

A *Activities for Learning, Inc.*

RIGHTSTART™ MATHEMATICS

by Joan A. Cotter, Ph.D.

LEVEL B LESSONS
FOR HOME EDUCATORS

Special thanks to Sharalyn Colvin, who converted *RightStart™ Mathematics: Grade 1 Lessons* into *RightStart™ Mathematics: Level B For Home Educators*.

Note: Rather than use the designation, K-4, to indicate a grade, levels are used. Level A is kindergarten, Level B is first grade, and so forth.

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Home Educators

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by Joan A. Cotter, Ph.D.

The following are items needed to complete the RightStart™ Mathematics Level B Lessons:

STATUS	ITEM	CODE
REQUIRED	<i>Level B Lessons</i>	T-B
REQUIRED	<i>Level B Worksheets</i>	W-B
RECOMMENDED	<i>Math Card Games</i> book	M
REQUIRED; either abacus	Classic AL Abacus - 8-1/2" x 9-1/2" hardwood frame & beads	A-CL
	Standard AL Abacus - 7-1/2" x 9-1/2" plastic frame & beads	A-ST
REQUIRED	Place Value Cards	P
REQUIRED	Base 10 Picture Cards	DP
REQUIRED	Abacus Tiles	AT
REQUIRED	Cards, Six Special Decks needed for Games	C
RECOMMENDED	Fraction Charts	F
RECOMMENDED	Basic Drawing Board Geometry Set	DS
REQUIRED	Geoboards	R3
REQUIRED	Color Cubes, 20-1" cubes in set	RH13
REQUIRED	Colored Tiles, apx 200 in set	RH2
RECOMMENDED	Casio Calculator SL-450	R4
REQUIRED	4" Geared Clock	R12
REQUIRED	Yellow is the Sun CD	CD Sun
REQUIRED	Geometry Reflector	RF
REQUIRED	Math Balance (Invicta)	R7
RECOMMENDED	Tally Sticks, apx 75 count	RH1
RECOMMENDED	Plastic Coins	R5

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RightStart™ MATHEMATICS: OBJECTIVES FOR LEVEL B

Name _____

Year _____

Teacher _____

Numeration

- Can recognize quantities 1 to 5 and represent it on abacus
- Knows even numbers to 20
- Knows odd numbers to 19
- Can identify even/odd numbers to 100
- Can count by 2s to 30
- Can count by 5s to 50
- Can count by 10s to 100

1ST QTR	2ND QTR	3RD QTR	4TH QTR
N/A			
N/A			
N/A			

Money

- Knows name and value of penny, nickel, dime, and quarter
- Can determine the value of three coins

Place Value

- Can trade 10 ones for 1 ten
- Can trade 10 tens for 1 hundred
- Can trade 10 hundreds for 1 thousand
- Knows 37 as 3-ten 7
- Can read four-place numbers
- Knows traditional names: e.g., 18 as eighteen as well as 1-ten 8

N/A			

Addition

- Understands addition as combining parts to form whole
- Can add 4-digit numbers
- Knows number facts equal to 10
- Knows number facts up to 18
- Can add 2-digit numbers mentally

N/A	N/A	N/A	
N/A	N/A	N/A	
N/A	N/A		

Subtraction

- Understands subtraction as missing addend
- Understands subtraction as separating
- Knows number facts subtracting from numbers up to 10

N/A			
N/A			
N/A			

Calculator

- Can add and subtract whole numbers

N/A	N/A	N/A	
-----	-----	-----	--

Problem Solving

- Can solve change problems
- Can solve combine problems
- Can solve equalize problems

N/A			
N/A	N/A	N/A	

Geometry

- Knows square is a special rectangle
- Knows parallel and perpendicular lines
- Knows what a reflection is

N/A			
N/A			

Time

- Knows days of the week and months of the year
- Can tell time to five-minute intervals
- Can tell time to the minute

N/A			
N/A	N/A		
N/A	N/A		

Measurement

- Can determine length with nonstandard measure
- Can find perimeter
- Can read scales with numbers missing

N/A	N/A	N/A	
N/A	N/A	N/A	
N/A	N/A	N/A	

Fractions

- Can divide into halves and fourths
- Knows unit fractions up to 1/10

N/A	N/A	N/A	

How This Program Was Developed

We have been hearing for years that Japanese students do better than U.S. students in math in Japan. The Asian students are ahead by the middle of first grade. And the gap widens every year thereafter.

Many explanations have been given, including less diversity and a longer school year. Japanese students attend school 240 days a year.

A third explanation given is that the Asian public values and supports education more than we do. A first grade teacher has the same status as a university professor. If a student falls behind, the family, not the school, helps the child or hires a tutor. Students often attend after-school classes.

A fourth explanation involves the philosophy of learning. Asians and Europeans believe anyone can learn mathematics or even play the violin. It is not a matter of talent, but of good teaching and hard work.

Although these explanations are valid, I decided to take a careful look at how mathematics is taught in Japanese first grades. Japan has a national curriculum, so there is little variation among teachers.

I found some important differences. One of these is the way the Asians name their numbers. In English we count ten, eleven, twelve, thirteen, and so on, which doesn't give the child a clue about tens and ones. But in Asian languages, one counts by saying ten-1, ten-2, ten-3 for the teens, and 2-ten 1, 2-ten 2, and 2-ten 3 for the twenties.

Still another difference is their criteria for manipulatives. Americans think the more the better. Asians prefer very few, but insist that they be imaginable, that is, visualizable. That is one reason they do not use colored rods. You can imagine the one and the three, but try imagining a brown eight—the quantity eight, not the color. It can't be done without grouping.

Another important difference is the emphasis on non-counting strategies for computation. Japanese children are discouraged from counting; rather they are taught to see quantities in groups of fives and tens.

For example, when an American child wants to know $9 + 4$, most likely the child will start with 9 and count up 4. In contrast, the Asian child will think that if he takes 1 from the 4 and puts it with the 9, then he will have 10 and 3, or 13.

Unfortunately, very few American first-graders at the end of the year even know that $10 + 3$ is 13.

I decided to conduct research using some of these ideas in two similar first grade classrooms. The control group studied math in the traditional work-book-based manner. The other class used the lesson plans I developed. The children used that special number naming for three months.

They also used a special abacus I designed, based on fives and tens. I asked 5-year-old Stan how much is $11 + 6$. Then I asked him how he knew. He replied, "I have the abacus in my mind."

The children were working with thousands by the sixth week. They figured out how to add four-place numbers on paper after learning how to do it on the abacus.

Every child in the experimental class, including those enrolled in special education classes, could add numbers like $9 + 4$, by changing it to $10 + 3$.

I asked the children to explain what the 6 and 2 mean in the number 26. Ninety-three percent of the children in the experimental group explained it correctly while only 50% of third graders did so in another study.

I gave the children some base ten rods (none of them had seen them before) that looked like ones and tens and asked them to make 48. Then I asked them to subtract 14. The children in the control group counted 14 ones, while the experimental class removed 1 ten and 4 ones. This indicated that they saw 14 as 1 ten and 4 ones and not as 14 ones. This view of numbers is vital to understanding algorithms, or procedures, for doing arithmetic.

