Contents

Section 1

7.1 Cramer's Rule **2**

7.2 Finding the n^{th} Term of an Arithmetic Sequence **6**

7.3 Combinations **10**

7.4 Radian Measure **14**

7.5 Quiz 1 . . . . . . **17**

Section 2

7.6 Applications of Composite Functions **18**

7.7 Conic Sections and Analytic Geometry: Parabolas **21**

7.8 Finding the n^{th} Term of a Geometric Sequence **25**

7.9 Probability **29**

7.10 Quiz 2 . . . . . . **32**

Section 3

7.11 Exponential Functions **33**

7.12 Logarithms **36**

7.13 Multiple Probabilities **39**

7.14 Radical Functions **42**

7.15 Review **47**

7.16 Test **49**

7.1

Cramer's Rule

Coefficient Matrix

$ax \bullet by = c$

$dx \bullet ey = f$

$$\begin{bmatrix} a & b \\ d & e \end{bmatrix}$$

Matrices can be used to find the solution to a system of equations. One technique, called Cramer's Rule, uses three determinants derived from the coefficients and constants of the system of equations. Before Cramer's rule can be used, all equations must be in standard form: $ax + by = c$, or $ax + by + cz = d$. The set of equations below is in standard form. Note the color of the coefficients and constants.

$$\begin{cases} ax \bullet by = c \\ dx \bullet ey = f \end{cases}$$

The coefficients of a system of equations forms the coefficient matrix. The determinant of the coefficient matrix is represented by "*D*".

Coefficient Matrix	Coefficient Matrix Determinant
$\begin{bmatrix} a & b \\ d & e \end{bmatrix}$	$D = \begin{vmatrix} a & b \\ d & e \end{vmatrix} = ae - bd$

To find the *x*-value determinant (*Dx*), replace the *x*-value coefficients with the constants (*c* and *f*) from the system of equations. To find the *y*-value determinant, replace the *y*-values with the constants.

Coefficient Matrix Determinant	*x*-Value Determinant	*y*-Value Determinant
$D = \begin{vmatrix} a & b \\ d & e \end{vmatrix} = ae - bd$	$D_x = \begin{vmatrix} c & b \\ f & e \end{vmatrix} = ce - bf$	$D_y = \begin{vmatrix} b & c \\ e & f \end{vmatrix} = bf - ce$

The *x* and *y*-values of the system can be found by dividing D_x and D_y by the determinant of the coefficient matrix *(D)*

$$x = \frac{D_x}{D} \qquad y = \frac{D_y}{D}$$

Example 1 **Use Cramer's Rule to solve the system:** $\begin{cases} x + 3y = 9 \\ 4x - 3y = 6 \end{cases}$

1. Write the coefficient matrix.

$$\begin{cases} 1x + 3y = 9 \\ 4x - 3y = 6 \end{cases} \quad \begin{bmatrix} 1 & 3 \\ 4 & -3 \end{bmatrix}$$

Note: Cramer's Rule does not work if the coefficient determinant is 0 because division by zero is undefined. If the coefficient determinant is 0, the two lines either have *no* intersection at all (parallel lines), or an infinite number of intersections (same line).

Example 1 Continued.

2. Evaluate the determinant of the coefficient matrix.

$$D = \begin{vmatrix} 1 & 3 \\ 4 & -3 \end{vmatrix} = -15$$

3. Evaluate the determinants of D_x and D_y

$$D_x = \begin{vmatrix} 9 & 3 \\ 6 & -3 \end{vmatrix} = -45 \qquad D_y = \begin{vmatrix} 1 & 9 \\ 4 & 6 \end{vmatrix} = -30.$$

4. Use Cramer's Rule to find the x value.

$$x = \frac{D_x}{D} = \frac{-45}{-15} = 3$$

5. Use Cramer's Rule to find the y value.

$$y = \frac{D_y}{D} = \frac{-30}{-15} = 2$$

The solution for the system is the ordered pair (3, 2).

Cramer's rule can also be used for systems of 3 or more variables. Use the determinant of the coefficient matrix for the denominator of each variable solution. And for each numerator, use the determinants of D_x, D_y, and D_z.

