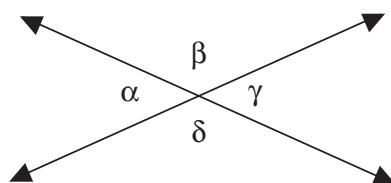


LESSON 6

Supplementary and Complementary Angles

Greek Letters

Figure 1

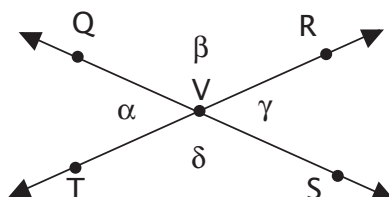


α = alpha
 β = beta
 γ = gamma
 δ = delta

Adjacent Angles - Angles that share a common side and have the same origin are called *adjacent angles*. They are side by side. In figure 1, α is adjacent to both β and δ . It is not adjacent to γ . In figure 1, there are four pairs of adjacent angles: α and β , β and γ , γ and δ , δ and α .

In figure 2, we added points so we can name the rays that form the angles. The common side shared by adjacent angles α and β is \overrightarrow{VQ} .

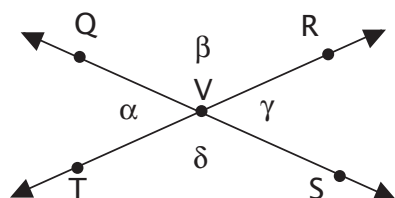
Figure 2



Given: $\overleftrightarrow{RT} \cap \overleftrightarrow{QS} = V$

Vertical Angles - Notice that $\angle \gamma$ is opposite $\angle \alpha$. Angles that share a common origin and are opposite each other are called *vertical angles*. They have the same measure and are congruent. $\angle \beta$ and $\angle \delta$ are also vertical angles.

Figure 2 (from previous page)

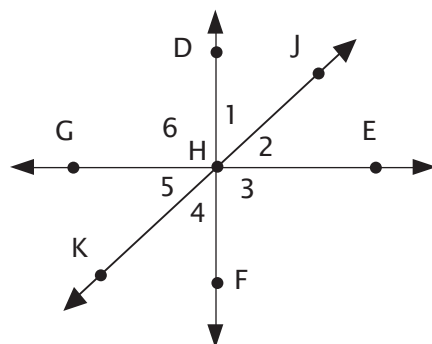


Given: $\overleftrightarrow{RT} \cap \overleftrightarrow{QS} = V$

If $m\angle\beta$ is 115° , then $m\angle\delta$ is also 115° . If this is true, then do we have enough information to find $m\angle\alpha$? We know from the information given in figure 2 that \overleftrightarrow{RT} and \overleftrightarrow{QS} are lines. Therefore, $\angle RVT$ is a straight angle and has a measure of 180° . If $\angle RVQ$ ($\angle\beta$) is 115° , then $\angle QVT$ ($\angle\alpha$) must be $180^\circ - 115^\circ$, or 65° . Since $\angle RVS$ ($\angle\gamma$) is a vertical angle to $\angle QVT$, then it is also 65° .

Supplementary Angles - Two angles such as $\angle\alpha$ and $\angle\beta$ in figure 2, whose measures add up to 180° , or that make a straight angle (straight line), are said to be supplementary. In figure 2, the angles were adjacent to each other, but they don't have to be adjacent to be classified as supplementary angles.

Figure 3

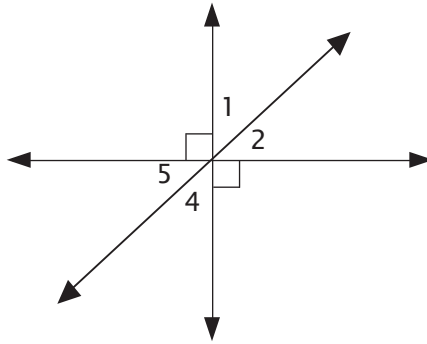


All drawings are in the same plane unless otherwise noted.