I asked the experimental class to mentally add $64 + 20$, which only 52% of nine-year-olds on the 1986 National test did correctly; 56% of those in the experimental class could do it.

Since children often confuse columns when taught traditionally, I wrote $2304 + 86 =$ horizontally and asked them to find the sum any way they liked. Fifty-six percent did so correctly, including one child who did it in his head.

This following year I revised the lesson plans and both first grade classes used these methods. I am delighted to report that on a national standardized test, both classes scored at the 98th percentile.

Some General Thoughts on Teaching Mathematics

1. Only five percent of mathematics should be learned by rote; 95 percent should be understood.
2. Teaching with understanding depends upon building on what the child already knows. Teaching by rote does not care.
3. The role of the teacher is to encourage thinking by asking questions, not giving answers. Once you give an answer, thinking usually stops.
4. It is easier to understand a new model after you have made one yourself. For example, a child needs to construct graphs before attempting to read ready-made graphs.
5. Good manipulatives cause confusion at first. If the new manipulative makes perfect sense at first sight, it wasn't needed. Trying to understand and relating it to previous knowledge is what leads to greater learning, according to Richard Behr and others.
6. Lauren Resnick says, "Good mathematics learners expect to be able to make sense out of rules they are taught, and they apply some energy and time to the task of making sense. By contrast, those less adept in mathematics try to memorize and apply the rules that are taught, but do not attempt to relate these rules to what they know about mathematics at a more intuitive level."
7. According to Arthur Baroody, "Teaching mathematics is essentially a process of translating mathematics into a form children can comprehend, providing experiences that enable children to discover relationships and construct meanings, and creating opportunities to develop and exercise mathematical reasoning."
8. Mindy Holte puts learning the facts in proper perspective when she says, "In our concern about the memorization of math facts or solving problems, we must not forget that the root of mathematical study is the creation of mental pictures in the imagination and manipulating those images and relationships using the power of reason and logic."
9. The only students who like flash cards are those who don't need them.
10. Mathematics is not a solitary pursuit. According to Richard Skemp, solitary math on paper is like reading music, rather than listening to it; "Mathematics, like music, needs to be expressed in physical actions and human interactions before its symbols can evoke the silent patterns of mathematical ideas (like musical notes), simultaneous relationships (like harmonies) and expositions or proofs (like melodies)."
11. "More than most other school subjects, mathematics offers special opportunities for children to learn the power of thought as distinct from the power of authority. This is a very important lesson to learn, an essential step in the emergence of independent thinking." (A quote from *Everybody Counts*)

12. Putting thoughts into words helps the learning process.
13. The difference between a novice and an expert is that an expert catches errors much more quickly. A violinist adjusts pitch so quickly that the audience does not hear it.
14. Europeans and Asians believe learning occurs not because of ability, but primarily because of effort. In the ability model of learning, errors are a sign of failure. In the effort model, errors are natural. In Japanese classrooms, the teachers discuss errors with the whole class.
15. For teaching vocabulary, be sure either the word or the concept is known. For example, if a child is familiar with six-sided figures, we can give him the word, hexagon. Or, if he has heard the word, multiply, we can tell him what it means. It is difficult to learn a new concept and the term simultaneously.
16. Introduce new concepts globally before details. This lets the children know where they are headed.
17. Informal mathematics should precede paper and pencil work. Long before a child learns how to add fractions with unlike denominators, she should be able to add one half and one fourth mentally.
18. Some pairs of concepts are easier to remember if one of them is thought of as dominant. Then the non-dominant concept is simply the other one. For example, if even is dominant over odd; an odd number is one that is not even.
19. Worksheets should also make the child think. Therefore, they should not be a large collection of similar exercises, but should present a variety.
20. In Japan students spend more time on fewer problems. Teachers do not concern themselves with attention spans as is done in the U.S.
21. In Japan the goal of the math lesson is that the student has understood a concept, not necessarily has done something (a worksheet).
22. The calendar should show the entire month, so the children can plan ahead. The days passed can be crossed out or the current day circled.
23. A real mathematical problem is one in which the procedures to find the answer or answers are not obvious. It is like a puzzle, needing trial and error. Emphasize the satisfaction of solving problems and like puzzles, of not giving away the solution to others.
24. Keep math time enjoyable. A person who dislikes math will avoid it. We store our emotional state along with what we've learned. A child under stress stops learning. If a lesson is too hard, end it and play a game. Try again another day.

RightStart™ Mathematics

There are 13 major characteristics that make this research-based program effective.

1. Refers to quantities of up to 5 as a group; discourages counting individually.
2. Uses fingers and tally sticks to show quantities up to 10; teaches quantities 6 to 10 as 5 plus a quantity, for example $6 = 5 + 1$.
3. Avoids counting procedures for finding sums and remainders. Teaches five- and ten-based strategies for the facts that are both visual and visualizable.
4. Employs games, not flash cards, for practice.
5. Once quantities 1 to 10 are known, proceeds to 10 as a unit. Uses the “math way” of naming numbers for several months; for example, “1 ten-1” (or “ten-1”) for eleven, “1-ten 2” for twelve, “2-ten” for twenty, and “2-ten 5” for twenty-five.
6. Uses expanded notation (overlapping) place-value cards for recording tens and ones; the ones card is placed on the zero of the tens card. Encourages a child to read numbers starting at the left and not backward by starting at the ones column.
7. Proceeds rapidly to hundreds and thousands using manipulatives and place-value cards. Provides opportunities for trading between ones and tens, tens and hundreds, and hundreds and thousands with manipulatives.
8. Only after the above work, about the fourth month of first grade, introduces the traditional English names for quantities 20 to 99 and then 11 to 19.
9. Teaches mental computation. Investigates informal solutions, often through story problems, before learning procedures.
10. Teaches four-digit addition on the abacus, letting the child discover the paper and pencil algorithm. This occurs in Level B. Four-digit subtraction is mastered in Level C.
11. Introduces fractions with a linear visual model.
12. Approaches geometry through drawing boards and tools.
13. Teaches short division (where only the answer is written down) for single-digit divisors, before long division. Both are taught in Level E.

Some Pointers

Kindergarten. Most of the kindergarten lesson plans have two distinct topics, which can be taught on alternate days.

Transition Lessons. These lessons are designed for children starting Levels C, D, or E (grades 2, 3, or 4) who have not been doing RightStart™ Mathematics previously. The lessons need to be studied before the regular manual, except where noted. The manual tells which lessons to use for the particular grade.

Counting. Counting needs to be discouraged because it is slow and inaccurate. It also interferes with understanding quantity and learning place-value.

Warm-up. The warm-up time is the time for quick review, memory work, and sometimes an introduction to the day's topics. The drawing board makes an ideal slate for quick responses.

Place value. In order to understand addition algorithms, place-value knowledge is essential. From the very beginning, the children are helped to see quantities grouped in fives and tens. Children can understand place value in first grade and even in kindergarten when it is approached as it is in this program.

