UNIT 3

Contents

3.1

Factoring By Grouping

Some polynomials may not have a common factor, but pairs of terms within the polynomial may share factors. In such cases, group those pairs and extract their common factors. If this creates a pair of identical binomials, the polynomial can be factored. The identical binomials can be factored out of the expression and the remaining factors form a second binomial factor. This process is called *factoring by grouping.*

It may be necessary to rearrange the terms to find terms with common factors.

Example 1 **Factor $6xy + 4x + 3y + 2$ by grouping.**

$6xy + 4x + 3y + 2$	Original polynomial.
$(6xy + 3y) + (4x + 2)$	Terms grouped into pairs with common factors.
$3y(2x + 1) + 2(2x + 1)$	Common factor extracted from each pair.
$(2x + 1)(3y + 2)$	The common parenthetical factor, $(2x + 1)$, factored out of the sum.

Sometimes when factoring by grouping, a pair of binomials will have identical terms but opposite signs, as in

$$(xy - 3x)(-4y + 12) = x(y - 3) + 4(-y + 3)$$

One of the binomials above has $+ y$ and -3 and the other has $-y$ and $+3$. To make both binomials the same, factor a (-4) instead of a $(+4)$ out of the right binomial. This reverses the signs of the terms inside the parentheses.

$$x(y - 3) + (-4)(y - 3) = (y - 3)(x - 4)$$

Example 2 **Factor $12xy - 9x - 8y + 6$ by grouping.**

$12xy - 9x - 8y + 6$	Original polynomial.
$(12xy - 9x) + (-8y + 6)$	Terms grouped into pairs sharing a common factor.
$3x(4y - 3) + (-2)(4y - 3)$	Common factors extracted from each pair. To reverse the signs, extract from the second binomial.
$(3x - 2)(4y - 3)$	The common parenthetical factor, $(4y - 3)$, factored out of the sum.

Factoring Trinomials in the Form $ax^2 + bx + c$

Trinomials whose first terms have a coefficient other than 1 are more difficult to factor because the leading coefficient increases the number of factor-pair possibilities. However, there is a technique that avoids having to test all the different possibilities.

Multiply the coefficient of the first and last terms, then find two factors of that product whose sum equals the coefficient of the middle term. Rewrite the polynomial, using the pair of factors to make a new set of middle terms. Factor the new polynomial by grouping to complete the factoring.

Example 3 **Factor $6x^2 + 11x - 10$.**

$(6)(-10) = -60$	Coefficients of the first and last terms multiplied.
1, −60; 2, −30; 3, −20; 4, −15; 5, −12; 6, −10 −1, 60; −2, 30; −3, 20; −4, 15; −5, 12; −6, 10	Factor pairs for −60 written down. Factor pair with a sum equal to the middle term of the trinomial (11) identified.
$6x^2 - 4x + 15x - 10$	The middle term of the trinomial ($11x$) rewritten as a sum using the factor pair from the last step.
$(6x^2 - 4x) + (15x - 10)$	Terms grouped into pairs sharing factors.
$2x(3x - 2) + 5(3x - 2)$	Common factors extracted.
$(3x - 2)(2x + 5)$	$(3x - 2)$ factored from the sum.

Check: $(3x - 2)(2x + 5) = 6x^2 + 11x - 10$.

To Factor Trinomials by Identifying Factor Pairs

1. Multiply the coefficients of the first and last terms of the trinomial (a · c).
2. List the possible two-factor combinations for that product.
3. Identify the factor pair whose sum equals the coefficient of the trinomial's middle term (b).
4. Rewrite the middle term of the trinomial as a sum of two terms using the factors identified in step 3 (order does not matter).
5. Factor the new polynomial by grouping.

Today's Lesson

Factor by grouping.

1. $6xy + 4x - 9y - 6$
2. $4xy - 3x - 4y + 3$
3. $2xy + 3x + 4y + 6$
4. $15xy - 6x - 5y + 2$
5. $9xy - 6x - 3y + 2$
6. $12xy + 3y - 8x - 2$

Factor by identifying factor pairs.