Given: \overleftrightarrow{DF} , \overleftrightarrow{GE} , and \overleftrightarrow{KJ} all intersect at H.
 $\overleftrightarrow{DF} \perp \overleftrightarrow{GE}$

Complementary Angles - We can observe many relationships in figure 3. $\angle 1$ is adjacent to both $\angle 6$ and $\angle 2$. Angle 3 and $\angle 6$ are vertical angles, as are $\angle 1$ and $\angle 4$. Angle 6 and $\angle 3$ are also right angles since $\overleftrightarrow{DF} \perp \overleftrightarrow{GE}$. The new concept here is the relationship between $\angle DHE$ and $\angle GHE$. Both of these are right angles because the lines are perpendicular; therefore their measures are each 90° . Then $m\angle 1 + m\angle 2 = 90^\circ$, and $m\angle 4 + m\angle 5 = 90^\circ$. Two angles whose measures add up to 90° are called complementary angles. Notice that from what we know about vertical angles, $\angle 1$ and $\angle 5$ are also complementary. Let's use some real measures to verify our conclusions.

Figure 4 (a simplified figure 3)



In figure 4, let's assume that $m\angle 1 = 47^\circ$. Then $m\angle 2$ must be 43° , since $m\angle 1$ and $m\angle 2$ add up to 90° . If $m\angle 1 = 47^\circ$, then $m\angle 4$ must also be 47° , since $\angle 1$ and $\angle 4$ are vertical angles. Also, $m\angle 5$ must be 43° . So $\angle 1$ and $\angle 5$ are complementary, as are $\angle 2$ and $\angle 4$. Remember that supplementary and complementary angles do not have to be adjacent to qualify.

It helps me to not get supplementary and complementary angles mixed up if I think of the *s* in straight and the *s* in supplementary. The *c* in complementary may be like the *c* in corner.

LESSON PRACTICE

Use the drawing to fill in the blanks.

1. $\angle AHC$ is adjacent to \angle _____ and \angle _____.

2. $\angle BHD$ is adjacent to \angle _____ and \angle _____.

3. $\angle FHB$ and \angle _____ are vertical angles.

4. $\angle FHC$ and \angle _____ are vertical angles.

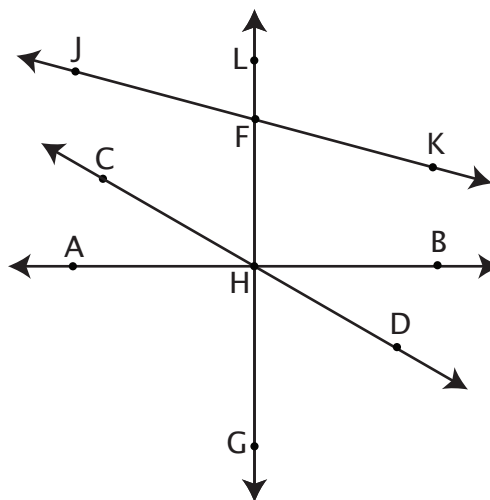
5. $\angle LFJ$ and \angle _____ are supplementary angles.

6. $\angle FHC$ and \angle _____ are complementary angles.

7. $\angle JFH$ and \angle _____ are supplementary angles.

8. $\angle BHD$ and \angle _____ are complementary angles.

9. If $m\angle CHA = 40^\circ$, then $m\angle BHD =$ _____.



Given: \overleftrightarrow{AB} , \overleftrightarrow{CD} , \overleftrightarrow{LG} , and \overleftrightarrow{JK} are straight lines. $m\angle FHB = 90^\circ$.

The drawing is a sketch and not necessarily to scale. Don't make any assumptions about the lines and angles other than what is actually given.

Use the drawing from the previous page to fill in the blanks.