Worksheets. The worksheets are designed to give the children a chance to think about and to practice the day's lesson. Some lessons, especially in the early grades, have no worksheet.

Games. Games, not worksheets or flash cards, are used for practice. They can be played as many times as necessary until memorization takes place. Games are as important to math as books are to reading.

Some games are incorporated in this manual. Extra games, found in the book, *Math Card Games*, are suggested in the Review and Practice lessons in Levels C to E. There are games for the child needing extra help, as well as for the advanced child.

Teaching. Establish with the children some indication when you want a quick response and when you want a more thoughtful response. Research shows that the quiet time for thoughtful response should be about three seconds. Avoid talking during this quiet time; resist the temptation to rephrase the question. This quiet time gives the slower child time to think. It also gives a quicker child time to think more deeply.

Encourage the child to develop perseverance. Avoid giving answers too quickly. Children tend to stop thinking once they hear the answer.

Help the children realize that it is their responsibility to ask questions when they do not understand. Do not settle for "I don't get it."

Number of lessons. It is not necessary that each lesson be done in one day. Sometimes two days may be more appropriate. However, do complete each manual before going to the next one.

Visualization. The ability to imagine or visualize is an important skill to develop in mathematics and other subjects as well. Often you are called upon to suggest to the children that they imagine a particular topic.

Questions. I really want to hear how this program is working. Please let me know any improvements and suggestions that you may have.

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www.alabacus.com

List of Appendix Pages to be Copied

FOR LESSON	COPY	NUMBER OF COPIES
Lesson 8	App. 2, 3, 4*	2, heavy paper
Lesson 9	App. 6	1
Lesson 17	App. 10	1, heavy paper
Lesson 18	App. 11	1, heavy paper
Lesson 25	App. 13	1, heavy paper
Lesson 29	App. 14	1, heavy paper
Lesson 30	App. 15	10
Lesson 38**	App. 17-18	1, optional
	App. 19	10, optional
	App. 20	25, optional
	App. 21	1
Lesson 44	App. 22***	12, legal size
Lesson 52	App. 23	1
Lesson 54	App. 24-26	1
Lesson 55	App. 27	1
Lesson 56	App. 28-30	1
Lesson 64	App. 33	2
Lesson 66	App. 34-39	1
Lesson 103	App. 44	1
Lesson 104	App. 45-46	1, heavy paper

Notes. Where only one copy is needed, you could use the copy in the appendix.

*If possible, use different colors for each appendix page.

**Lesson 38 needs at least a dozen children to make the fractal.

***Be sure the extra space is at the *top* of the page of the calendar.

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Lesson 4*	Quantities 1 to 7, Taps, and Writing Numbers
Lesson 5*	Quantities 1 to 8
Lesson 6*	Quantities 9 and 10
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Lesson 10*	Solving Problems
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Lesson 12*	Mastering Partitioning 5
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Lesson 21*	Writing More Equations & Parallel
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Lesson 23*	Introducing Tens
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* A child who has completed all of Level A may skip these lessons.

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Lesson 45	Working with Teens
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Lesson 106 (1 or 2 days)	Graphing
Lesson 107	Final Test
	Practice Record & Practice Sheets
	Tests
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Lesson 1

Quantities 1 to 3

OBJECTIVES

1. To review terms *left* and *right*
2. To recognize quantities 1 to 3 without counting
3. To learn finger sets and tally marks for quantities 1 to 3
4. To realize how the number of objects change
5. To practice ordinal counting to 3

MATERIALS

Music for "Yellow is the Sun" (end of manual), optional
 A calendar (see Appendix pg. 1 for suggestions)
 Various collections of 1 to 3 interesting objects, toys or dolls
 Three trays, or sheets of construction paper
 4 to 5 tally sticks (craft sticks, also called Popsicle sticks)

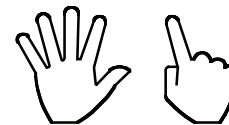
WARM-UP

Start teaching the following song. During the words, "Six is five and one," the child shows 5 with her *left* hand and 1 with her *right* hand as shown below. Continue with the remaining verses.

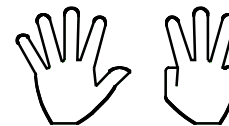
Note: While the correct hand is important in showing quantities, which fingers on a hand is not important.

Yellow is the Sun*Yellow is the sun.**Six is five and one.**Why is the sky so blue?**Seven is five and two.**Salty is the sea.**Eight is five and three.**Hear the thunder roar.**Nine is five and four.**Ducks will swim and dive.**Ten is five and five.*

Joan A. Cotter



Showing 6.



Showing 8.

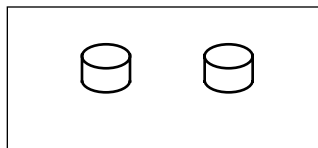
ACTIVITIES

Left and right. Ask the child to raise her right hand. For many children, their right hand is the one with which they *write*. The hand that is left is their *left* hand. Then ask her to raise her left hand. For further practice, ask her to point to her right or left foot, eye, or other body parts.

Quantity of 2. Place 2 objects on a tray or sheet of paper near the child. Ask her to show the *number* with her fingers on her left hand. It does not matter which fingers she uses as long as she uses her left hand. (Left hands are used for numbers less than 5 to correspond to the abacus and tally marks.) Ask her how many she sees. [2] Ask her if she needed to count. [no]



Note: Left hands are used for numbers less than 5 to correspond to the abacus and tally marks. An additional reason is to reinforce reading left to right.

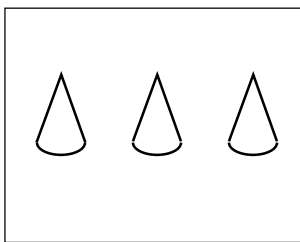


Showing 2 objects with tally sticks (Popsicle or craft sticks).

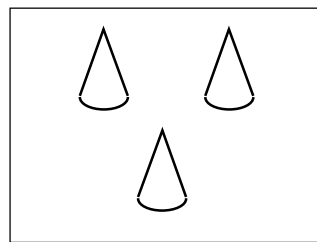
With the pile of tally sticks nearby, ask the child to take 3 sticks. Now tell her, Lay out the same number of tally sticks as objects on the tray. [2] Be sure the tally sticks are laid out vertically.



Quantity of 3. Place 3 different objects in a line on a second tray; ask the child, How many do you see? [3] Show the number with your fingers and with the tally sticks. Rearrange the objects as shown and again ask how many and to show it. [3] Ask her, Did you need to count? [no]



3 objects in a row.

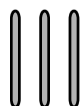
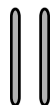
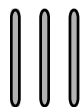


3 objects scattered.

Quantity of 1. On the third tray place 1 object (choose an object larger than those used previously). Ask the child, How many do you see? [1] Show it with your fingers and the tally sticks.

Finding given quantities. Now tell the child, Look around the room and name 1 of something. [for example, the table] Encourage her to look for and name additional single objects. Repeat for finding 2 and 3 objects.