Example 2 Find the solution to the system: $\begin{cases} 2x + y + z = 1 \\ 6x - y + 2z = 4 \\ 3x + y + z = 2 \end{cases}$

1. Write the coefficient matrix.

$$\begin{cases} 2x + 1y + 1z = 1 \\ 6x - 1y + 2z = 4 \\ 3x + 1y + 1z = 6 \end{cases} \quad \begin{bmatrix} 2 & 1 & 1 \\ 6 & -1 & 2 \\ 3 & 1 & 1 \end{bmatrix}$$

2. Evaluate the determinant of the coefficient matrix.

$$D = \begin{vmatrix} 2 & 1 & 1 \\ 6 & -1 & 2 \\ 3 & 1 & 1 \end{vmatrix} = 3$$

3. Evaluate the determinants of D_x and D_y and D_z

$$D_x = \begin{vmatrix} 1 & 1 & 1 \\ 4 & -1 & 2 \\ 2 & 1 & 1 \end{vmatrix} = 3 \quad D_y = \begin{vmatrix} 2 & 1 & 1 \\ 6 & 4 & 2 \\ 3 & 2 & 1 \end{vmatrix} = 0 \quad D_z = \begin{vmatrix} 2 & 1 & 1 \\ 6 & -1 & 4 \\ 3 & 1 & 2 \end{vmatrix} = -3$$

4. Use Cramer's Rule to find the x value.

$$x = \frac{D_x}{D} = \frac{3}{3} = 1$$

5. Use Cramer's Rule to find the y value.

$$y = \frac{D_y}{D} = \frac{0}{3} = 0$$

6. Use Cramer's Rule to find the z value.

$$y = \frac{D_z}{D} = \frac{-3}{3} = -1$$

The solution for the system is the point (1, 0, –1).

Today's Lesson

Use Cramer's Rule to solve the systems.

1. $\begin{cases} 2x - y = -5 \\ x - 2y = -7 \end{cases}$

2. $\begin{cases} 5x - 4y = 28 \\ x + 4y = -4 \end{cases}$

3. $\begin{cases} 3x + 2y = 3 \\ 3x - y = -6 \end{cases}$

4. $\begin{cases} 2x - y - z = 5 \\ x - 2y - z = 5 \\ x - y - z = 4 \end{cases}$

5. $\begin{cases} 2x + y + 4z = 4 \\ 3x + y + 4z = 7 \\ 3x + 2y + 4z = 5 \end{cases}$

REVIEW

Graph the rational function. 6.14

6. $f(x) = \dfrac{2x - 1}{2x + 3}$

Determine the range and standard deviation of these data sets. 6.13

7. 5, 7, 5, 5, 8, 4, 5

8. A doctor's office measured the BMI (body mass index) of eight men:
23.8, 23.2, 24.6, 26.2, 23.5, 24.5, 21.5, 31.4

Find the answers. 6.6

9. An avid fisherman wants to try out his 8 new lures. In how many orders could he do this?

10. The Western musical system uses 12 different notes. How many different ways could those notes be played in order before duplicating any?

Simplify. 6.1

11. $\sqrt[3]{4\sqrt[4]{2}}$

12. $\sqrt{\sqrt{\sqrt{2}}}$

13. $\sqrt{x\sqrt[3]{x}}$

Multiply the following matrices if possible. 5.6

14. $\begin{bmatrix} 6 & 3 \\ 5 & 1 \\ 2 & 1 \\ 9 & 5 \end{bmatrix} \begin{bmatrix} 8 \\ -2 \end{bmatrix}$

15. $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \end{bmatrix} \begin{bmatrix} 1 & 5 \\ 2 & 6 \\ 3 & 7 \\ 4 & 8 \end{bmatrix}$

Solve the equations. 4.14

16. $\dfrac{3x}{x - 1} = \dfrac{2x^2}{x + 1}$

17. $\dfrac{4}{3x} + \dfrac{8}{x} = \dfrac{2}{5} - \dfrac{x}{6x}$

18. $\dfrac{5}{x + 5} + \dfrac{3}{x - 2} = \dfrac{4}{x^2 + 3x - 10}$

Simplify. 4.8

19. $\frac{2}{4+\sqrt{2}}$

20. $\frac{5x}{2-\sqrt{3x}}$

21. $\frac{2-\sqrt{x}}{2+\sqrt{x}}$

Write the first five terms of a sequence, given the first term (a_1) and the common difference (d). 6.4

22. $a_1 = -6,\ d = \frac{5}{6}$

23. $a_1 = -15,\ d = 2.5$

Find the common difference (if any) of the sequences. 6.4

24. 1, 1, 2, 3, 5

25. −23, −21.5, −20, −18.5, −17

Extra Practice

Use Cramer's Rule to solve the systems.

26. $\begin{cases} 2x - 3y = 13 \\ x + 4y = -10 \end{cases}$

27. $\begin{cases} 3x + y = 1 \\ x + 4y = 15 \end{cases}$

28. $\begin{cases} 3x + y - 2z = -1 \\ 4x + 2y + 2z = 14 \\ x + y + z = 6 \end{cases}$