Grades 7–10

ALGEBRA LAB GEAR

Algebra 1

- Polynomial Arithmetic
- Equations and Identities
- Quadratics
- Factoring
- Graphing Connections

By Henri Picciotto

Didax
395 Main Street
Rowley, MA 01969
www.didax.com

Contents

Correlation to the Common Core Standards

CCSS Standard	Description	Activities
5.OA.3	Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane.	2.3A, 2.3B
5.MD.5a	Relate volume to the operations of multiplication and addition and solve real-world and mathematical problems involving volume. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as values, e.g., to represent the associative property of multiplication.	3.1C, 3.1E
6.NS.6a	Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., $-(-3) = 3$, and that 0 is its own opposite.	4.1A, 4.1B
6.NS.7a	Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram.	4.2A, 4.2B
6.EE.2c	Write, read, and evaluate expressions in which letters stand for numbers. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations).	5.2A
6.EE.3	Apply the properties of operations to generate equivalent expressions.	(Lesson) 3.6, 4.2C
6.EE.6	Use variables to represent numbers and write expressions when solving real-world or mathematical problems; understand that a variable can represent an unknown number, or depending on the purpose at hand, any number in a specified set.	1.1B, 1.1C
6.EE.9	Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.	5.4A
6.G.4	Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.	2.2A
7.G.6	Solve real-world and mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.	1.1A, 2.2B, 2.2C
8.EE.1	Know and apply the properties of integer exponents to generate equivalent numerical expressions.	3.5B, 3.5D
8.EE.7	Solve linear equations in one variable.	4.3A
8.EE.7a	Solve linear equations in one variable. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).	6.4B

CCSS Standard	Description	Activities
8.EE.7b	Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.	1.3A, 1.3B, 3.2A, 3.2B
8.EE.8a	Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.	(Lesson) 5.3
8.EE.8b	Solve systems of two linear equations in two variables algeraically, and estimate solutions by graphing the equations. Solve simple cases by inspection.	4.4B
8.SP.3	Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept.	5.4B
A.SSE.2	Use the structure of an expression to identify ways to rewrite it.	6.3A, 6.3B
A-APR.1	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.	3.1D, 3.4A, 3.4B, 6.2A–E
F-IF.8a	Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.	6.5B, 6.5C, 6.6A
Not correlated		1.1D, 1.2A, 1.2B, 2.1A, 2.1B, 3.1A, 3.1B, 3.2C, 3.3A–D, 3.5A, 3.5C

Multiplying, Factoring, and Dividing Polynomials

The Main Concepts

This section covers multiplication of polynomials and several related topics. This is the most important use of Lab Gear. We start with “Make a Rectangle” and “Make a Box,” geometric puzzles for which all students can readily understand the instructions. These serve as an introduction to the geometric representation of multiplication in the corner piece, using the formulas for area of a rectangle and volume of a box.

Once this foundation has been laid, we can proceed to the introduction of the distributive property and to several topics related to multiplication: factoring, division, squaring, cubing, and simplifying fractions.

In a way, factoring is the key to this section, since the initial “make a rectangle” problems are in fact informal factoring problems. In addition to the pedagogic reason given above for starting that way, there is another more fundamental reason: a full understanding of multiplication and the distributive property is not possible without some conceptual grasp of factoring. Similarly, one cannot fully understand addition without understanding reversed problems of the type “find two numbers that add up to 10” or “what plus 1.41 equals 10?” This is the principle of reversibility.

On the other hand, do not get too concerned about having your students become fast and accurate trinomial factoring experts. While factoring is important on a conceptual level, it is becoming obsolete as a skill. Like other symbolic manipulations, it will be performed primarily by electronic means in the near future.

When working with the Lab Gear in this section, we do not use minus. Beginning in Lesson 3.2, we also work in the multiplication table format, what students often call “the box,” using both + and –. This is because the corner piece model of multiplication for a multiplication like $(y - 1)(y - 2)$, while mathematically correct, is somewhat tricky. On the other hand, the multiplication table format readily generalizes to minus. We tackle the modeling of minus in the corner piece in Section 6.