10. If $m\angle JFL = 65^\circ$, then $m\angle KFH = \underline{\hspace{2cm}}$.

11. If $m\angle FHB = 90^\circ$, then $m\angle FHA = \underline{\hspace{2cm}}$.

12. If $m\angle CHA = 40^\circ$, then $m\angle FHC = \underline{\hspace{2cm}}$.

13. If $m\angle LFJ = 65^\circ$, then $m\angle LFK = \underline{\hspace{2cm}}$.

14. If $m\angle FHB = 90^\circ$, then $m\angle AHG = \underline{\hspace{2cm}}$.

Use the letters to match each term to the best answer.

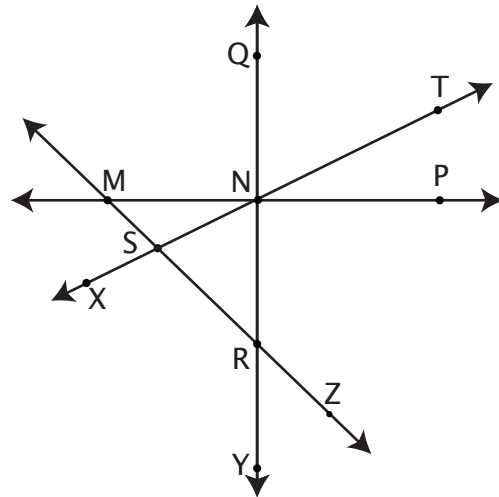
- | | |
|-------------------------------|---------------------------------|
| 15. β ____ | a. share a common ray |
| 16. adjacent angles ____ | b. alpha |
| 17. supplementary angles ____ | c. always have the same measure |
| 18. α ____ | d. add up to 90° |
| 19. complementary angles ____ | e. add up to 180° |
| 20. vertical angles ____ | f. beta |

LESSON PRACTICE

6B

Use the drawing to fill in the blanks.

1. $\angle MNS$ is adjacent to \angle _____ and \angle _____.
2. $\angle QNT$ is adjacent to \angle _____ and \angle _____.
3. $\angle SRN$ and \angle _____ are vertical angles.
4. $\angle MNS$ and \angle _____ are vertical angles.
5. $\angle QNP$ and \angle _____ are supplementary angles.
6. $\angle QNT$ and \angle _____ are complementary angles.
7. $\angle NRZ$ and \angle _____ are supplementary angles.
8. $\angle MNS$ and \angle _____ are complementary angles.



Given: All lines that appear to be straight lines are straight lines.
 $m\angle QNP = 90^\circ$.

The drawing is a sketch and not necessarily to scale. Do not make any assumptions about the lines and angles other than what is actually given.

Use the drawing from the previous page to fill in the blanks.

9. If $m\angle MNS = 35^\circ$, then $m\angle SNR =$ _____.
10. If $m\angle MNS = 35^\circ$, then $m\angle TNP =$ _____.
11. If $m\angle QNP = 90^\circ$, then $m\angle PNR =$ _____.
12. If $m\angle MSN = 95^\circ$, then $m\angle NSR =$ _____.
13. If $m\angle SRN = 40^\circ$, then $m\angle YRZ =$ _____.
14. If $m\angle XNY = 55^\circ$, then $m\angle QNT =$ _____.

Fill in the blanks with the correct terms.

15. The name of the Greek letter α is _____.
16. Two angles whose measures add up to 90° are called _____.
17. Two angles whose measures add up to 180° are called _____.
18. The name of the Greek letter γ is _____.
19. Intersecting lines form two pairs of _____ angles.
20. The name of the Greek letter δ is _____.

SYSTEMATIC REVIEW

6C

Use the drawing to fill in the blanks.

1. $\angle 1$ is adjacent to \angle _____ and \angle _____.

2. $\angle 1$ and \angle _____ are vertical angles.

3. $\angle AFE$ and \angle _____ are vertical angles.

4. \angle _____ is a straight angle.

5. \angle _____ is an obtuse angle.