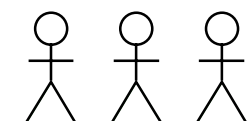
Disagreeing. Place 3 objects on one of the trays and put 2 objects on the other tray. Point to each tray in turn and ask if she agrees that there are 3 (pencils) on this tray. It is very important that the child feels comfortable disagreeing with you.



Changing quantities. Start with 3 objects; ask the child to show the number. Now ask her to watch. Remove an object and ask her to show the new number. [2] Ask the child if the number has changed. [yes] Remove another object and ask her to show the number. Replace 2 objects and ask her to show the number. [3] Rearrange the objects and ask for the number and ask if the number changed. [no]

Summarize by saying, So, let's think what we know about numbers. Does size make a difference? [no] Does color make a difference? [no], Does how they are arranged make a difference? [no]. What makes the number change? [only adding or removing]

Ordinal numbers. This activity can be used to help the child incorporate learning with playing. Ask the child to place 3 toys or dolls in a row. Ask her, Which toy is first in the row? Encourage her to observe that there are answers; let her decide where the beginning of the row should be. Encourage her to answer in complete sentences: Dolly is first. Then ask, Who is second? Who is third? Also ask inverse questions, In what position is Dolly?



Who is first in line?

Repeat with other toys forming the row.

Lesson 2

Quantities 1 to 5

- OBJECTIVES**
1. To learn finger sets and tally marks for quantities 4-5
 2. To increase quantities by 1, using 1 more
 3. To continue ordinal counting to 5

- MATERIALS**
- Music for "Writing Numbers" (end of manual), optional
 At least 10 objects, not all the same
 Two trays
 Five tally sticks (craft or Popsicle sticks)

- WARM-UP**
- Continue teaching "Yellow is the Sun" (Lesson 1).
 Begin teaching him the following rhyme to help in writing numbers. Use the motions shown in parentheses.

Writing Numbers

Before you write a number, stop
 (with writing hand, show stop)

And take a hop to the top.
 (raise hand in the writing position with the first 2 fingers resting relaxed on the thumb at *hop to top*.)

Start at the left to make 2, 3, or 4,
 (move writing hand in position to *left* and pound air at 2, 3, 4)

Also 7, which you can't ignore.
 (pound air at 7)

Start at the right for 5, 6, 8, or 9.
 (move to *right* and pound air at 5, 6, 8, 9)

Don't lift your pen and you'll do just fine.

Zero and one—not left, not right—a riddle.

They start at the top, smack in the middle.
 (move to *middle*)

Joan A. Cotter

Note: Be sure to teach writing numbers as suggested in this manual, and not according to a writing/reading program. Then the child's numbers will match the cards for games, place-value cards, and worksheets.

- ACTIVITIES**
- Quantities 1 to 4.** Review quantities 1 to 3 by showing objects to the child and asking him to show the results with the fingers on his left hand and with tally sticks and to say it.

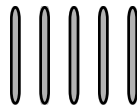


Introduce 4 objects and practice the same way with 1 to 4 objects.



The quantity 5. Tell the child to show 5 fingers and ask, Why is 5 is special? [whole hand] Say the numbers 1 to 5 in random order and ask him, Show it with your fingers as quickly as you can.

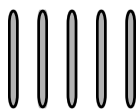
Ask the child, Take 5 tally sticks and lay them out. Which tally stick is in the middle? Ask them to remove a stick and ask, Does 4 have a middle? [no] (Our middle finger is also the tallest.)



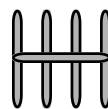
5 has a middle.

Challenge the child by asking, Do you think you could see 5 objects without counting? Practice naming and showing fingers with up to 5 objects. Show 5 objects and ask him to say the corresponding numbers.

Next tell him that because 5 is hard to see with the tally sticks, we are going to do something special. Ask him to watch while you take the last tally stick and place it across the other 4. Demonstrate rearranging the sticks so the fifth stick covers the other 4 as shown on the next page. (The horizontal position is used because a tally



5 tally sticks with-
out grouping.



5 tally sticks with
grouping.

stick will not fit diagonally and young children find it easier.) Tell him that from now on, that is how we will show 5.

Practice. Show various numbers of objects up to 5 and ask the child to represent the number with the sticks. He need not disassemble his arrangements before making the next number; it is better if he merely alters his arrangement. For example, if he sees 2 objects and the next time he sees 4, he needs only to add 2 more sticks.

Also ask him to write lines, like the tally sticks, on the drawing board to show the numbers. Tell him they are called *tally marks*.

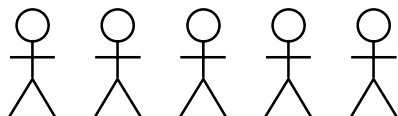
Be sure that each vertical line is written starting at the top. For writing the horizontal mark, a right-handed child should start at the left and a left-hand child at the right.

Practice the inverses: show tally marks up to 5 and ask the child to name them or show fingers. Also, say the numbers and ask him to show the corresponding tally marks.

1 more. Start with 1 object on a tray. Ask the child to show how many by using his fingers or tally sticks. Add 1 more object and ask, What is 1 and 1 more? [2] Show him how to raise the number of fingers to start and then to raise 1 more. Encourage fingers, tally marks, and oral responses. Add 1 more and repeat the question. What is 2 and 1 more? [3]

Then ask the questions without the objects. Vary the wording with problems, such as, I have 2 pencils and I get 1 more. How many do I have now? [3] Or, Josh has 3 books. Maria has 1 more book than Josh. How many books does Josh have? [3] How many books does Maria have? [4]

Ordinal numbers. Continue with ordinal numbers up to 5. (The “thir” and “fif” sounds are especially important later for pronouncing the names of 13, 15, 30, and 50). Ask the child to place 5 dolls or toys in a row. First ask him to identify what is first, second, third, fourth, or fifth. Then ask for them in random order. Also ask for the inverse, for example, In what position is Dolly? Rearrange or bring up 5 different toys and change the beginning to the other end. Ask the same questions as before. Then ask the child to solve this riddle. What is in the same position no matter where the beginning is? [the 3rd doll or toy]



Who is second in the row?

Lesson 3

Quantities 1 to 6 and the Abacus

- OBJECTIVES**
1. To learn finger sets and tally marks for the quantity 6
 2. To represent 1 to 5 quantities on the abacus
- MATERIALS**
- Abacus
 - 6 tally sticks
- WARM-UP**
- Continue teaching “Yellow is the Sun” (Lesson 1) and “Writing Numbers” (Lesson 2).
- ACTIVITIES**
- Quantities 1 to 5.** Review the finger sets 1 to 5 by asking the child to show quickly with her fingers numbers 1 to 5 in random order. Repeat 1 to 5 with the tally sticks.
- Quantity 6.** Show 6 with your fingers by raising 5 fingers on 1 hand and 1 finger on the other hand. Ask her to do the same. Then ask the child what number it is. [6]



Showing 6 with fingers and tally sticks.

Next demonstrate 6 with tally sticks as shown above.