Pacing and Selection

There is a lot in this section, and you should not expect to do it all in a few days. Once you have decided how much of it you want to cover, plan to alternate between one week of concentrated effort on one lesson and a few activities from this book and a week of other work. Doing the work in this section will take time, but because your students will have a more solid understanding, you will need less drill than you may have assigned in the past on topics such as the distributive property and factoring. In any case, be sure to do Lessons 3.1 and 3.2 and their activities early in the year.

The following lessons and activities are essential:

- **Lesson 3.1,** Activities 3.1A and 3.1D
- **Lesson 3.2,** Activities 3.2A, and 3.2B
- **Lesson 3.3,** Activities 3.3A, 3.3B, and 3.3C
- **Lesson 3.4,** Activity 3.4C
- **Lesson 3.5,** Activities 3.5B and 3.5C

The Transition to Symbolic Algebra

The general progression within each topic area in this section is:

1. Build with the Lab Gear.
2. Sketch.
3. Use the multiplication table format.

Students are often asked to sketch corner piece figures for the various types of problems in the activities. By encouraging them to use pictorial representations as a substitute for work with the blocks, you help them make the transition from the concrete environment of the Lab Gear to the multiplication table format and the world of symbols.

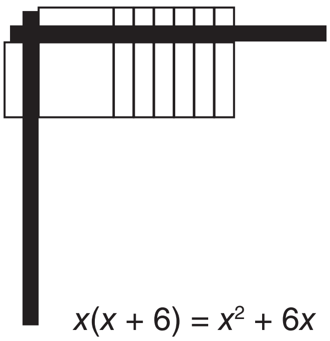
About the Lessons

- **Lesson 3.1, The Geometry of Multiplication.** This lesson introduces the geometric model for multiplication, laying the groundwork for the whole section.
- **Lesson 3.2, The Distributive Property.** This lesson helps students generalize what they learn from Lab Gear multiplication, so that they can correctly multiply using symbols. Mathematically, the distributive property is the central structure underlying the entire section.
- **Lesson 3.3, Factoring.** This lesson focuses on the concept of factoring, with a particular emphasis on the inverse relationship with distributing.
- **Lesson 3.4, Dividing by a Monomial.** This lesson is about the distributive property of division over addition and subtraction.
- **Lesson 3.5, Perfect Square Trinomials.** This lesson introduces the identity for the square of a sum, which becomes important for completing the square in the context of quadratic equations and functions.
- **Lesson 3.6, Multiplying, Factoring, Dividing.** This lesson summarizes the whole section.

The Geometry of Multiplication

The purpose of this lesson is to model multiplication and factoring with the Lab Gear and to introduce the corresponding symbolic notation.

Parentheses



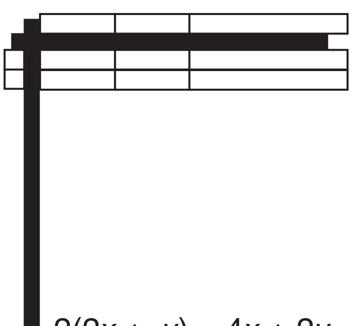
$x(x + 6) = x^2 + 6x$

- Ask the students to make a rectangle with x^2 and $6x$, and then to measure it in the corner piece.
- Ask the students to write an equation in the form *length* · *width* = *area* to describe the figure.
- Tell the students that beginners in algebra will often write this multiplication $x \cdot x + 6$ or $x + 6 \cdot x$. However, mathematicians agree that multiplication comes before addition, so that the first expression means "multiply x by x, then add 6" and the second expression means "multiply 6 by x, and add that to x." Build these two expressions with the Lab Gear on the screen.

$x \cdot x + 6$ $\qquad$ $x + 6x$

- Explain the correct use of parentheses to write the original multiplication: $x(x + 6)$ or $(x + 6)x$.

Make a Rectangle



$2(2x + y) = 4x + 2y$

- Ask students to make a rectangle with $4x + 2y$, and to write a *length* · *width* = *area* equation for the rectangle.
- Show that 2 is a common dimension of the subrectangles made by $4x$ and $2y$. Say that 2 is called a common factor of each of the original two terms $4x$ and $2y$. A factor of a monomial (or term) is a monomial that divides it evenly. $2 \cdot 2x = 4x$, so 2 is a factor of $4x$, and $2 \cdot y = 2y$, so 2 is a factor of $2y$.

> If rectangles have a common dimension, their areas have a common factor.