6. $\angle 2$ and \angle _____ are complementary angles.

7. If $m\angle 2 = 50^\circ$, then $m\angle 1 =$ _____. Why?

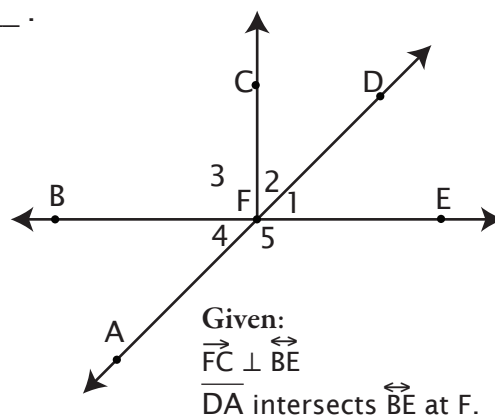
8. If $m\angle 2 = 50^\circ$, then $m\angle 4 =$ _____. Why?

9. $\angle 5$ and \angle _____ are supplementary angles.

10. If $m\angle 4 = 40^\circ$, then $m\angle 5 =$ _____. Why?

11. Name two acute angles.

12. Name two right angles.



From now on, we will assume lines that look straight to be straight lines. Do not make any assumptions about the size of the angles.

Follow the directions.

13. Draw a line segment $1\frac{1}{2}$ inches long. Then draw its perpendicular bisector using compass and straightedge.

14. Draw a 52° angle and bisect it.

Fill in the blanks with the correct terms.

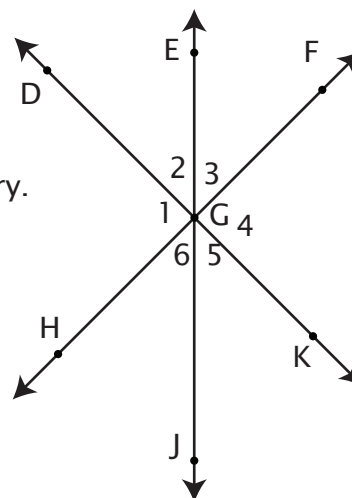
15. Two lines forming a right angle are said to be _____ to each other.
16. A right angle has a measure of _____ $^\circ$.
17. A straight angle has a measure of _____ $^\circ$.
18. The measures of two complementary angles add up to _____ $^\circ$.
19. The measures of two supplementary angles add up to _____ $^\circ$.
20. The intersection of two sets with no elements in common is the _____ set.

SYSTEMATIC REVIEW

6D

Use the drawing to tell if each statement is true or false.

1. $\angle 2$ and $\angle 5$ are vertical angles.
2. If $\overleftrightarrow{FH} \perp \overleftrightarrow{DK}$, then $\angle 2$ and $\angle 3$ are supplementary.
3. $\angle 3$ and $\angle 4$ are adjacent angles.
4. $\angle FGK$ is known to be a right angle.
5. \overrightarrow{GJ} is the common side for $\angle JGK$ and $\angle KGF$.
6. $\angle 2$, $\angle 3$, and $\angle 5$ appear to be acute.



Given:

\overleftrightarrow{DK} , \overleftrightarrow{EJ} , and \overleftrightarrow{FH} intersect at G.

Lines that look straight are straight. Do not make any other assumptions.

Use the drawing to fill in the blanks.

