UNIT 8

Contents

Section 1

Section 2

Section 3

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8.1

The Gauss-Jordan Method of Solving Two Equation Systems

Remember:
Use the inverse property of multiplication to create 1's.

$\frac{1}{7}\cdot\frac{7}{1}=1 \quad \frac{2}{3}\cdot\frac{3}{2}=1$

$6\cdot\frac{3}{4}=1$

Remember:
Use the inverse property of addition to create 0's.

$\frac{2}{3}+(-\frac{2}{3})=0 \qquad -6+6=0$

Hint:
The Gauss-Jordan method should be done column by column. It works best if the first step in each column is to get a 1 in the appropriate spot. That 1 can then be used to get 0's in the rest of the column. Multiply the row that has the 1 by the opposite of the element that needs to be 0. Then add the transformed row to the row that needs a zero.

The Gauss-Jordan method solves systems of equations by reducing the coefficients of each term in the system to only one term and a constant left in each equation. When completed each equation holds a different variable. The answers can be read directly from the right side of each equation.

The Gauss-Jordan method begins by forming an augmented matrix. An augmented matrix has only the coefficients and constants of the equations.

Original system | **Augmented matrix** (Coefficients | constants)

$$\begin{cases}3x-1y=3\\1x+2y=8\end{cases} \rightarrow \left[\begin{array}{cc|c}3 & -1 & 3\\1 & 2 & 8\end{array}\right]$$

The vertical line separating the coefficients from the constants is optional but helpful.

The Gauss-Jordan method transforms the coefficient side of the augmented matrix into a pattern of diagonal 1's with zeros everywhere else. The transformed matrix for a 2-variable looks like this:

2 variable systems: $\left[\begin{array}{cc|c}1 & 0 & a\\0 & 1 & b\end{array}\right]$

This pattern is accomplished using three basic operations:

1. Any two rows can be swapped.
2. Any row can be multiplied by a non-zero number.
3. The elements of any row can be added to another row—or the multiple of another row.*

*When the multiple of a row is added to another row, the multiplication does not change the values of the variables in the system of equations and therefore does not change the values of the row.

Example 1 **Use the Gauss-Jordan method to solve the system of equations.**

Step 1: get 1 as the first element in column 1. $\begin{cases}3x-1y=3\\1x+2y=8\end{cases}$

$\left[\begin{array}{cc|c}3 & -1 & 3\\1 & 2 & 8\end{array}\right]$ Write the system as an augmented matrix.

$\left[\begin{array}{cc|c}1 & 2 & 8\\3 & -1 & 3\end{array}\right]$ Switch rows 1 and 2 to get a 1 as the first element in column 1.

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Example 1 **Continued.**

Step 2: get 0 in row 2 of column 1

This can be achieved by multiplying row 1 by –3, which makes 0 when the result is added to row 2.

$-3\,(1\ \ 2\ |\ \ 8)\quad -3r_1$ Row 1 multiplied by –3.

$$\begin{array}{rr|r} 3 & -1 & 3 \\ -3 & -6 & -24 \\ \hline 0 & -7 & -21 \end{array} \quad -3r_1 + r_2$$

Multiplied row 1 added to row 2 to get a 0 in row 2, column 1.

Transformed row 2.

The second element in column 1 is now transformed.

$$\left[\begin{array}{rr|r} 1 & 2 & 8 \\ 0 & -7 & -21 \end{array}\right]$$

Step 3: get 0 in row 1 column 2

This can be achieved by multiplying row 2 by $\frac{2}{7}$ and adding the result to row 1.

$\frac{2}{7}\,(0\quad -7\ |\ -21)\quad \frac{2}{7}\,r_2$ Row 2 multiplied by $\frac{2}{7}$.

$$\begin{array}{rr|r} 1 & 2 & 8 \\ 0 & -2 & -6 \\ \hline 1 & 0 & 2 \end{array} \quad \frac{2}{7}r_2 + r_1$$

Multiplied row 2 added to row 1 to get 0 in row 1, column 2.

Transformed row 1.