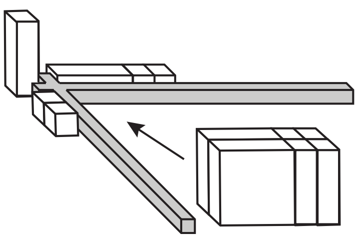
- Ask students to make rectangles with each of the following collections of blocks. In each case, they should write the *length · width = area* equation, and identify the common factor.

  $3x + 15$ $4y + 8$ $5x + 20$ $3x + x^2 + xy$ $5x + 2x^2$

  (Answers: $3(x + 5)$, $4(y + 2)$, $5(x + 4)$, $x(3 + x + y)$, $x(5 + 2x)$)

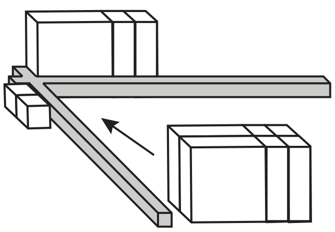
- Discuss the two alternate solutions for $2x^2 + 4x$ (common factor x, or common factor $2x$)

Make a Box

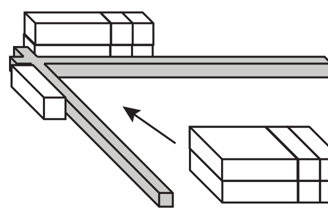


$2 \cdot x \cdot (x + 2)$

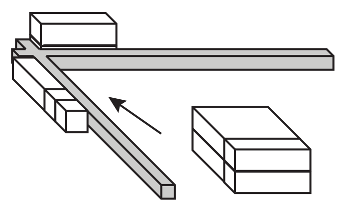
- Now show how the common factor 2 in $2x^2 + 4x$ leads to a box. If the three dimensions are measured separately, the box can be described by an equation in the form *length · width · height = volume*, $2 \cdot x \cdot (x + 2) = 2x^2 + 4x$.
- Describe how the collection of blocks can be arranged into a box and measured in three more ways: $2(x^2 + 2x)$, $x(2x + 4)$, and $2x(x + 2)$. These three arrangements show the common factors of 2, x, and $2x$ respectively. Geometrically, they represent the multiplication *area of base times height equals volume*.



$2(x^2 + 2x)$



$x(2x + 4)$



$2x(x + 2)$

> It is difficult to demonstrate the above example on the screen, since it involves three-dimensional arrangements. You might have the students gather around a central desk or table for a demonstration with the student blocks. Or you can show the process to representatives of groups, and have them show their neighbors.

- Have students make boxes (or rectangles) for each of the following collections of blocks. In each case they should write one or more multiplications and identify the common factors. Encourage them to compare answers with one another, or to look for alternate solutions because each collection has many different solutions. (Do not worry at this stage about which version is "the most factored." This is a level of sophistication most students are probably not ready for.)

  $12x + 12$ $3x^2 + 6x$ $2xy + 10x$

- These require 3-D blocks: $x^3 + 2x^2 + x^2y$ $2x^2y + 4xy$

> Note that when multiplying in the corner piece, *the dimension of the product is the sum of the dimensions of the factors*. It is not crucial for students to understand this fact, but many find it interesting. You may lead a discussion of it by showing that the x^3 block, for example, can be shown as the product of three x's, each of which is one-dimensional, or as the product of a one-dimensional x and a two-dimensional x^2. Some students may build it as $x^2 \cdot x^2$, but that is one dimension too many.

- Wrap up this lesson by asking students if they have found ways to look carefully at the terms of an expression and find the common factor and the multiplication without the Lab Gear. Even though not all the students may see this yet, it is important to begin this discussion because it lays the groundwork for the introduction of the distributive property in the next lesson.

About the Activities

- **Activity 3.1A, Make a Rectangle.** These geometric puzzles provide practice in writing multiplication equations of the form *length* · *width* = *area*. A wrap-up of the completed activity may include a discussion of the common factor in each exercise.
- **Activity 3.1B, Make a Box.** These geometric puzzles provide practice in writing multiplication equations of the form l*ength* · *width* · *height* = *volume*. Encourage students to build boxes rather than rectangles in this activity.
- **Activity 3.1C, Make a Box Using the 3-D Blocks.** If you have the 3-D blocks in your classroom, use this activity to reinforce the connection between geometric volume and algebraic multiplication equations.
- **Activity 3.1D, Multiplying in the Corner Piece.** In this activity, students build multiplication problems with the Lab Gear blocks and the corner piece, and then record the product. This work prepares students for the discussion in Lesson 3.2 about the distributive property and symbolic multiplication.
- **Activity 3.1E, Multiplication Using the 3-D Blocks.** This activity is similar to the preceding one, but requires the 3-D blocks.