Practice with the child by: (1) showing 1 to 6 with the tally sticks and asking her to show fingers and say the number, (2) showing fingers and asking her to make the tally sticks and name the amount, and (3) saying numbers and asking her to show fingers and tally sticks.

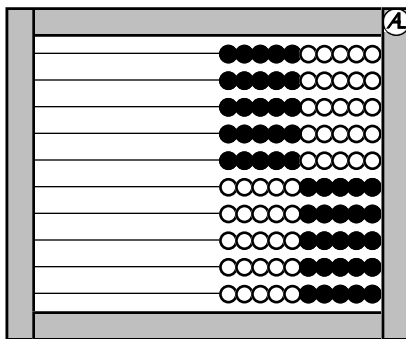
Also ask the child to close her eyes and see the 6 in her mind.

Abacus rules. Show the child the abacus. Engage her in deciding the ground rules. Include handling it carefully and quietly, using it flat on a surface without sliding it around, moving the beads only with fingers, and storing it in the designated place.

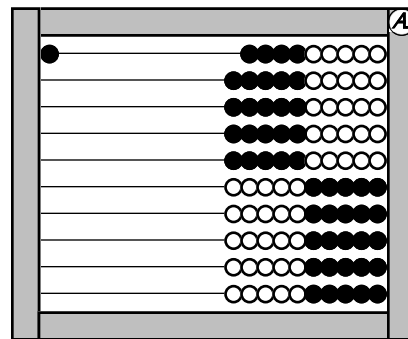
Give the abacus to the child and allow time for free exploration; encourage her to make designs.

Entering quantities. Tell the child that now she will learn to enter numbers 1 to 5 on the abacus. Ask her to place the abacus with the wires horizontal and the dot on the top right. She *clears* the abacus by lifting the left side to allow the beads to fall toward the right side, the side with the dot, and returning it to the flat surface quietly. See the left figure at the top of the next page.

Show the child one object and ask her to show with her fingers how much it is. Demonstrate *entering* 1 by sliding one bead on the top wire to the left edge as shown in the right figure on the next page.

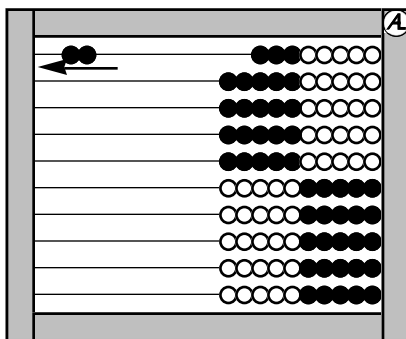


The abacus cleared; that is, set to zero.

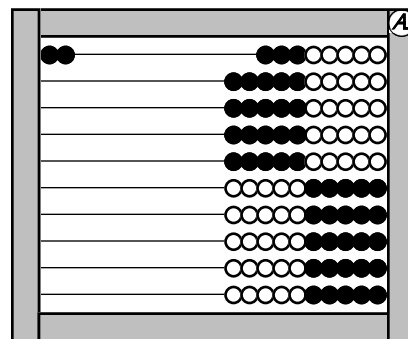


The quantity 1 entered on the abacus.

Ask her to clear the abacus. Show her 2 objects and ask her to show with her fingers how much it is. Demonstrate entering 2 by sliding 2 beads together on the top wire toward the left edge. See the figure below. Do *not* count the beads; enter them as a unit, not individually.



Entering 2 as a unit on the abacus.



The quantity 2 entered on the abacus.

Repeat for 3 and 4, always entering the whole amount without counting and clearing between numbers. Continue with 5. After she has entered 5 ask, What is special about 5? [a whole hand, all the dark-colored beads on a wire]

Practice. State the numbers from 1 to 5 at random and ask the child to enter the correct quantity. Show tally sticks for numbers 1 to 5 and ask her to enter the same number on the abacus. Repeat for finger sets.

Now enter quantities 1 to 5 on the abacus and ask her to show them with tally marks and fingers and to name them.

Lesson 4

Quantities 1 to 7, Taps, and Writing Numbers

OBJECTIVES

1. To learn to detect the number of taps from 1 to 4
2. To learn finger sets and tally marks for quantity 7
3. To learn to write the numbers 1, 2, 3, 4, and 7

MATERIALS

A tool for making audible taps or tones that the child can hear
 7 tally sticks
 Abacus
 Worksheet 1, Writing Numbers Starting at the Left

WARM-UP

Continue teaching “Yellow is the Sun” (Lesson 1) and “Writing Numbers” (Lesson 2).

Teach or review the nursery rhyme, “One, Two, Buckle my Shoe.” This rhyme will be used later for teaching other skills.

One, Two, Buckle my Shoe*One, two,**Buckle my shoe.**Three, four,**Shut the door.**Five, six,**Pick up sticks.**Seven, eight,**Lay them straight.**Nine, ten,**A big, fat hen.*

Note: Listening to the number of taps engages the child’s auditory sense.

Some children benefit from tapping dots with their fingers.

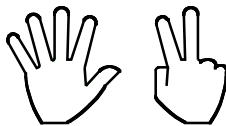
However, not all children succeed with tapping. If it is too hard for your child, just skip the tapping activities.

ACTIVITIES

Taps. Ask the child to listen carefully so he can hear the sound. Tap once with the tapping tool and ask him how many taps he heard. [1] Next tap it twice, about one second apart, and ask how many taps he heard. [2] Repeat for 3 and 4.

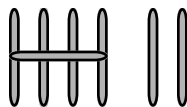
Tap 1 to 4 times in random order and ask the child to show what he heard with his tally sticks.

Quantity 7. Review quantity 6 by asking, How do you show being 6 years old with your fingers? Ask the child to describe what 6 looks like. [all fingers on the left hand and a 1 on the right] Next ask, If you have a birthday and you are 7, how would you show it? What does 7 look like? [the whole left hand and 2 with the right]



Ask him to show 4 with his tally sticks, then 6, and then 7. Ask him to construct the following numbers, each time starting with the current number: 6, 4, 7, 5, 6, 3, 4, 6, 7, 6, and 1.

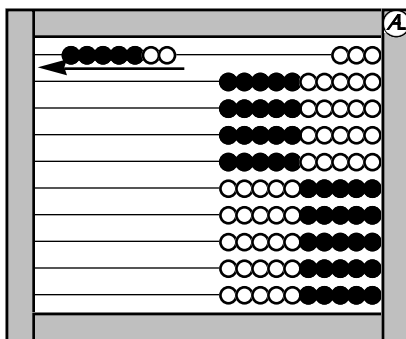
Ask him to close his eyes and see 5. Then ask him to add 2 more and to say how many he sees now. [7]



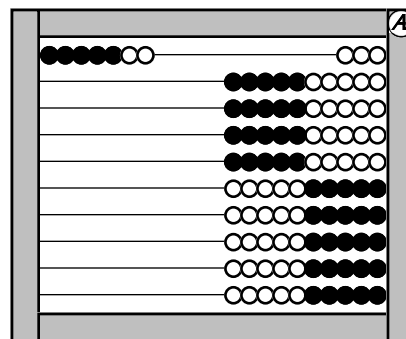
Showing 7 with tally sticks.