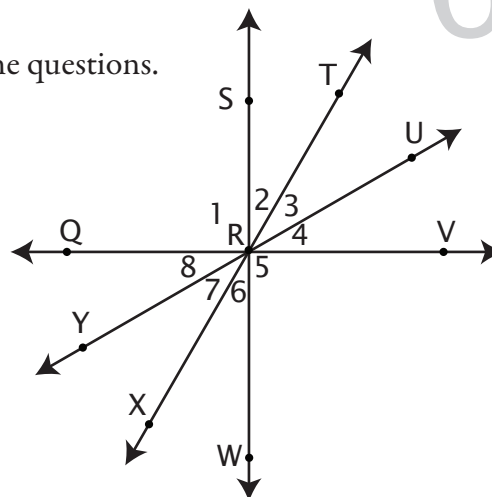
7. If $m\angle 3 = 39^\circ$, then $m\angle 6 = \underline{\hspace{2cm}}$. Why?
8. If $\overleftrightarrow{FH} \perp \overleftrightarrow{DK}$ and $m\angle 3 = 39^\circ$, then $m\angle 2 = \underline{\hspace{2cm}}$. Why?
9. If $\overleftrightarrow{FH} \perp \overleftrightarrow{DK}$, then $m\angle 1$ and $m\angle 4$ are each $\underline{\hspace{2cm}}$. Why?
10. If $m\angle 1$ is 90° , then it is a(n) $\underline{\hspace{2cm}}$ angle.
11. If the measures of $\angle 4$ and $\angle 1$ add up to 180° , they are called $\underline{\hspace{2cm}}$ angles.
12. $m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 + m\angle 5 + m\angle 6 = \underline{\hspace{2cm}}^\circ$.

Use the letters to match each description to the correct term.

- | | |
|----------------------------------------------|------------------|
| 13. Greek letter beta _____ | a. α |
| 14. less than 90° _____ | b. complementary |
| 15. measures add up to 90° _____ | c. δ |
| 16. Greek letter alpha _____ | d. obtuse |
| 17. Greek letter gamma _____ | e. acute |
| 18. between 90° and 180° _____ | f. β |
| 19. measures add up to 180° _____ | g. γ |
| 20. Greek letter delta _____ | h. supplementary |

SYSTEMATIC REVIEW

Use the drawing to fill in the blanks or answer the questions.



1. Name a line containing \overrightarrow{RV} .
2. Name a line segment contained in \overleftrightarrow{RT} .
3. If all eight angles were congruent, rather than as given, what would the measure of each be?

Given: $\overleftrightarrow{SW} \perp \overleftrightarrow{QV}$

All four straight lines intersect at R.

4. Since $m\angle 1$ is 90° , what is $m\angle 2 + m\angle 3 + m\angle 4$?
5. $\angle 4 + \angle 5$ is a(n) _____ angle.
6. Are $\angle 1$ and $\angle 5$ supplementary?
7. Are $\angle 1$ and $\angle 5$ complementary?
8. Are $\angle 1$ and $\angle 5$ vertical angles?
9. If $\angle 2 \cong \angle 3 \cong \angle 4$, then $m\angle 8 =$ _____ $^\circ$.

Remember the drawing is a sketch.

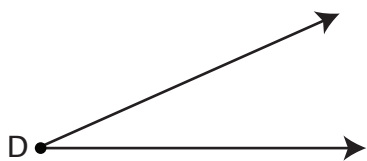
Use the measurements given in the questions, even if the drawing appears to be different.

10. $\angle 6 \cong \angle$ _____.
11. $\angle 2$ and $\angle 3$ are _____ angles (size).
12. If $m\angle 2 = 25^\circ$, and $m\angle 4 = 35^\circ$, then $m\angle 3 =$ _____.
13. If $m\angle 2 = 25^\circ$, and $m\angle 4 = 35^\circ$, then $m\angle YRX =$ _____.
14. Which ray is the common side for $\angle SRQ$ and $\angle QRX$?

15. Draw the perpendicular bisector of the given line segment.



16. Draw a ray that bisects the given angle.



Sharpen your algebra skills!

Be very careful when squaring negative numbers.