7. $4x^2 + 8x - 5$
8. $3x^2 + x - 14$
9. $12x^2 + 5x - 2$
10. $6x^2 + x - 2$
11. $4x^2 + 12x + 5$
12. $4x^2 + 8x + 3$

REVIEW

Solve by completing the square. 2.14

13. $x^2 - 3x - 1 = 0$
14. $6x^2 + 3x + 6 = 0$
15. $x^2 - 6x + 2 = 0$

Use linear programming to solve the following problem. 2.13

16. Christopher wants to grow corn and soybeans on his 300-acre farm. He must grow at least 80 acres of corn and at least 30 (but no more than 140) acres of soybeans. If the profit from corn per acre is \$266 and for soybeans is \$306, how many acres of each crop should he grow to maximize his profits?

Solve the quadratic equations. 2.11

17. $(x + 1)^2 = 5$
18. $4x^2 + x = 0$
19. $7x^2 + 5x - 2 = 0$

Simplify and add or subtract as directed. 2.6

20. $3\sqrt{18} - 7\sqrt{2} + 5\sqrt{8}$
21. $7\sqrt[3]{40} - 10\sqrt[3]{135}$
22. $3\sqrt{5x} - \sqrt{25x}$

Simplify the radicals. 2.6

23. $\sqrt[3]{-250x^5y^7}$
24. $\sqrt[3]{-32m^5n^6o^3}$
25. $\sqrt{500x^7yz^4}$

Factor by taking out the GCF. 2.1

26. $20x^2y^2z - 200xy^2z^2 - 50xyz$
27. $7m^3n - 14m^2n^2o + 7m^3n^2$
28. $5m^2n + 10mn - 15mo$

Factor the difference of squares. 2.1

29. $36y^2 - 1$
30. $x^2 - 16$
31. $4x^4 - 100$

Factor the perfect square trinomials. 2.1

32. $9x^2 - 36x + 36$ **33.** $x^4 + 2x^2 + 1$ **34.** $x^{10} - 10x^5 + 25$

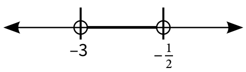
Factor by identifying factor pairs. 2.1

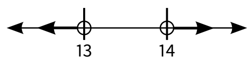
35. $x^2 - 11x + 28$ **36.** $x^2 - 8x - 33$ **37.** $x^2 - 14x - 32$

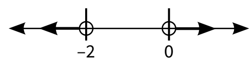
Solve and graph the compound inequalities. 1.11

38. $-\frac{1}{2}x + 2 \geq 5$ or $5x - 2 > 3$ **39.** $7x + 7 \leq 28$ and $-x < 4$ **40.** $-5 < (2x + 1) < 3$

Identify as *conjunction* or *disjunction,* then write an inequality for the given graph. 1.11

41. 

42. 

43. 

Fill in the correct set notation. 1.1

44. Set W ____?____ Sets V and Z.

45. Set Y ____?____ Set V.

46. Set X ____?____ Set Y.

47. 7 ____?____ Sets V and W.

48. 7 ____?____ Sets Y and Z.

49. Set W ________ Set Y = set Z.

Set $V = \{0, 2, 3, 4, 5, 8, 10\}$
Set $W = \{3, 5\}$
Set $X = \{0, 2, 6\}$
Set $Y = \{1, 7, 9\}$
Set $Z = \{1, 3, 5, 7, 9\}$

Extra Practice

Factor by grouping.

50. $6xy + 21x + 4y + 14$ **51.** $12ab - 8a + 21b - 14$ **52.** $12mn - 12n + 6m - 6$

Factor by identifying factor pairs.

53. $6x^2 + 7x - 3$ **54.** $6x^2 + 7x - 20$ **55.** $5a^2 + 32a - 21$