Ask him to see 5 again in his mind and then to remove 1 stick; ask what he sees now. [4]

Quantities 1 to 7 on the abacus. Ask the child to enter 2 apples, clear and enter 5 oranges, clear and enter 6 bananas, clear and enter 7 grapes. Let him name a quantity of fruit up to 7 and enter it on his abacus.



The quantity 7 being entered.



The quantity 7 entered.

Practicing with a partner. Tell the child that he will practice with you (or another partner if more than 1 child is working on the lesson). He makes a quantity from 1 to 7 with tally sticks, which you enter on the abacus. After about 2 minutes, ask him to trade materials with you. Spend 2 more minutes with this practice.

Writing numbers. Tell the child that now he will practice writing number 1, which starts in the middle, and the numbers that start at the left. Emphasize that each number starts at the top. Demonstrate writing a “1” on the drawing board.

Ask him to practice it on the table using the first 2 fingers of his writing hand. Drawing board practice is also good because mistakes can quickly be erased. He can also practice in rectangular trays covered with baking soda or light sand.

For many children the following phrases are helpful. He is to stop for a moment after each action, as indicated by commas.

- 1: Straight down
- 2: Around, and over.
- 3: Around, and around.
- 4: Down, over, and make a 1.
- 7: Over, and down.

NOTE It is very important that a child learn to write numbers clearly and efficiently. In standard fonts, a 9 is an inverted 6; this is very confusing for many children, especially those with dyslexia. The numerals used here are designed to be easy to learn to write. Also, they are used on the worksheets, the basic number cards, the place-value cards, and the fraction charts.

Worksheet 1. Give the child this worksheet for practice in writing these numbers; reminding him to start at the dots. You might keep extra copies available so the child can practice writing his numbers at other times.

7	7	4	4	3	3	2	2	1	1
7	7	4	4	3	3	2	2	1	1
7	7	4	4	3	3	2	2	1	1
7	7	4	4	3	3	2	2	1	1
7	7	4	4	3	3	2	2	1	1
7	7	4	4	3	3	2	2	1	1

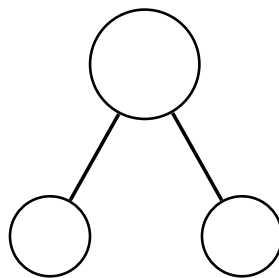
7	7	4	4	3	3	2	2	1	1
7	7	4	4	3	3	2	2	1	1
7	7	4	4	3	3	2	2	1	1
7	7	4	4	3	3	2	2	1	1
7	7	4	4	3	3	2	2	1	1
7	7	4	4	3	3	2	2	1	1

Name _____

Lesson 9

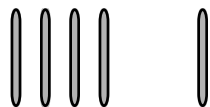
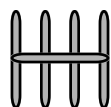
Part-Whole Circles

- OBJECTIVES**
1. To visualize 1 more or 1 less
 2. To introduce the part-whole circles
- MATERIALS**
- A set of part-whole circles (Appendix pg. 6)
 - Tally sticks
 - 20 Base-10 cards (These cards are 1 1/2 inches on each side.) On the *backs* of the cards, write a number from 0 to 9. Make two sets. Or use two of each number 0 to 9 from the green basic number cards.
 - Worksheet 3, Partitioning
- WARM-UP**
- Continue teaching “Yellow is the Sun” (Lesson 1) and “Writing Numbers” (Lesson 2), using the actions.
- Practice counting backward with the abacus stairs.
- Ask the child to recite only the numbers in “One, Two, Buckle my Shoe” (Lesson 4) and to accent 2, 4, 6, 8, and 10. Repeat. Then you and she alternate saying the numbers. Repeat but have her start.
- To review the terms *yesterday* and *tomorrow*, show her today’s date on the calendar and ask her what day it is. Then ask her to find yesterday and to name it. Repeat for tomorrow.
- NOTE**
- Research has shown that the part-whole concept helps children in problem solving. The child is to write the numbers in the problem in the appropriate circle as she solves the problem.
- ACTIVITIES**
- Part-whole circles.*** Ask the child to construct a 5 with tally sticks. Then ask her to break the 5 and to put *part* of it aside. Ask what the whole amount, or all of it, was before breaking it into parts. [5] Then ask her what parts she has. [possible splits: 2 & 3, 1 & 4, 4 & 1, 3 & 2, 0 & 5, and 5 & 0] Help her discuss if zero is a possible split; she may not accept it, which is fine.
- Place the paper part-whole circle (shown below) where the child can see it. Explain that the larger circle is for the *whole* and the smaller circles are for the *parts*.

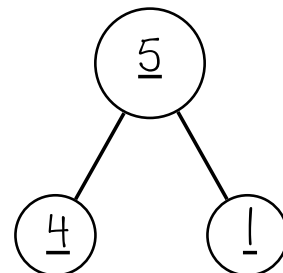


The part-whole circle set.

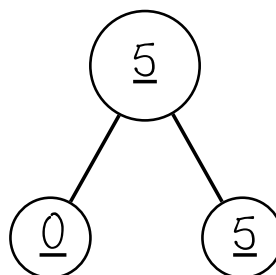
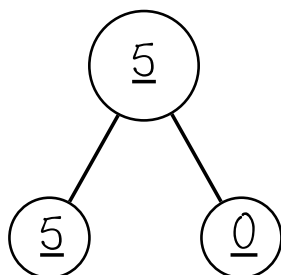
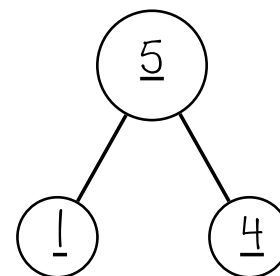
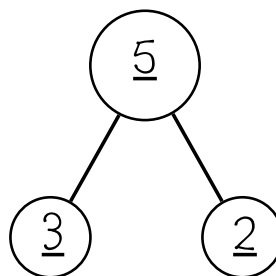
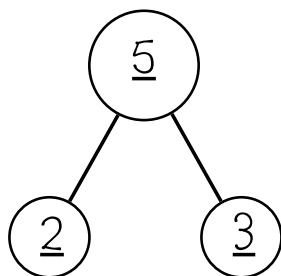
Also set out the 20 cards with numbers, 0 to 9. Ask the child to find the number showing the whole [5] and to put it in the large circle. Ask her to find the numbers showing the parts and to place them in the small circles as shown on the next page.



The whole, 5, broken into parts of 4 and 1.



Ask the child for another way to *partition*, or break, the 5. Ask her to change the numbers to show the new parts. Continue asking until she cannot find any more ways. The “0” partitions may not occur to her. If so, ignore them. Some possibilities are shown below.



The other ways to partition 5.