Make a Rectangle

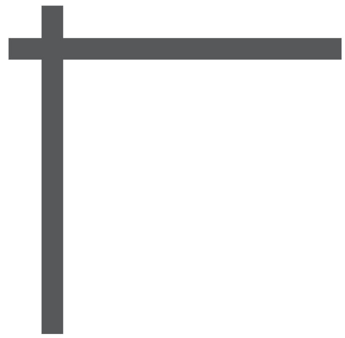
For each of the polynomials shown below:

a. Build the polynomial and arrange the blocks into a rectangle.
b. Sketch the rectangle in the corner piece.
c. Write an equation of the form *length* · *width* = *area*.

1. $x^2 + 3x$

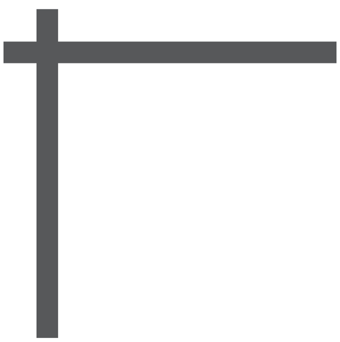
Equation: ______________________________

2. $5x + 5y$



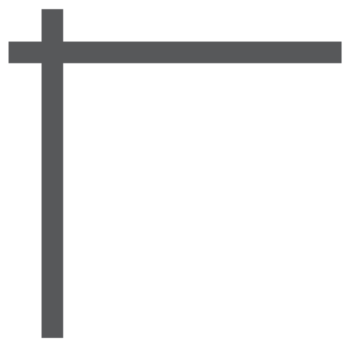
Equation: ______________________________

3. $3y + 6x$



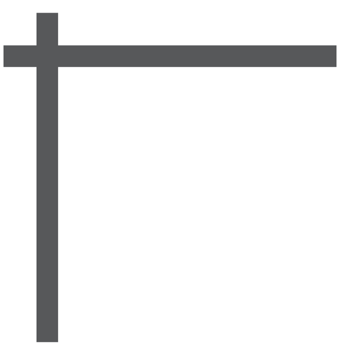
Equation: ______________________________

4. $4x + 6 + 2y$



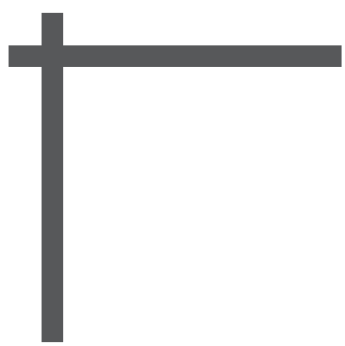
Equation: ______________________________

5. $xy + x^2 + 2x$



Equation: ______________________________

6. $4y + 8x + 20$



Equation: ______________________________

Make a Box

For each of the polynomials shown below:

a. Build the polynomial and arrange the blocks into a rectangle.
b. Make a 3-D sketch of the box.
c. Complete the equation volume = *length · width · height*.

1. $4y + 4 =$ ______________________

2. $2xy + 2y =$ ______________________

3. $6y + 18 =$ ______________________

4. $2xy + 2x + 2x^2 =$ ______________________

5. $4x^2 + 8x =$ ______________________ = ______________________

   Find two different ways.

Make a Box Using 3-D Blocks

For each of the polynomials shown below:

a. Build the polynomial and arrange the blocks into a box.
b. Make a 3-D sketch of the box.
c. Complete the equation *volume* = *length* · *width* · *height*.

1. $x^2y + 4xy =$ ________________

2. $x^3 + 5x^2 =$ ________________

3. $xy^2 + x^2y + xy + x^2 =$ ________________

4. $y^3 + x^2y + 2xy^2 =$ ________________

5. $2x^2y + 2xy^2 =$ ________________ = ________________ = ________________
   Find three different ways.