EXAMPLE 1 $(-5)^2 = (-5)(-5) = +25$

EXAMPLE 2 $-(8)^2 = -(8)(8) = -64$

EXAMPLE 3 $-6^2 = -(6)(6) = -36$

17. $(-7)^2 =$

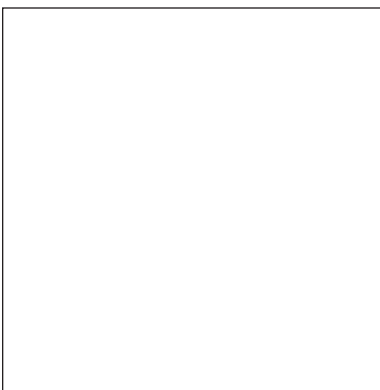
18. $-(15)^2 =$

19. $-12^2 =$

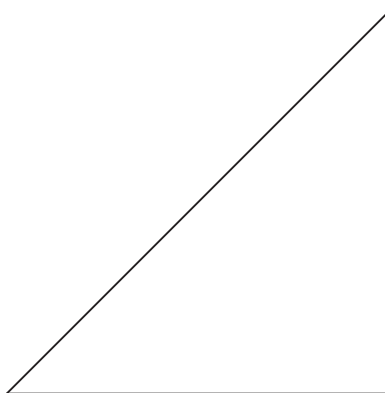
20. $-(9)^2 =$

Here are some more figures you may use to practice your bisection skills.

1. Draw the perpendicular bisectors of each line inside the square.



2. Using dotted lines or a different colored pencil, bisect each angle in the original square.
3. Draw the perpendicular bisectors of each side of the triangle. You have marked off two line segments on each side of the triangle. Now construct a perpendicular bisector for each of those line segments. What kinds of shapes do you see inside the large triangle?



4. If you wish, draw other shapes and construct bisectors as you did above. Try parallelograms, trapezoids, octagons, and other kinds of triangles for interesting results.

Read and follow the directions.

5. Lindsay's base pay is X dollars an hour. For every hour of overtime she works, she gets her base pay plus $.5X$. Last week she worked six hours of overtime. Let P be her total overtime pay for the week, and write an equation to find P .

6. Lindsay's base pay is \$8 an hour. Use the equation you wrote in #9 to find her total overtime pay for the week.

Circle your answer.

1. Two angles whose measures add up to 180° are called:

A. straight
B. complementary
C. acute
D. obtuse
E. supplementary

2. Vertical angles are:

A. supplementary
B. complementary
C. congruent
D. adjacent
E. obtuse

3. $m\angle XYZ = 35^\circ$. What is the measure of its complement?

A. 145°
B. 55°
C. 35°
D. 65°
E. 125°

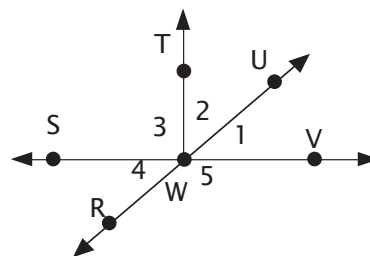
4. $m\angle GEF = 40^\circ$. What is the measure of its supplement?

A. 60°
B. 50°
C. 140°
D. 320°
E. 40°

5. Angle A is 20° and angle B is 70° . What is their relationship?

A. supplementary
B. vertical
C. reflexive
D. coplanar
E. complementary

Use this diagram for #6–10.



Given: $\overrightarrow{WT} \perp \overleftrightarrow{SV}$; $\overleftrightarrow{RU} \cap \overleftrightarrow{SV}$ at W.

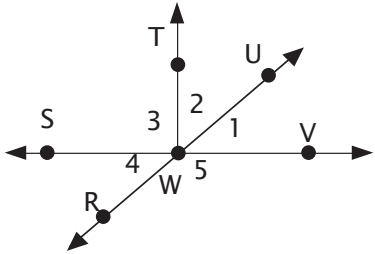
6. $\angle 1$ is adjacent to:

A. $\angle 1$
B. $\angle 2$ and $\angle 5$
C. $\angle 3$
D. $\angle 4$
E. $\angle 2$

7. The sum of $m\angle 1$ and $m\angle 2$ is:

A. 90°
B. 180°
C. 45°
D. 360°
E. can't tell from information given

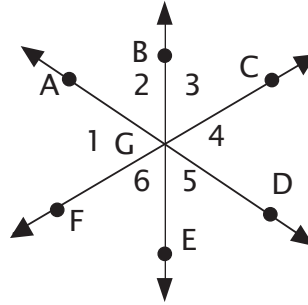
Use this diagram for #6–10.