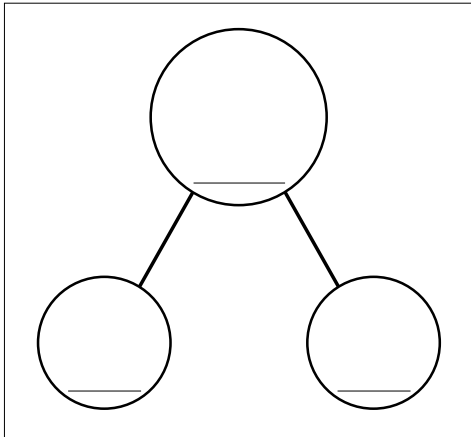
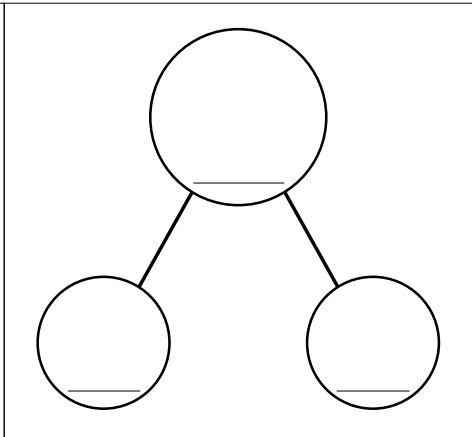
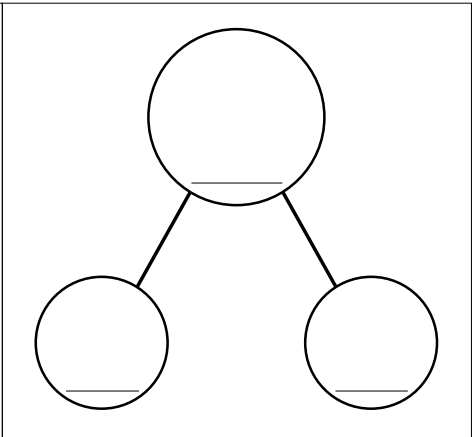
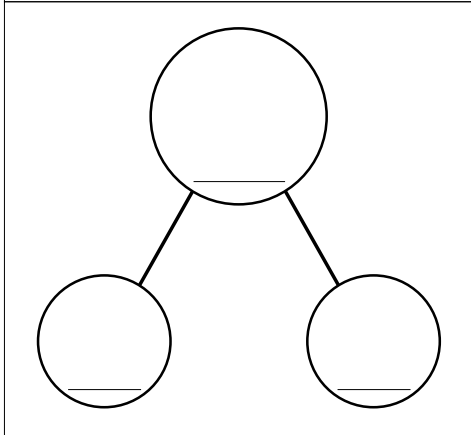
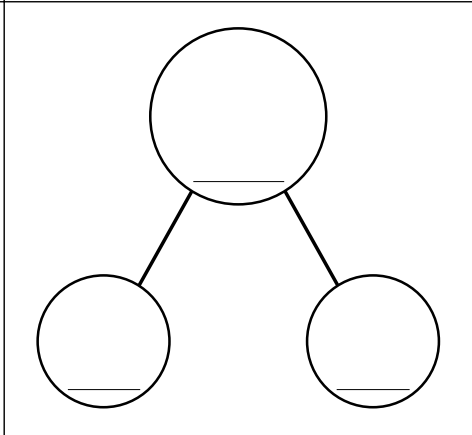
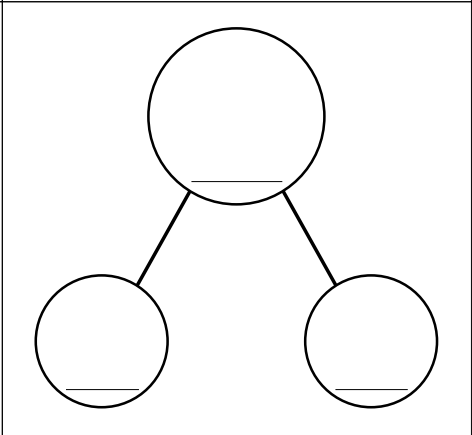
Partitioning 6. Next ask the child to partition 6. Guide her through the same steps. This time you might want to draw several part-whole circle sets on the drawing board. When she finds the double [3 and 3], ask if 5 had a double. [no] Ask her to think about why it doesn't.

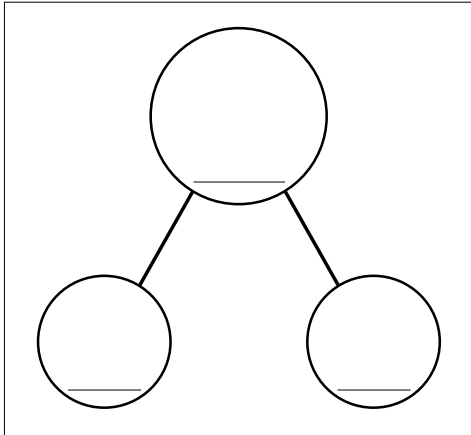
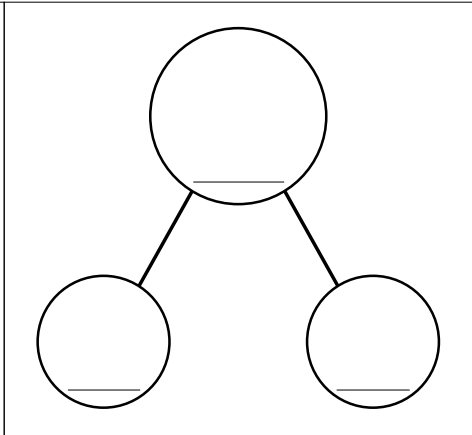
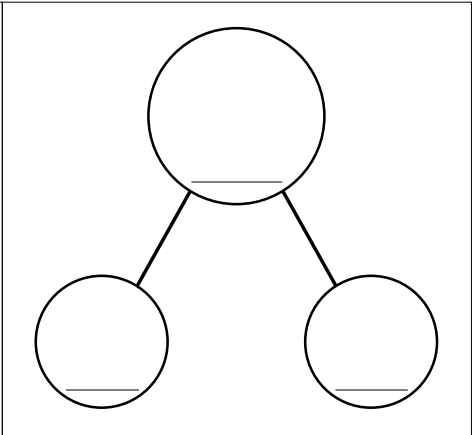
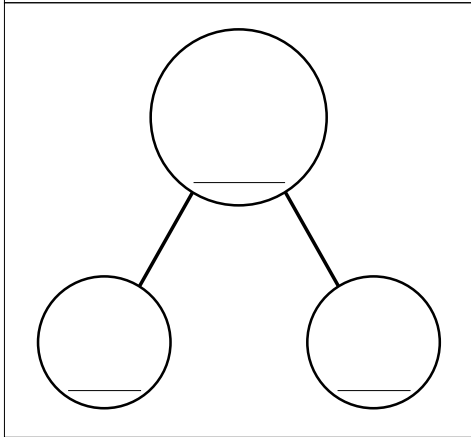
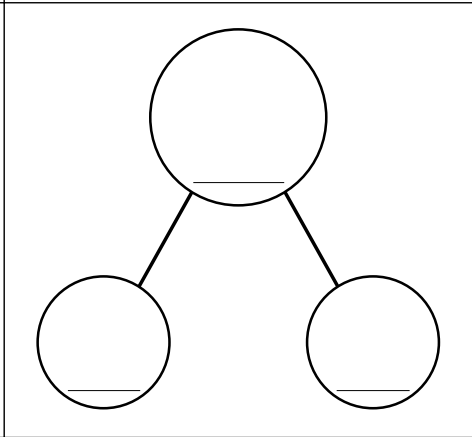
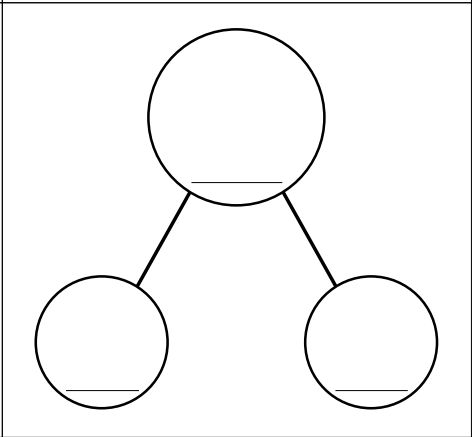
Summarize. Ask, What do we put into the large circle? [the whole amount, or all of it] Ask, What do we put into the smaller circles? [the parts]

