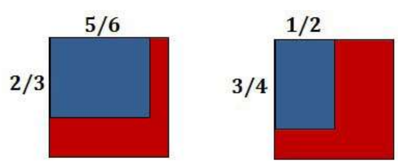


At the left above we have a square with dimensions 1 by 1. Its area is one square unit. In the middle, in blue, we section off a rectangle with dimensions $\frac{2}{3}$ and $\frac{1}{2}$. We need to find the area of this blue portion.

At the right, we draw vertical lines and extend the horizontal line as shown. We see that we have a total of 6 equal small rectangles, which together make up the area of the square. Since the area of the square is 1, each small rectangle has the area $\frac{1}{6}$ square units.

Now, the blue portion consists of two of those rectangles. Hence, its area is $\frac{2}{6}$ square units. We notice that the area of the blue portion, $\frac{2}{6}$, is also the product of the two sides of the blue rectangle, $\frac{1}{2} \times \frac{2}{3} = \frac{2}{6}$. In general, the area A of a rectangle with fractional sides $\frac{a}{b}$ and $\frac{c}{d}$ will be the product of its sides, e.g., $A = \frac{a}{b} \times \frac{c}{d}$.

The area of each square below is 1 square foot.



a. What is the area of the rectangle with dimensions $\frac{2}{3}$ by $\frac{5}{6}$?

b. What is the area of the rectangle with dimensions $\frac{3}{4}$ by $\frac{1}{2}$?

What we have learned:
The area, A, of a rectangle with fractional sides $\frac{a}{b}$ and $\frac{c}{d}$ will be the product of its sides. Hence, $A = \frac{a \times c}{b \times d}$.