Given: $\overrightarrow{WT} \perp \overrightarrow{SV}$; $\overleftrightarrow{RU} \cap \overleftrightarrow{SV}$ at W.

8. The measure of $\angle UWV$ is:
- 45°
 - 30°
 - 90°
 - 35°
 - can't tell from information given
9. $\angle 4$ and what other angle are vertical angles?
- $\angle 3$
 - $\angle 4$
 - $\angle 2$
 - $\angle 1$
 - $\angle TWV$
10. $\angle SWT + \angle TWU + \angle UWV =$
- 180°
 - 360°
 - 90°
 - 100°
 - can't tell from information given

Use this diagram for #11–15.

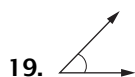


Given: \overleftrightarrow{FC} , \overleftrightarrow{AD} , \overleftrightarrow{BE} intersect at G.

- if the quantity in column I is greater.
- if the quantity in column II is greater.
- if the two quantities are equal.
- if the relationship cannot be determined from the information given.

Write the correct letter in the blank.

	I	II	
11.	$m\angle 2$	$m\angle 5$	_____
12.	$m\angle 4 + m\angle 5$	136°	_____
13.	180°	$m\angle 2 + m\angle 3$	_____
14.	$m\angle 2$	$m\angle 3$	_____
15.	185°	sum of the measures of 2 right angles	_____



Systematic Review 5E

1. f
2. e
3. b
4. c
5. g
6. a
7. d
8. false: Use a compass and a straightedge
9. true
10. false: The two parts are congruent.
11. false: The line will be perpendicular only if it forms a 90° angle.
12. true
13. Use a ruler to check.
14. Use a ruler to check.
The segment on each side of the bisector should measure $\frac{7}{8}$ in.
15. Use a protractor to check.
16. Use a protractor to check.
 $\angle XYG$ and $\angle ZYG$
should each measure 10° .
17. $24Q + 18Y = 30$
 $6(4Q + 3Y) = 6(5)$
 $4Q + 3Y = 5$
18. $-14Q - 21D = -42$
 $-7(2Q + 3D) = -7(6)$
 $2Q + 3D = 6$

$$\begin{aligned}
 19. \quad 16X - 8 &= 56 \\
 8(2X - 1) &= 8(7) \\
 2X - 1 &= 7 \\
 2X &= 7 + 1 \\
 2X &= 8 \\
 X &= \frac{8}{2} = 4
 \end{aligned}$$

$$\begin{aligned}
 20. \quad 22X + 33 &= 44 \\
 11(2X + 3) &= 11(4) \\
 2X + 3 &= 4 \\
 2X &= 4 - 3 \\
 2X &= 1 \\
 X &= \frac{1}{2}
 \end{aligned}$$

Lesson Practice 6A

1. $\angle AHG$, $\angle CHF$
2. $\angle FHB$, $\angle GHD$
3. $\angle AHG$
4. $\angle GHD$
5. $\angle LFK$ or $\angle JFH$
6. $\angle CHA$
7. $\angle HFK$ or $\angle JFL$
8. $\angle DHG$
9. 40° : vertical angles
10. 65° : vertical angles
11. 90° : supplementary angles
12. 50° : complementary angles
13. 115° : supplementary angles
14. 90° : vertical angles
15. f
16. a
17. e
18. b
19. d
20. c

Lesson Practice 6B

1. $\angle MNQ$, $\angle SNR$
2. $\angle MNQ$, $\angle TNP$
3. $\angle YRZ$
4. $\angle TNP$
5. $\angle QNM$ or $\angle PNR$