Worksheet 3. Give her this worksheet, which consists of 6 sets of blank part-whole circles. Suggest that she use any numbers up to 10, except 5 or 6. She can break them into the parts she chooses.

ENRICHMENT Which numbers have doubles? [2, 4, 6, 8, and 10]

Name _____

Lesson 26

More Hundreds & Building Rectangles

OBJECTIVES

1. To partition hundreds
2. To read and write hundreds
3. To form new rectangles from existing rectangles

MATERIALS

Dot cards
 11 abacus tiles
 Place-value cards
 Math journal
 1 small set of five rectangles
 1 sheet of construction paper in a contrasting color (optional)

WARM-UP

Continue teaching the months of the year in groups of threes.

Play the new version of the Comes After game: say three numbers and ask what comes next: 7, 8, 9. [10]

Flash the various dot cards for about two seconds and ask the child to state the number of dots.

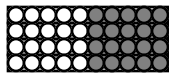
NOTE

A child who is unable to determine the number of dots needs to play the Memory game matching dot cards with basic number cards.

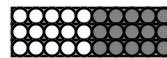
ACTIVITIES

Review. Show the 300 card and ask the child to build the number with abacus tiles. Repeat for 500 and 900.

Partitioning hundreds. Ask the child to build 400. Then say, Now we will partition 400 into hundreds. Move the top abacus tile to a new pile adjacent to the existing 400 as shown and ask the child to write the equation. [$400 = 300 + 100$] Move another tile to



Partitioning 400 into $300 + 100$.

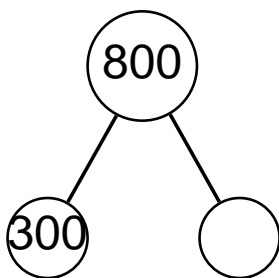


the pile on the right and ask the child to say and write the equation. [$400 = 200 + 200$] Continue to $400 = 0 + 400$. If necessary, say that “0-hundred” is really zero and is written as “0.”

Tell the child that he is to partition a hundred number and write it in his math journal. If the child is struggling with the concept give him a simpler number, like 400 or 500; if not, give him 10-hundred or 9-hundred.

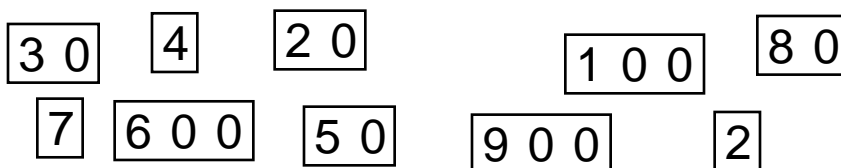
Problem. (The following problem is a change problem with a missing part.) Eight hundred trees are to be planted in a park. Three hundred trees have already been planted. How many more trees must still be planted. Read it twice to the child.

Use a part-whole circle set and ask, Are the 800 trees a whole or a part? [whole] Write 800 in the whole circle. Continue with, Are the 300 trees a whole or a part? [part] Write it in a part circle. Let the child think about solving the problem. [500]



Game. Play the Can You Find game with the place-value cards. Ask the child to lay out all the hundreds, tens, and ones cards face up in no particular order as shown on the next page. Say, Can you

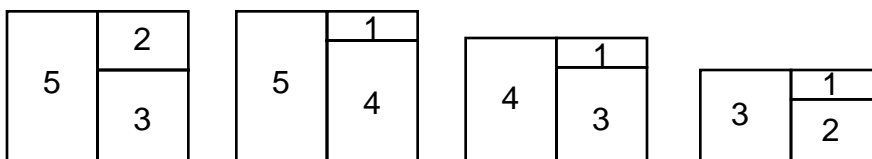
find 7-ten? The child finds the correct card and places it near the bottom of his workspace. Continue with the other numbers in random order.



Some of the place-value cards laid out for the game of Can You Find.

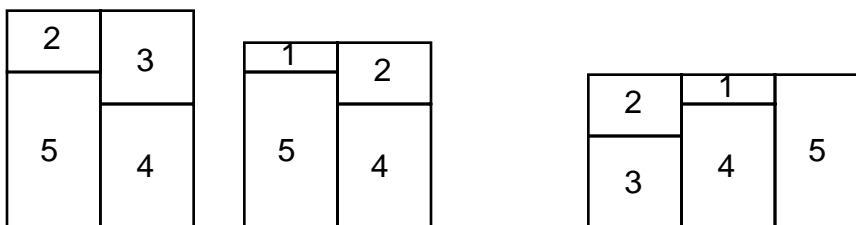
Making rectangles. Ask the child to lay out the five rectangles in order from smallest to largest. Tell him to give them number names. Write 1 on the smallest, 2 on the next smallest, 3 on the next, 4 on the next, and 5 on the largest.

Ask him to make new rectangles using three of his rectangles. When he has made one, record the results on the writing board using the numbers. Challenge him to find new ways, besides putting them into rows or columns. [There are 10 different combinations of putting them in a row and 4 other arrangements as shown below.] Merely interchanging the same rectangles in a row or column does not constitute a new arrangement.



Some of the ways to construct new rectangles using 3 of the 5 basic rectangles.

Now ask him to use four rectangles to construct new rectangles. [There are five different combinations of putting them in a row and two other arrangements as shown below.]



New rectangles using 4 of the five basic rectangles.

New rectangle using all five basic rectangles.

Ask him to use all five rectangles to construct new ones. [There is one way of putting them in a row and one other arrangement.] See the right figure above.

If the child notices, discuss the pattern of the numbers.

Displaying the rectangles. If desired, ask the child to construct a rectangle and paste it on the construction paper. You might include some designs.