The first element in column 2 is now transformed.

$$\left[\begin{array}{rr|r} 1 & 0 & 2 \\ 0 & -7 & -21 \end{array}\right]$$

Step 4: get 2 of column 2 to be 1

This can be achieved by multiplying row 2 by $-\frac{1}{7}$.

$-\frac{1}{7}\,(0\quad -7\ |\ -21)\quad -\frac{1}{7}\,r_2$ Row 2 multiplied by $\frac{1}{7}$.

$0\quad 1\ |\ \ 3$ Transformed row 2.

Complete matrix is transformed

$$\left[\begin{array}{rr|r} 1 & 0 & 2 \\ 0 & 1 & 3 \end{array}\right]$$

Matrix with a diagonal pattern of 1's.

Step 5: find solution

(2, 3) Solution is values in last column.

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Today's Lesson

Use the Gauss-Jordan method to solve each system.

1. $\begin{cases} x - 2y = 1 \\ x + 4y = 13 \end{cases}$ **2.** $\begin{cases} 2x - 3y = -18 \\ 3x + y = -5 \end{cases}$

REVIEW

Sketch the graphs for the functions. 7.14

3. $f(x) = \frac{1}{2}\sqrt{x-1} - 3$ **4.** $f(x) = \sqrt[3]{x-1} - 1$

State whether the function is increasing or decreasing. Then graph. 7.11

5. $y = \left(\frac{1}{3}\right)^x$

Find the answers. 7.6

6. Sound travels @ 1,100 feet per second, modeled by the function $g(x) = 1{,}100t$ (time in seconds). The sound from a lightning bolt emanates equally in all directions – a circle, whose area can be modeled by the function $f(x) = \pi r^2$. Write a composite function to express the total area of land where the sound from a lightning bolt could be heard in t seconds. Hint: the radius would be the distance traveled by the sound.

7. A local clothing store is advertising an additional 20% off of their already 40% discounted merchandise. Write functions for the original discount, the second discount, and the composite function for the chain discount. Then calculate the final price of an article of clothing originally marked for $45.

Solve the systems using Cramer's Rule. 7.1

8. $\begin{cases} 2x + 3y = 25 \\ 3x - 2y = 5 \end{cases}$ **9.** $\begin{cases} 2x + y = 2 \\ 3x - y = 18 \end{cases}$ **10.** $\begin{cases} x + y + z = 4 \\ x - 2y + z = -5 \\ x + y + 2z = 3 \end{cases}$

Find the answers. 6.6, 6.9

11. A photographer is taking photos of five siblings. How many different ways can the photographer line up the siblings?

12. How many five letter "words" are possible from the twenty-six letters in the English alphabet?

13. ${}_8P_5$

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Simplify. 6.1

14. $\sqrt{2\sqrt[5]{2}}$

15. $\sqrt{27\sqrt{3}}$

16. $\sqrt[3]{x^2\sqrt{y}}$

Multiply the following matrices if possible. 5.6

17. $\begin{bmatrix} -10 & 8 & -6 \\ -2 & -5 & -5 \end{bmatrix} \begin{bmatrix} -6 & 3 \\ 3 & -5 \\ -3 & -6 \end{bmatrix}$

18. $\begin{bmatrix} -9 & 1 \\ 1 & -7 \end{bmatrix} \begin{bmatrix} 4 \\ 2 \end{bmatrix}$

Given that $f(x) = x - 3$ and $g(x) = 5x^2 - 1$, find the following. 6.2

19. $(f \circ g)(3)$

20. $(f \circ g)(x)$

21. $(g \circ f)(x)$

Convert to fractional notation and simplify. Express answers in radical notation, extracting roots when possible. 5.12

22. $\sqrt[3]{m^2} \div \sqrt{m}$

23. $\sqrt{y} \bullet \sqrt[4]{y^2}$

24. $(\sqrt[5]{a^2b^3c^4})^3$

Extra Practice

Use the Gauss-Jordan method to solve each system.

25. $\begin{cases} x + 2y = 11 \\ 4x - y = 8 \end{cases}$

26. $\begin{cases} 2x - 5y = -9 \\ 3x + 2y = -4 \end{cases}$

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